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Two- and three-dimensional holey phononic crystals with unit cells of resonators

Yan-Feng Wang[1,2], Yue-Sheng Wang[1] , and Litian Wang[2]
1. Beijing Jiaotong University, Institute of Engineering Mechanics, Beijing, China, 100044;
2. Østfold University College, Department of Mechanical Engineering, Halden, Norway, 1757.

Introduction: Widely interests have been devoted to studies on the propagation behaviors of the elastic waves in phononic crystals (PCs). These new artificial structures exhibit bandgaps in their spectra, where the propagation of waves is fully prohibited. The bandgaps in PCs may have potential applications in acoustic isolation, noise suppression and vibration attenuation, etc. While much reported work is focused on the PC systems with convex (circular or regular-polygonal [1]) holes. However, we noticed that, by introducing non-convex holes, photonic crystals may display a broad stopband or a dual-stopband, (or even multi-stopband). These works motivate investigations on the PCs with non-convex holes. In this paper, we will study the bandgaps of 2D PCs with cross-like (a kind of non-convex) holes in a square lattice (Figure 1). We will also extend our work to study 3D holey PCs with resonators in a simple cubic lattice (Figure 2). The study in this paper is relevant to the optimal design of the bandgaps in light porous materials.

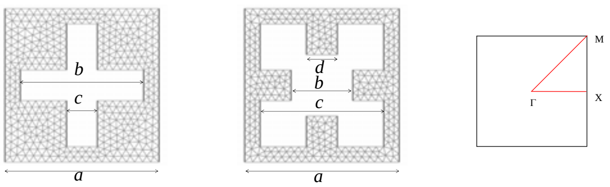

Figure 1. Cross-sections and finite element models of the unit cells of the 2D phononic crystals with (a) "+"- and (b) "x"-holes as well as their associated Brillouin zones.

Methods: We apply the Acoustic Module operating under the 2D/3D plane strain Application Mode. The free boundary condition is imposed on the surface of the hole, and the periodic Bloch boundary conditions on the two opposite boundaries of the unit cell [2-6], yielding the relationship between the displacements $\mathbf{U}(\mathbf{r})$for nodes on the boundaries:

$$\mathbf{U}(\mathbf{r}+\mathbf{a}) = e^{i(\mathbf{k}\cdot\mathbf{a})}\mathbf{U}(\mathbf{r})$$

The unit cell is meshed by using the default mesh. Still we require the Hermitian transpose of constraint matrix and in symmetry detection in the advanced solver parameter settings. The model built in COMSOL is saved as a MATLAB-compatible '.m' file. The file is programmed to let the wave vector $\mathbf{k}$ sweep the edges of the irreducible Brillouin zone, so that we can obtain the whole dispersion relations.

Results: No bandgap appears in the 2D PC system with the square holes if the symmetry of the holes is the same as that of the lattice. However if the square holes are replaced with the cross-like holes (Figure 3), large bandgaps at lower frequencies are generated [2-3]. For 3D PCs, no bandgap appears in the systems with cubic or spherical holes. When the proposed six-necked or one-necked resonators are introduced (Figure 4), complete bandgaps in a low frequency range are generated [4]. The influences of the geometry of the cross-like holes and resonators on the bandgaps are discussed. Based on the vibration modes at the bandgap edges, spring-mass models and spring-pendulum models are developed to explain the mechanism of the bandgap generation [3-4].

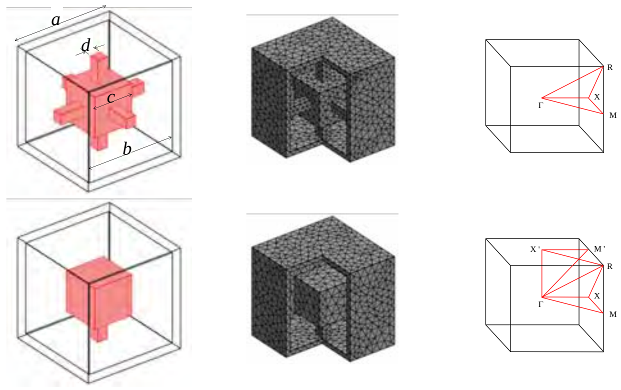

Figure 2. The unit cells of two kinds of 3D PCs and their finite element models as well as their associated Brillouin zones.

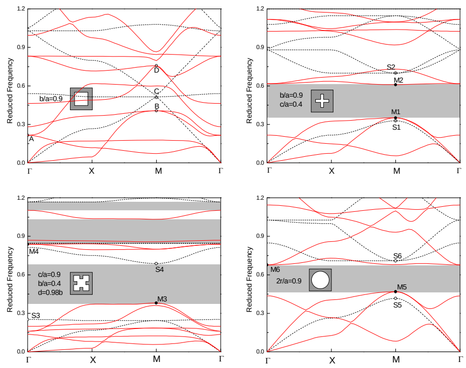

Figure 3. Band structures of the phononic crystal systems with (a) square, (b) "+"- , (c) "x"- and (d) circular holes. The red solid and black dashed lines represent the mixed and shear wave modes, respectively.

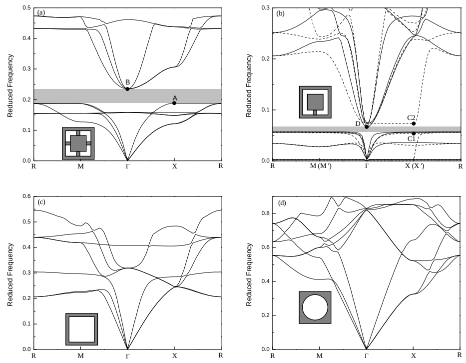

Figure 4. Band structures of 3D holey phononic crystal in a simple cubic lattice with (a) the six-necked resonators; (b) the one-necked resonators, (c) cubic holes, and (d) spherical holes. The insets show the cross-section of the corresponding unit cell.

Conclusions: We show in this paper that by careful design of the geometry of the resonators, complete bandgap with relatively low center frequency can be obtained for 2D and 3D PCs with resonators. The generation of the bandgap is due to the local resonance of the unit cell [2-6]. Spring-mass and spring-pendulum models are developed to predict the boundaries of the complete bandgap. The predicted results are in general agreement with the numerical results.

References:

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