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Lamb Waves and Dispersion Curves in Plates and its Applications in NDE Experiences Using Comsol Multiphysics

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Abstract: A model for numerically obtaining lamb wave modes and dispersion curves in plates is presented. It is shown that COMSOL Multiphysics can be employed to simulate the behavior of guided waves in dispersive plates, which is useful for NDE applications.
The simulation results will be used to obtain a time-offset acceleration profile where the dispersive lamb modes of propagation are implied. The MASW (Mutichannel analysis of surface waves) is then employed for data processing making explicit the presence and character of the lamb different propagation modes. The curves are then interpreted. The link between MATLAB and COMSOL for transforming the simulation in interpretable curves is very convenient for routine comparison between numerical and analytical or experimental solutions.

Keywords: Lamb waves, Dispersion curves, MASW, Numerical modeling.

1. Introduction

The dispersive behavior of different structural elements is very interesting in NDE applications. Specifically, in acoustic evaluation where a stratified media is presented, techniques like Impact-Echo method (Sansalone, 1985) is not enough for studying the thickness of different layers.

The reason of the above is that, across the plates, different frequency components of the input signal travels at different velocities (Graff, 1975). Thus, the idea of a specific frequency traveling up and down across the thickness of the plate makes no sense because guided waves phenomena grow up in the layer.

The behavior of guided waves in plates was first studied by Lamb (1905). Since then, many researchers have been working in this field over years. Recently works (Ryden, 2004) (Park, 2000) have shown how to combining spectral analysis and dispersive phenomena to determinate thickness and velocity profiles in stratified media.

In this paper we are going to use Comsol Multyphisics to generate pseudo-experimental data which can be analyzed over the MASW (Multichannel Analysis of surface waves) method for getting dispersion curves. A Matlab code is then employed for data processing showing how Comsol can be very useful for the study of guided waves in plates and for comparison with the experimental data.

2. Lamb Waves

As we said, Lamb derived the dispersion relation for different waves traveling across the plane of a free plate. According to him, only some frequency-velocity pairs can propagate through it. This pairs can be obtained from the dispersion relations:

$$\frac{\tan\left(\frac{\beta h}{2}\right)}{\tan\left(\frac{\alpha h}{2}\right)} = -\frac{4\alpha\beta k^2}{\left(k^2-\beta^2\right)^2} \qquad (1)$$

and

$$\frac{\tan\left(\frac{\beta h}{2}\right)}{\tan\left(\frac{\alpha h}{2}\right)} = -\frac{\left(k^2-\beta^2\right)^2}{4\alpha\beta k^2} \qquad (2)$$

Where

$$\alpha^2 = \frac{\omega^2}{V_p} - k^2 \quad ; \quad \beta^2 = \frac{\omega^2}{V_s} - k^2 \qquad (3)$$

In the expressions above, h is the thickness of the plate, ω the angular frequency, k the wave number and V_p and V_s the compression and shear wave velocities respectively. Equations (1) and (2) predict two different modes of propagation. One of them is symmetric (1) and the other antisymmetric (2). These modes propagate across the midplane of the plate (Figure 1).

Two fundamental modes A_0 and S_0 and the corresponding higher order modes can be seen in Figure 1. From the dispersion curves shape, Lamb predicted four different waves in the plate: the bending wave, shear wave, quasi longitudinal wave and Rayleigh wave. In the high frequencies limit, Figure 1 shows all that of them travel at the Rayleigh wave velocity and at the shear wave velocity.

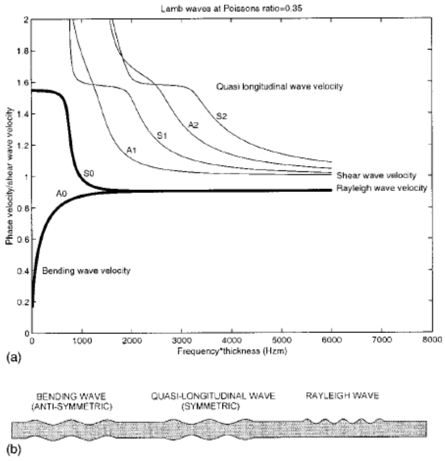

Figure 1. Normalized dispersion curves for a free plate (Ryden Thesis, 2004).

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3. Multichannel Analysis of Surface Waves (MASW)

Different surface waves analysis methods for getting experimental dispersion curves have been performed over the years. MASW method is one of them and we are going to explain its basis here.

Taking into account the features of the two dimensional Fourier Transform (2DFFT), the spatial and temporal variables can be related with the spatial and temporal frequencies and thus, obtain the experimental dispersion curves (because the phase velocity is the ratio between both). To do that, a large number of receivers located at the surface plate are necessary. The receivers record the acceleration field generated by an excitation signal (in general a hammer impact, Figure 2). The distance between each receiver must be the same and, it should be chosen according the Nyquist criteria for the spatial frequency. Since the spatial frequency is related with the minimum wavelength, an estimation of it will be necessary for instance, with the compression wave velocity of the plate. From the above, one can chose the distance between receivers and the length over the surface which is going to be evaluated.

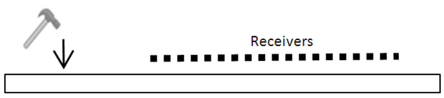

Figure 2. MASW experience scheme.

Similar considerations made for the spatial sampling are also necessary for temporal sampling and good frequencies detection.

Under Figure 2 configuration, an x-t profile can be measured. This profile defines a two dimensional function

$$g(x,t)$$

The Fourier transform with respect to the time variable can be written as

$$G(x,\omega) = \int g(x,t)e^{-i\omega t}dt \quad (4)$$

The function $G(x,\omega)$ can be now considered as a product of two different terms. The amplitude and the phase spectrums

$$G(x,\omega) = A(x,\omega)P(x,\omega) \quad (5)$$

The amplitude spectrum $A(x,\omega)$ contains information related with the attenuation and spherical divergence, however, the phase spectrum $P(x,\omega)$ contains all information about the dispersion properties (park et al, 2001). Therefore we have

$$G(x,\omega) = e^{-i\theta x}A(x,\omega) \quad (6)$$

Where $\theta = \omega/v_{ph}$, is the ratio between frequency and phase velocity. Applying now the Fourier Transformation to the spatial coordinate we get

$$T(w,\phi) = \int A(x,\omega)\, e^{-i(\theta-\phi)x}dx \quad (7)$$

The function $T(\omega,\phi)$ is a two dimensional function obtained from 2DFT. For a given frequency ω, $T(\omega,\phi)$ will be maximum if

$$\theta = \phi = \omega/v_{ph} \quad (8)$$

And thus, the phase velocity can be determined from the transformed variables. The function $T(\omega,\phi)$ is very useful because its Amplitude contains the information about which pairs of transformed variables transport the most energy. Then, with the chang new variables (8) we obtain an intensity function $I(v_{ph},\omega)$ which will have picks over the dispersion curves predicted for the plate.

Because of the above, MASW method can be used to obtained dispersion curves derived from experimental data, plotting and image of the function T we get the k-f representation and then, doing the change variables, we perform finally the v_{ph}-f representation.

3. Use of COMSOL Multiphysics

The experimental procedure in MASW method needs to apply a suddenly force and to record the acceleration in different points of the surface plate. Taking into account the elastic nature of the body, the Solid Mechanics Module of Comsol is used to solve the time dependent problem. This kind of study allows solving the elasticity equations for a linear elastic solid in the time domain.

A 2D model has been performed simulating the cross section of a steel plate (4x0.1 m). We have as Local Model Definitions 40 acceleration probes (measuring the y axis acceleration component) located on the surface plate. A point on the surface has been defined to locate the impact source. All distances included are shown in Figure 3

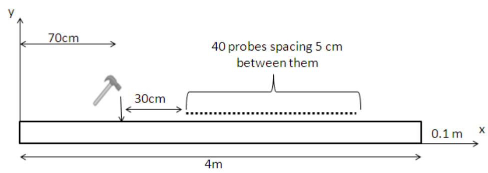

Figure 3. Simulation configuration

According with Sansalone (1985) the impact has been simulated with a sine pulse force applied to the surface. Different experiences with a steel plate showed us that the duration of a hammer impact on the steel surface rounds 200 microseconds (Figure 4). Due to it, as last Local Definitions, a sine function was included.

A point load equal to the sine function defined above is acting only during the first 200 microseconds. Finally, we have used a triangular mesh for our domain.

The Nyquist criteria apply to our receiver spacing (5cm), gives us a minimum wavelength $\lambda_{min} = 10cm$. Moreover, the time dependent solver has been set up manually, forcing the solver to evaluate solutions from 10 to 10 microseconds, which give us a sampling frequency $f_s = 50kHz$. Figure 1 indicates one frequency value above all A_0 and S_0 frequencies travels at Rayleigh wave velocity. The physical reason for this is that the high frequencies, which correspond with low wavelengths, are not able to "see" the bottom of the plate and, therefore, travels at Rayleigh wave velocity as if the medium was a half-space. Our temporal and spatial sampling criteria have been chosen according with the above consideration for see, at least, this frequency value. Finally, the simulation time has been 4 ms, which is enough for detecting all phenomena imply in the process.

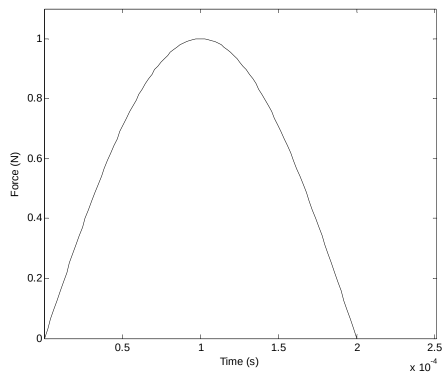

Figure 4. Temporal **s**ine pulse used for simulating the impact point force applies to the surface plate.

The material properties used in the model are shown in Table 1

Density	Compression wave velocity	Shear wave velocity
8030(kg/m^3)	5659 (m/s)	3120 (m/s)

Table 1: steel plate elastic constants used.

The signals obtained with the model will be then employed in a post-processing data over Matlab code develops on the basis of MASW method.

4. Results and discussion

While the model is running, the maximum and minimum displacement zones created in the plate due to the dispersive Lamb waves can be seen (Figure 5). Simulation results obtained in the time dependent study performed with Comsol are used to get the time-offset profile which defines our two dimensional function, see Figure 6.

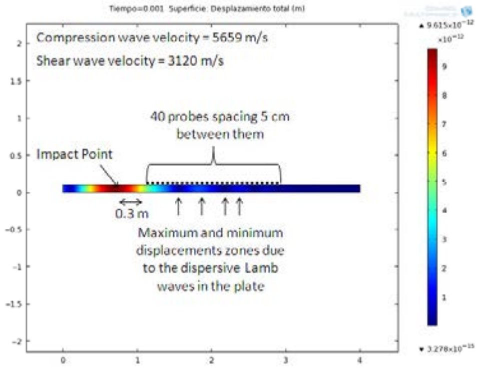

Figure 5. Simulation image taken at 1 ms. Maximum and minimum zones due to the dispersive waves are visible**.**

In the data we have a matrix which represent our two dimensional (temporal and spatial) discrete function. A 2D Fast Fourier Transform Matlab code is running over the data matrix. As program output returns another matrix, which is the amplitude of the spectrum function . A color image can them plotting. As we said in section 2, the maximum zones of follow the dispersion curves of the layer in the wave number-frequency (k-f) representation (Figure 7)

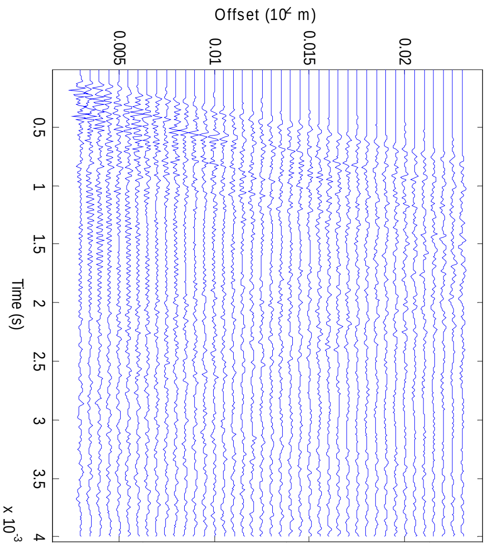

Figure 5. Pseudo-experimental (x,t) profile obtained from the Comsol simulation.

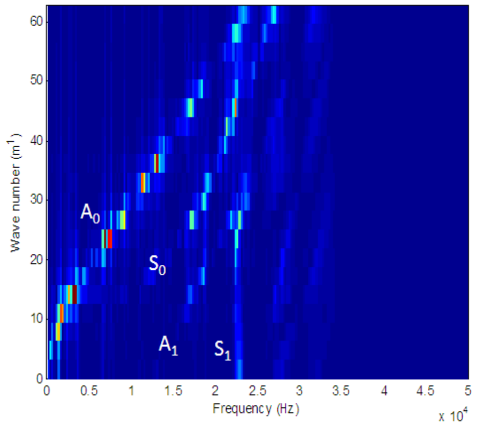

Figure 7. Dispersion Curves obtained from Comsol results in k-f representation.

Figure 7 shows a good accuracy dispersion curves according with Ryden (2004). To get dispersion relations in the phase velocity-frequency representation, a transformation code is necessary. For this work we have developed a changing variable code based in an interpolate criteria.

Figure 8 display the dispersion curves obtained in the v_{ph}-f representation, normalized with respect to shear wave velocity and the plate thickness. Comparing the curves obtained from Comsol data and theoretical behavior from Figure 1, we can see a very good accuracy between Comsol results and the expected. As we predicted, it is highly significant the presence of the limiting frequency above all higher fundamental modes frequency components travels at Rayleigh wave velocity.

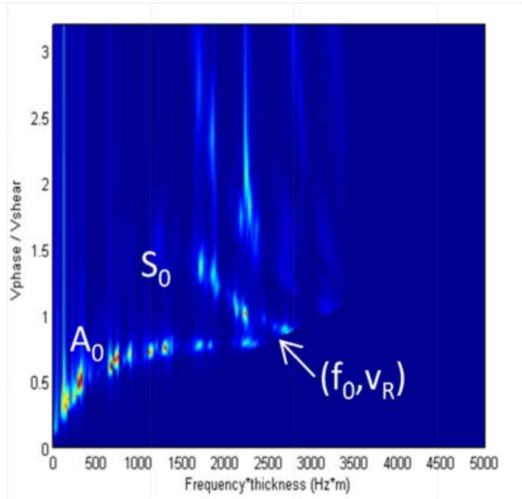

Figure 8. Dispersion Curves obtained from Comsol results in v_{ph}-f representation.

Another important aspect is how the velocity value at the limiting frequency is slightly below of the shear wave velocity. This is consistent with the theoretical limit corresponding at Rayleigh wave velocity because, according with Graff (1975), the R-wave velocity is about 80% of the S-wave velocity.

Finally, it should also be noted how the beginning of the S_0 mode is not detected. This result was also expected because this phase velocity-frequency pairs correspond to a higher wavelength values which are not measured under our experimental configuration.

5. Conclusions

As a conclusion, this Paper shown as Comsol Multyphysics simulation software is very useful for the study and obtaining dispersion curves in plates. The results show how the simulation predicts the theoretical behavior correctly. Therefore, Comsol can be used to compare experimental results with theoretical curves and, most importantly, to predict how different situations affect the dispersion relations for instance: boundary conditions, sources and receivers locations, different thickness layer, matched layers, or different component materials.

Theoretically, Lamb dispersion relations haven't analytical solutions. Therefore algorithms based on roots finding are necessary for implementing solutions. That is the reason why Comsol give us a solution to a problem the is useful in acoustic NDE experiences.

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