Some Clinical and Computational Studies on Haemodynamics in Stenosed Artery

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Abstract: Stenoses in arteries may cause critical flow conditions depending upon their severity, which may lead to stroke and heart-attack. A 2D axis-symmetric model of the carotid artery has been considered with varied shapes for the same degrees of constrictions. The incompressible Navier-Stokes equation has been considered as the governing equation of the fluid flow and it has been solved with varying flow parameters using standard software package COMSOL®. The radial velocity profiles at various points of the flow field, the centerline velocity plot and the centerline pressure plots have been obtained for clinical validation.

Keywords: Stenosis, clinical validation, numerical simulation

1. Introduction

Stenosis is the abnormal narrowing or constriction present in the blood vessel due to the deposition of cholesterol, fatty materials, cellular waste etc. It may happen in all large or small arteries, three major sites being Coronary artery, Carotid artery, and Peripheral artery. In our work we only emphasize on common carotid artery stenosis. The geometry of plaque may vary in shape from a simple to complex structure and also in dimension. Flow through these complex structures is commonly associated with flow separation, stagnation, recirculation and secondary vortex motion that influence quick depositions in the adjacent areas of the existing plaque. Plaque rupture can also be possible which may lead to clogging of small ruptured parts in smaller artery or arterioles.

Computational Fluid Dynamics (CFD) and experimental analyses have been used to study and model stenosis and its haemodynamics in various arteries since 1990's. Ku and others have made detailed studies on the fluid mechanics of vascular system hemodynamic changes due to stenoses [1, 2]. Johnston and Kilpatrick (1991) studied the effect of geometrical irregularities in the wall of a stenosed artery for Reynolds numbers from 20 to 1000 [3]. Tang *et al* [1995-1998] used 3D models for steady viscous flow in an elastic stenotic tube with various stenosis stiffness and pressure conditions [4]. In past experiments blood flow has been considered both as Newtonian or Non-Newtonian fluid depending upon the radius of the blood vessel. Haemodynamic studies have been made for both steady and pulsatile flows. However the stenosis geometries used in the above studies have been idealized models using definite curves like the cosine curve and other smooth and irregular curves. In this study, all the geometrical and flow parameters have been used after studying more than 130 actual clinical Doppler Ultrasound images of patients of varying age and sex in order to obtain a more accurate picture of changes in blood flow through such constricted arteries.

2. Physical Model

The statistical analysis of the 130 patients reveals two dominant geometries with varied dimensions:

i. Curved shape (Fig 1a)
ii. Rectangular shape (Fig 2a)

In the following study, an axis-symmetric geometry has been developed by considering the carotid artery to be a long straight pipe with radius $R_0=D_0/2$ and length $L_0= 875R_0$, where D_0 (pipe diameter) is taken as 0.0057m (validated from different medical books). The stenoses have been modeled based on the above two real-life geometries. In both of the models considered (Figs. 1b and 2b), there is a maximum constriction of 62%, which may be considered as moderate as a constriction of less than 50% is considered mild and above 70% is considered severe in most medical literature. The flow of blood at the inlet of the stenosis is fully developed. Sufficient length of the artery downstream of the stenosis has been taken so that the blood coming out of the constricted region becomes fully developed at the outlet of the artery.

Fig1a: Doppler Ultrasound image of curved stenosis

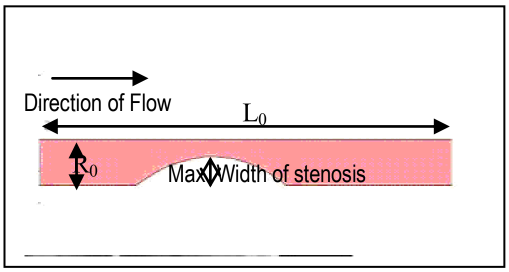

Fig 1b. Model of curved stenosis used in study

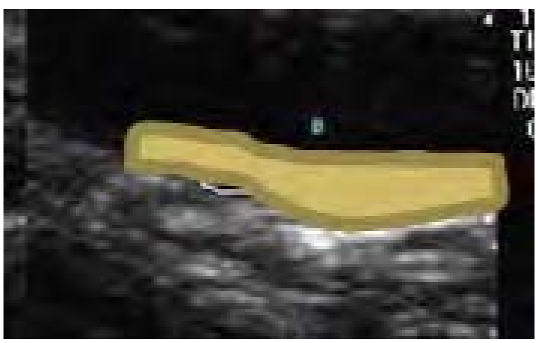

Fig 2a: Doppler Ultrasound image of rectangular stenosis

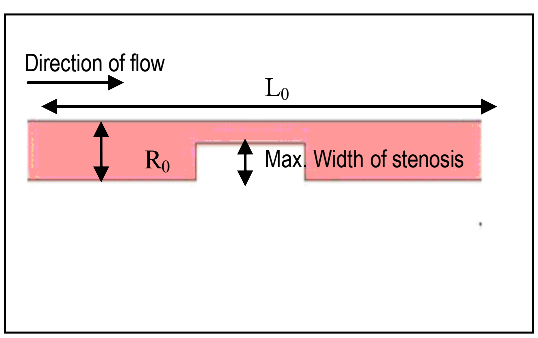

Fig 2b: Model of rectangular stenosis used in study

3. Mathematical Model

3.1 Governing Equation

The blood flow can be considered to be Newtonian when flowing through large arteries [1]. In this study, as the common carotid artery has been taken, the flow of blood has been considered Newtonian, laminar, steady-state and incompressible. Also the artery wall has been considered rigid.

The incompressible Navier-Stokes Equations along with the continuity equation have been used as the governing equation for modeling the fluid flow.

$$\rho(\mathbf{u}.\nabla)\mathbf{u} = \nabla.[-p\mathbf{I} + \nu(\nabla u + (\nabla u)^T)] \quad \text{(i)}$$

$$\nabla.\mathbf{u} = 0 \quad \text{(ii)}$$

where u is the axial velocity, p is the axial pressure, ν is the dynamic viscosity and ρ is the density of blood. Equation (i) is the momentum balance equation and equation (ii) is the continuity equation. In the current study the density of blood has been taken as 1050 Kg/m^3 and the dynamic viscosity as 0.00345 Pa.s.

3.2 Boundary Equations

The imposed boundary conditions are:

a) An axial symmetry condition at the axis.

b) A fully developed velocity profile at the inlet. The equation of the velocity profile is parabolic as expected in laminar flow:

$$u(r) = \bar{u}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

where,

u(r)= radial velocity at an arbitrary radius

$\bar{u}$ =mean velocity

R= the radius of the artery

r= the radius at which the velocity is to be obtained.

c) A zero pressure with no viscous stress condition at the outlet.

d) A no-slip condition at all the walls.

i.e. $\mathbf{u = 0}$

4. Numerical Procedure

Standard Finite Element CFD based software COMSOL® 3.5a has been used for the solution of the problems. Steady-state analysis was performed to solve the incompressible Navier-Stokes Equation within the Fluid Dynamics section of the COMSOL® Multiphysics Application Mode. The solver type used was stationary and the solver used was Direct (PARDISO).

5. Mesh details and grid sensitivity test

A free mesh consisting of triangular elements has been used in the study with the maximum possible refinement. In the curved constriction 14900 elements and for the rectangular constriction 15369 elements have been used for solution of the problems. The mesh has been refined in the vicinity of the constrictions so as to present a more accurate picture of their effects on the blood flow.

Grid sensitivity tests for all the simulations have been taken. For both the shapes there have been no noticeable changes in results when the grids have been refined above the values mentioned. So the above refinement of meshes is used in our subsequent studies.

5. Code validation

In the absence of any standardized data regarding the haemodynamics of stenosed arteries, the code has been validated by studying the flow through a straight pipe without any constriction. The fluid flow is fully developed after a certain distance from the inlet. The radial velocity profile of the fully developed flow is parabolic and the maximum velocity is the centerline velocity and its value is twice the mean velocity. All these results are fully compliant with the known results of classical fluid mechanics.

6. Results

In the available literature, blood has found to flow with Reynolds Number (*Re*) as low as 100 to values of 1000. So in this study, for all the stenosis geometries considered, the flow has been studied with *Re* 100, 400, 800 and 1000. In all the above models, a zone of recirculation and an irreversible pressure rise have been observed at the outlet of the stenosis. The following points have been observed

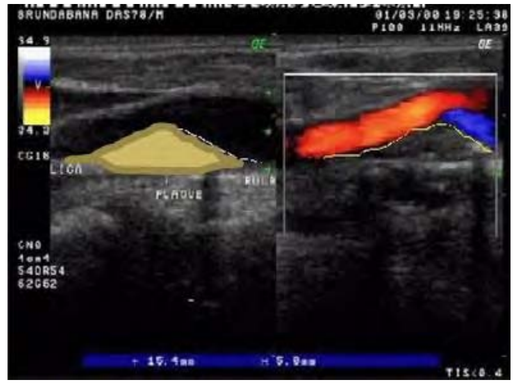

Fig3a: Actual Doppler Ultrasound image

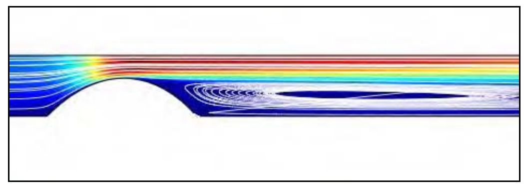

Fig3b: Simulated image from COMSOL®

by studying the simulated results of the rectangular and curved stenoses:

6.1 Comparison of clinical and computational images: Similarities in flow pattern has been observed on comparing the actual Doppler Ulstrasound images and the images obtained from the flow simulation using COMSOL®. In both the cases the flow of blood is from left to right.

Fig.3a shows the actual Doppler Ultrasound image of a patient. The blue coloured region corresponds to a zone of recirculation or back-flow. The same result is obtained from the flow simulation where a well-defined eddy can be observed (Fig 3b).

6.2Centerline pressure plot: From the centerline pressure plot, it has been observed that at the inlet of the stenosis the pressure fall is higher for higher values of Reynolds Number (*Re*). Even negative pressure values have been found in case of *Re*= 1000. But at the outlet of the stenosis, the flows with higher *Re* show higher values of pressure. So, the irreversible pressure rise increases with increasing values of *Re*. (Fig. 4)

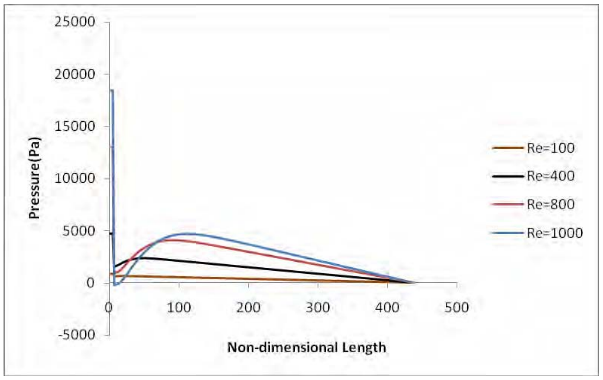

Fig4: Centerline pressure plots of a curved stenosis at different values of *Re*

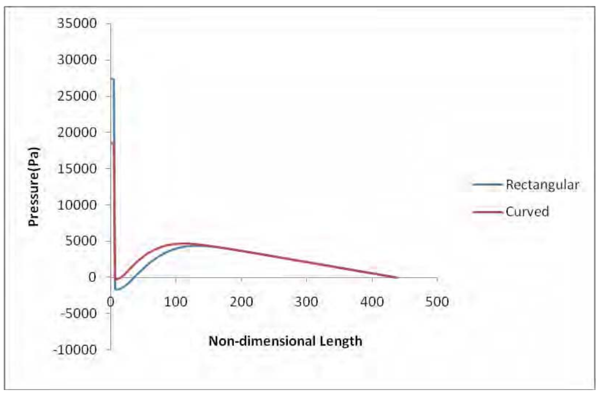

Fig5: Centerline pressure plots of Rectangular and curved stenoses at *Re*=1000

When comparing the pressure profiles of rectangular and curved stenoses at a fixed Reynolds Number, the irreversible pressure rise has been found to be higher in case of the rectangular geometry. At *Re*=1000, the pressure rise for the rectangular stenosis is found to be 23% higher than the curved geometry. (Fig. 5)

6.2Radial velocity field: As seen in Fig. 6 the radial velocity field at the outlet of the stenosis shows negative velocity and the maximum value of the negative velocity is higher for higher values of *Re*. (For the rectangular stenosis, at the stenosis inlet, the maximum velocity has been found to shift from the centerline).

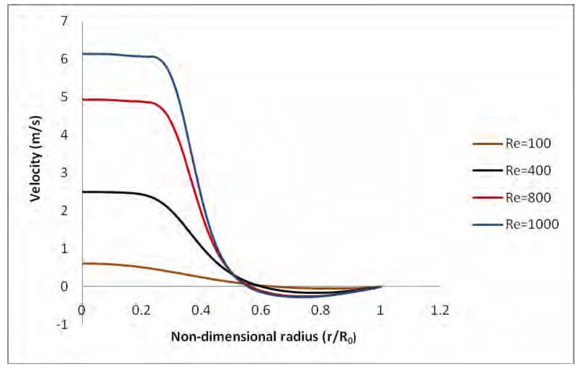

Fig6: Radial velocity profiles of a curved stenosis just beyond the end of the stenosis at different *Re*

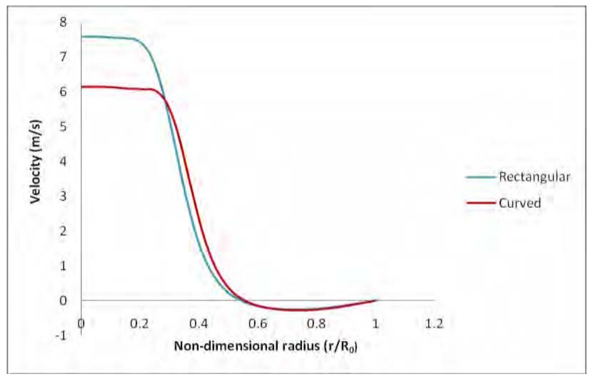

Fig7: Radial velocity profiles of Rectangular and Curved stenoses just beyond the outlet of the stenosis at *Re*=1000.

The comparison of the radial velocity profiles at the stenosis outlet (Fig.7) of the above geometries reveals that the maximum velocity in case of rectangular profile is higher than the curved geometry.

6.3Centerline velocity plot: From the centerline velocity plot (Fig 8), the maximum velocity in the entire sub-domain has been found in a zone near the outlet of the stenosis for all the values of *Re* used. Also the length of reattachment of the flow after the flow separation has been found to increase with increasing values of Reynolds Number.

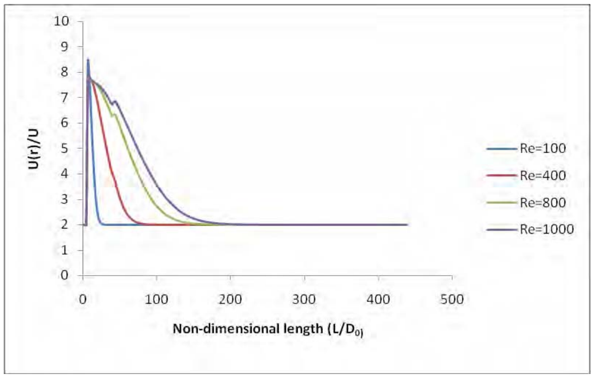

Fig8: Centerline velocity profiles of a Curved stenosis at different values of *Re*

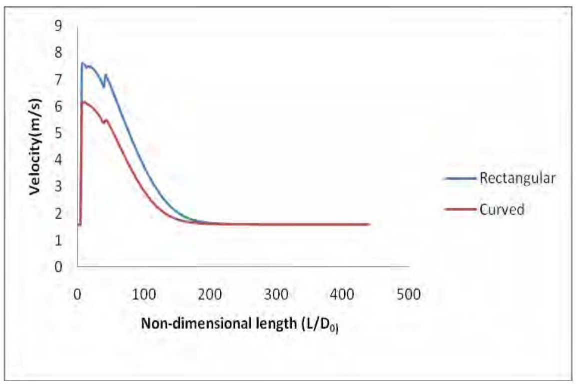

Fig9: Centerline velocity profiles of Rectangular and Curved stenosis at *Re*=1000

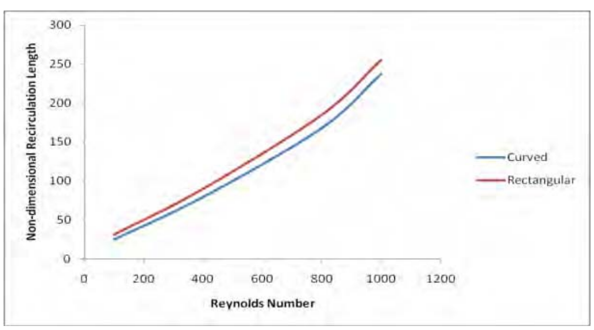

Fig10: Recirculation Length vs. Reynolds Number

The reattachment length of the rectangular stenosis is around 10% more than the curved stenosis, as can be concluded from Fig. 9.

6.4Recirculation length vs. Reynolds Number: The recirculation lengths of both the rectangular and curved stenoses plotted against the respective Reynolds Numbers show an almost linear variation. From this graph (Fig 10) it can be seen very clearly that the recirculation lengths of the rectangular stenosis is higher than a curved stenosis for the same value of *Re*.

7. Conclusions

In the present work the flow of blood through stenosed arteries has been studied by considering blood to be a Newtonian fluid and the flow to be laminar by varying the Reynolds Number,

From the close similarity of Figs 3a and 3b, it can be concluded that inspite of all the idealizations imposed during the numerical analysis, the current study qualitatively agrees with the actual situation. Based on this realization, further studies can be carried out by complicating the geometries and the flow conditions by lifting the idealizations gradually so as to obtain a more accurate picture.

For fixed stenosis geometry, as the pressure rise increases with increasing Reynolds Number, the heart has to supply even more pressure to overcome this adverse pressure gradient. Thus the effort of the heart increases, leading to angina (pain in the heart). Also as the recirculation zone is higher for higher values of the flow velocity, the tendency of the stenosis to propagate increases. This is because the stenosis aggravates with higher lengths of re-attachment.

From the comparison of the rectangular and curved stenosis, it is inferred that as the extent of the recirculation length is longer in case of the rectangular stenosis than the curved one, the rectangular stenosis has a higher tendency to propagate. This is expected from the shapes of the stenoses. The rectangular stenosis presents itself as a bluff body in the line of flow of blood while the curved stenosis is more streamlined. Also that the irreversible pressure rise for the rectangular geometry is higher proves that it is more severe than a curved stenosis of the same maximum constriction due to causes already discussed.

It can also be predicted from the above study that as depositions continue to occur downstream of a curved stenosis, it will eventually approach a rectangular stenosis, if enough time is available. So, the adverse effects of a stenosis essentially increase with time.

8. References

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