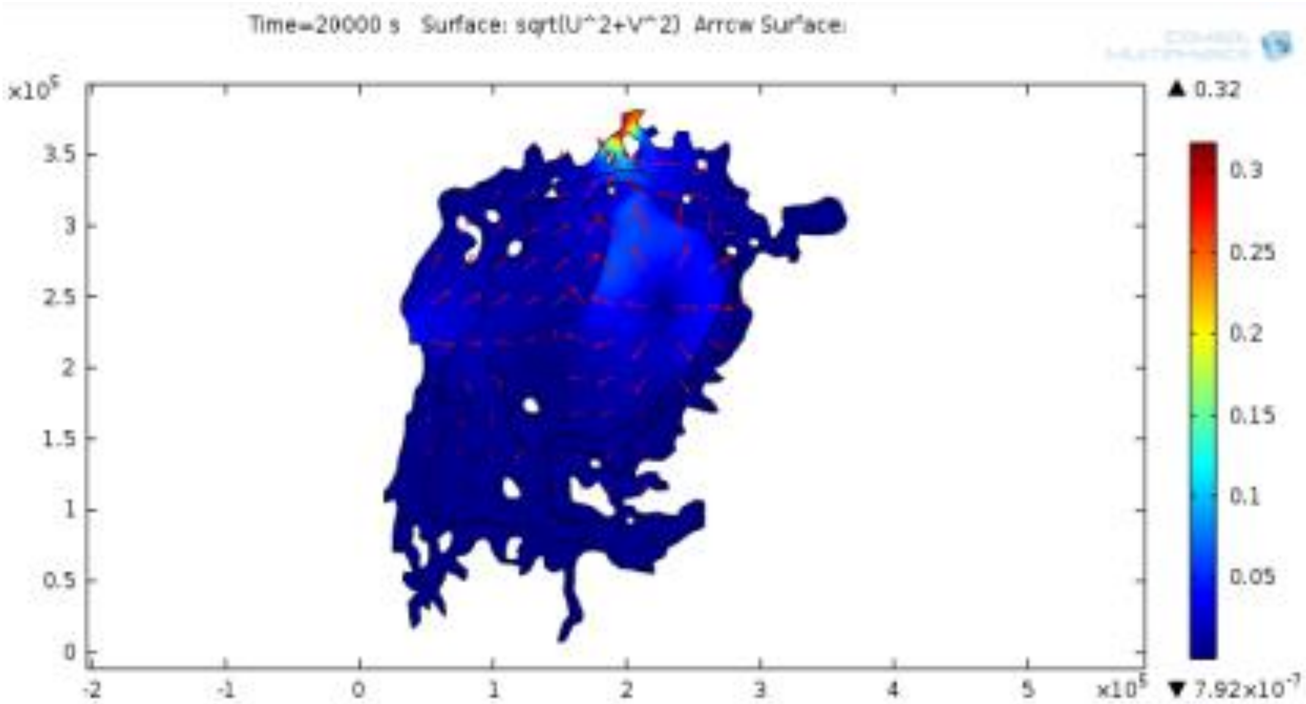
Hydrodynamics of Lake Victoria

Vertically integrated flow models in COMSOL Multiphysics



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COMSOL CONFERENCE 2014 BANGALORE

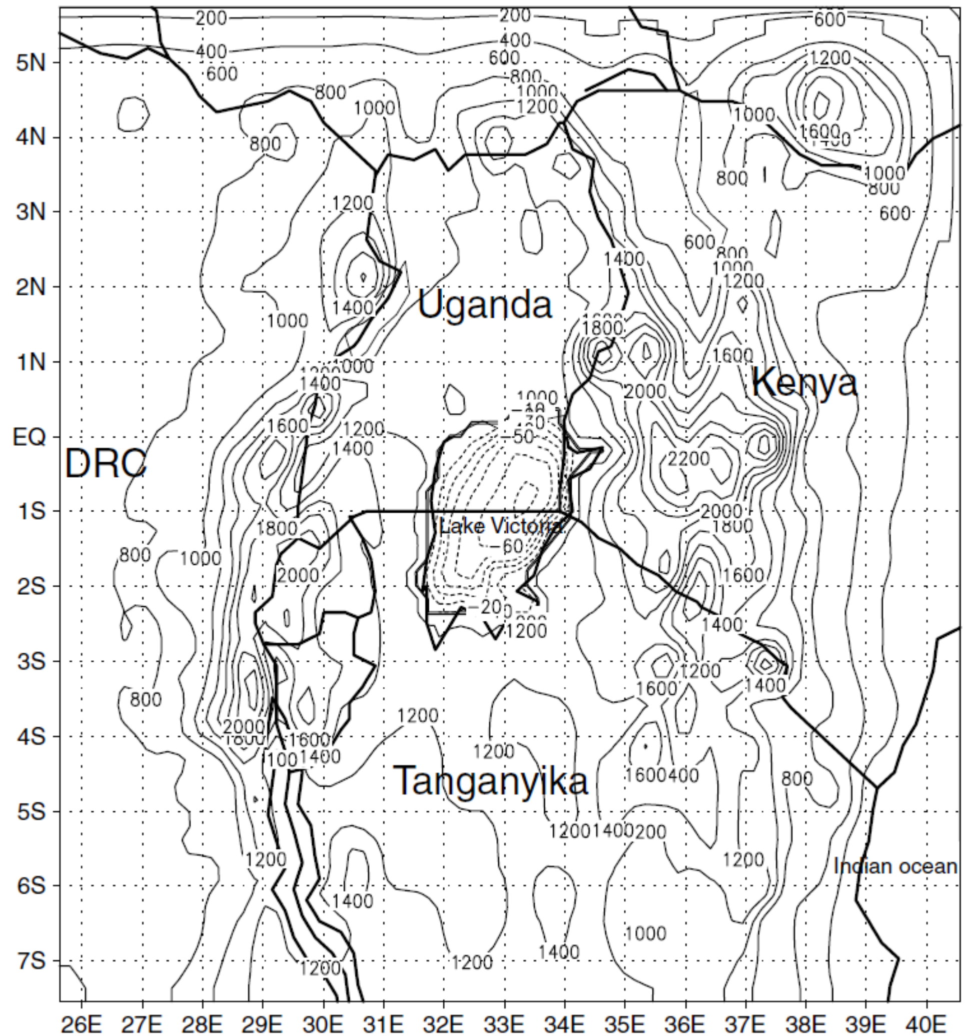
Excerpt from the Proceedings of the 2014 COMSOL Conference in Bangalore

Introduction

- Computational simulation plays an important role in science and engineering. It can be seen as a translation of real-world physical laws into their virtual form.
- Today researchers are using simulation broadly when developing new products or optimizing designs. For simulation they are using very basic programming language as well as various high-level packages with advanced methods implemented.
- Complete physical packages describing what happen in the real world are very demanding now-a-days.
- COMSOL Multiphysics is a flexible platform, allowing users to model all relevant physical aspects of their designs.

Lake Victoria

- Lake Victoria affects the social, economic, environmental and political livelihood of human, animal, plants in East Africa
- Second largest freshwater body in the world and the largest tropical lake in Africa.
- Hydrodynamic processes unique due to closeness to equator.
- 1,134 m above sea level, volume 2,760 km³, area 68,828 km²



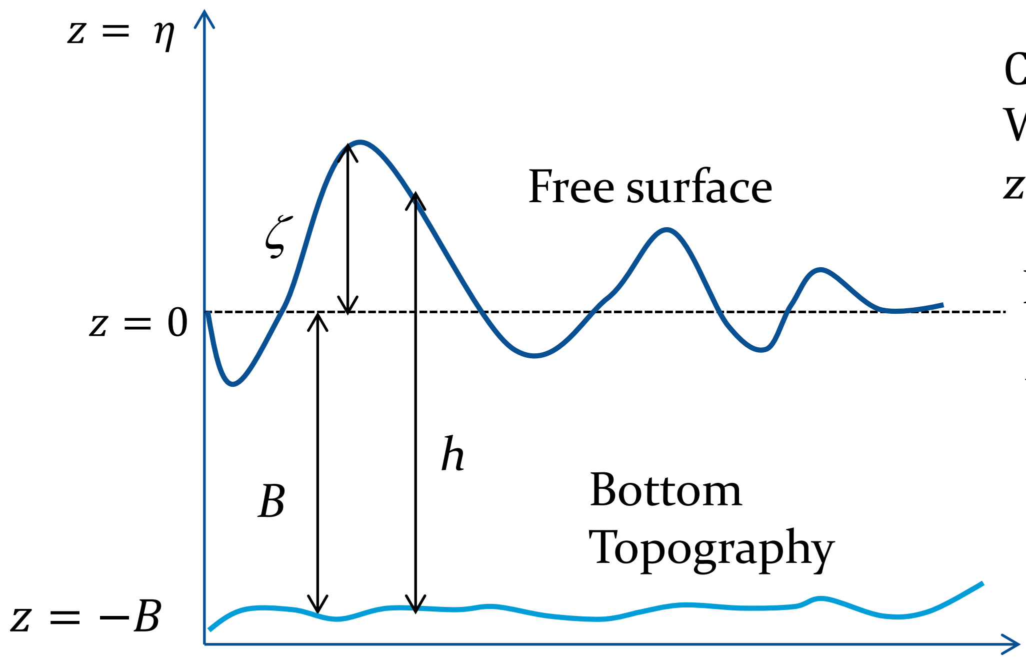
Hydrodynamics of Lake Victoria

- Knowledge of lake hydrodynamics is important for sustainable and environmentally friendly development of water resources.
- Frequently occurring geological processes are directly related to the hydrodynamics.
- Hydrodynamics :
  - Water velocities
  - Circulation patterns
  - Mixing and dispersion
  - Temperature, and density stratification etc.
- Complete Coriolis effects (longitudinal and vertical)

Model Introduction

- Lake Victoria is shallow: average depth 40 m and maximum depth 84 m:Horizontal scale of motion much larger than that of the vertical scale.
- First step: vertically integrated St.Venant shallow water model to look at the effects of bottom topography on large-scale flow patterns and the water level variation.
- The Comsol coefficient form PDE represents streamline artificial viscosity, Coriolis forces, and bottom friction, with boundary conditions representing river in- and outflow.
- Accurate hydrodynamic simulation requires realistic grid representation of boundary geometry .
- Conservative formulation in terms of the conserved quantities (mass and momentum) is only needed, if shocks (hydraulic jumps) might appear..
- Standard streamline artificial viscosity acts only in the direction of fluid velocity; gravity waves isotropic also need dissipation so we use isotropic viscosity scaled by largest signal speed

Vertically Integrated St.Venant Shallow Water Model



Celerity $\zeta(x,y,t)$:
Wave height from reference datum
$z = 0$

Bathymetry $B(x, y)$

Water depth: $h = B(x,y) + \zeta(x,y,t)$

Shallow water PDE

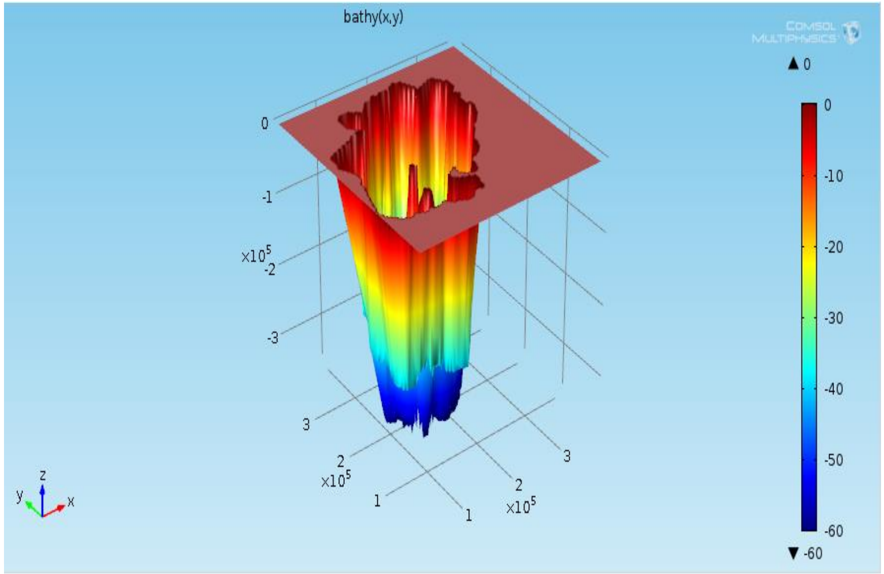
- Standard vertically integrated SWEs:
- h water depth, g gravitational acceleration
- (u,v), (x,y) velocity components,

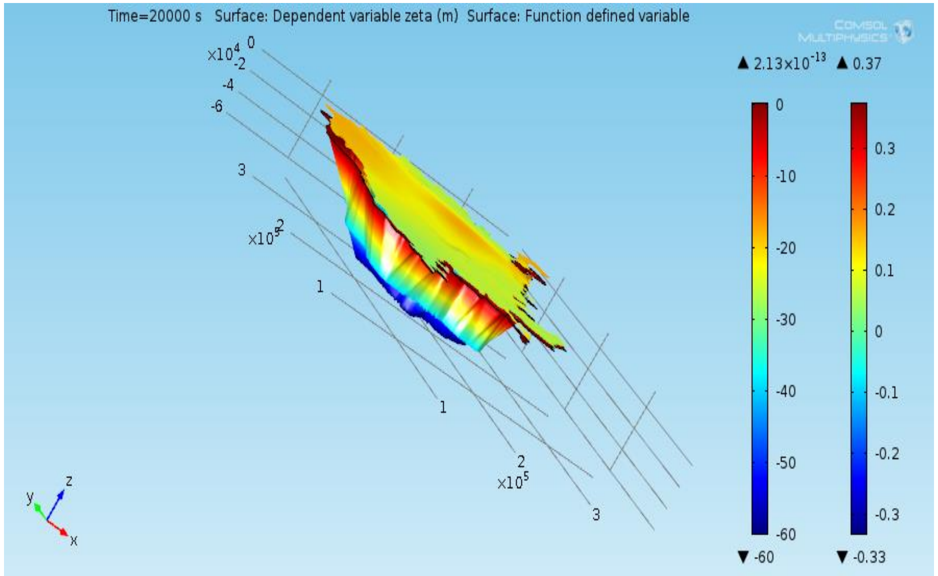
$$u_t + uu_x + vu_y + g(h_x + b_x) = div(v_A \nabla u) + fcx - CuV$$
$$v_t + uv_x + vv_y + g\left(h_y + b_y\right) = div(v_A \nabla v) + fcy - CvV$$
$$h_t + (hu)_x + (hv)_y = div(v_A \nabla h) + P - E$$
$$V = \sqrt{u^2 + v^2}\,,\ v_A = \mu\Delta\left(\sqrt{gh} + V\right),$$
$$Volume\ source = P - E$$

$b(x,y)$: bathymetry, Δ : local mesh size (h in Multiphysics),
μ : non-dimensional user defined artificial viscosity,
fcx, fcy : Coriolis force components
C : Bottom friction coefficient.

Bathymetry and Shoreline in COMSOL

- Import of DEM (Digital Elevation Map) file failed
- Create matlab function $z(x, y)$ for bathymetry.
- The shoreline should move with change in water level.





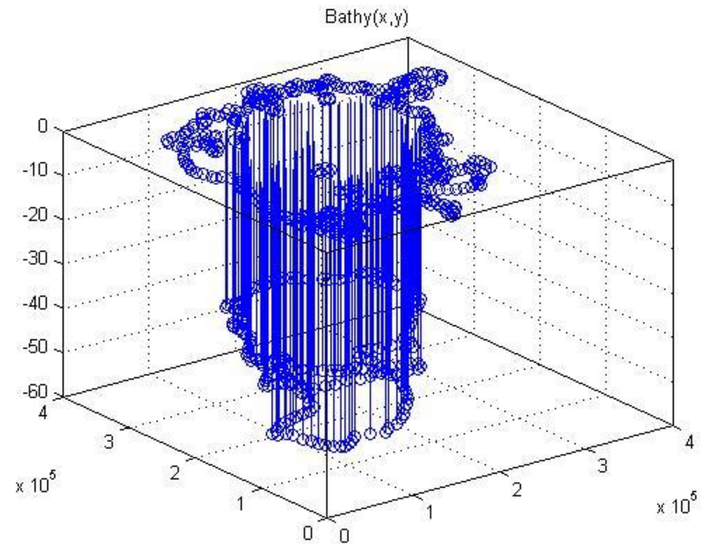
Bathymetry, I

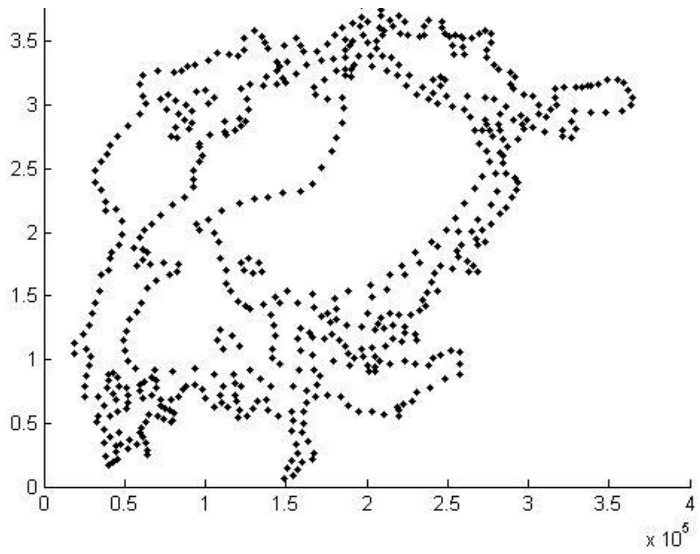
Data: points on 0, 40, and 60 m iso-depth curves from Dr. Richard Wait

1. Create Kriging interpolation function to for depth from this set of points – DACE Matlab ToolBox
2. Evaluate *Nx* x *Ny* dense rectangular grid,
3. Import as "2D Grid File"

Issue:
Interpolator becomes either very un-smooth or too smooth; Note that db/dx, db/dy is needed!

Reason: Initial point set anisotropic
short distances between points on curves,
long distances between curves



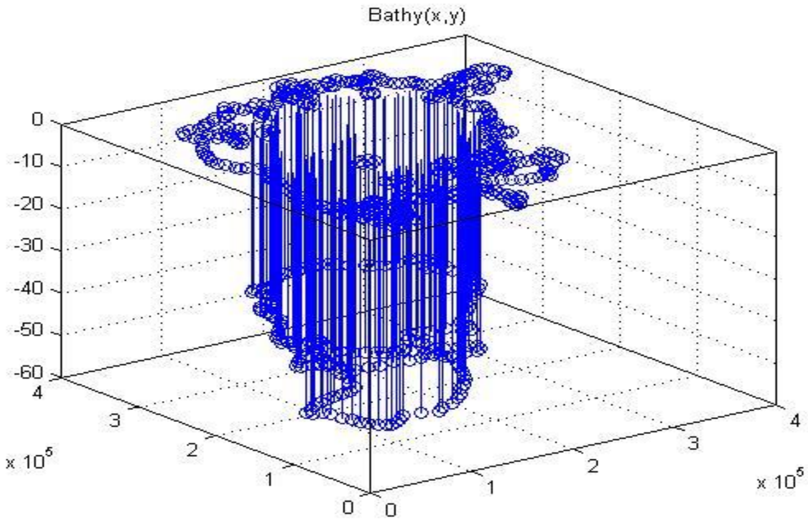


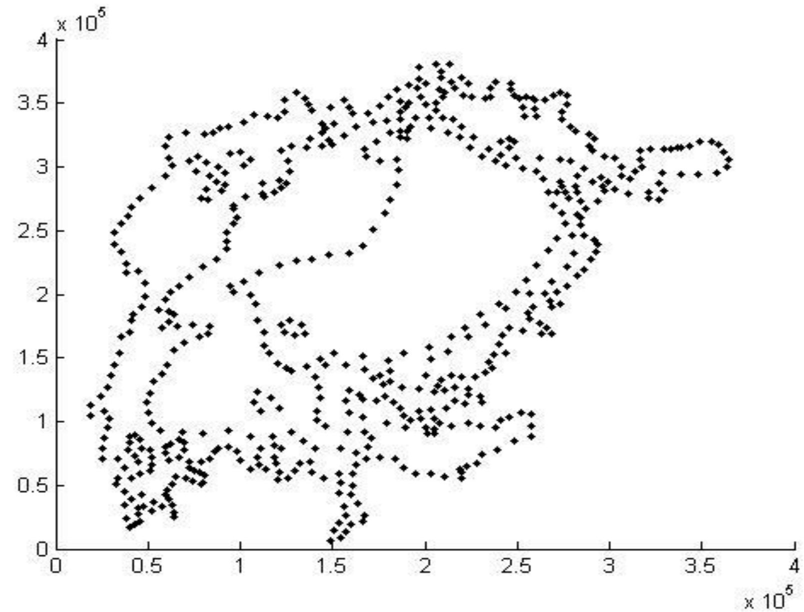
Bathymetry, II

Data: points on 0, 40, and 60 m iso-depth curves

1. Connect points into ordered sets defining shoreline and islands; 20 curves
2. Delaunay triangulation of convex hull
3. Linear interpolation over the triangles to form *Nx* x *Ny* dense rectangular grid,
4. Smooth by Gaussian filter to give continuous gradient
5. Import as "2D Grid File"

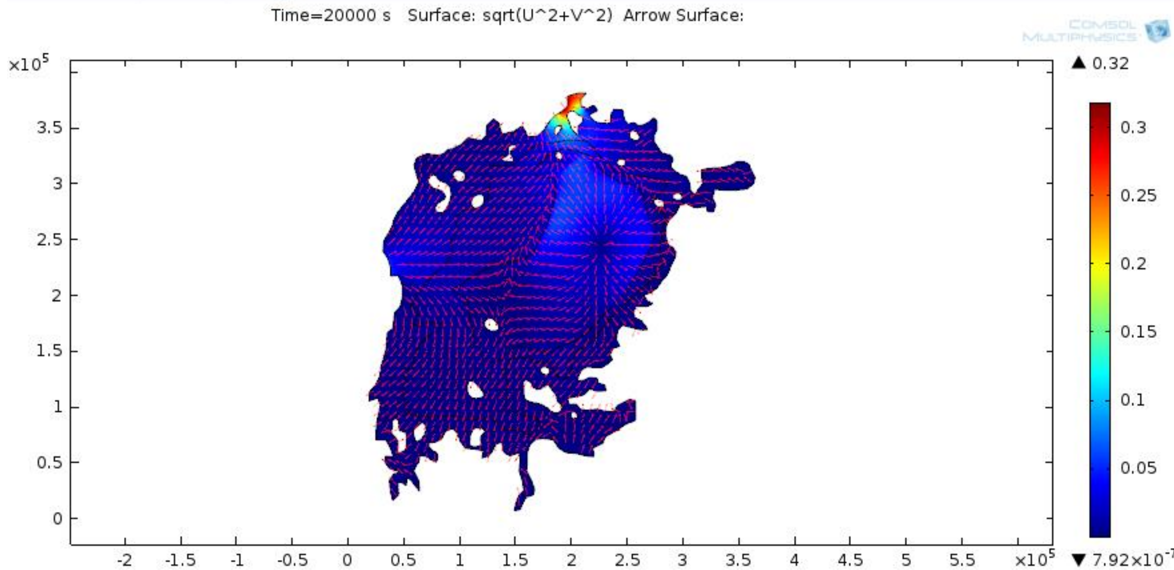
Issue:
triangles with all corners z = 0:
Add points manually at depth 20 m until all triangles of water domain have at least one non-zero depth corner

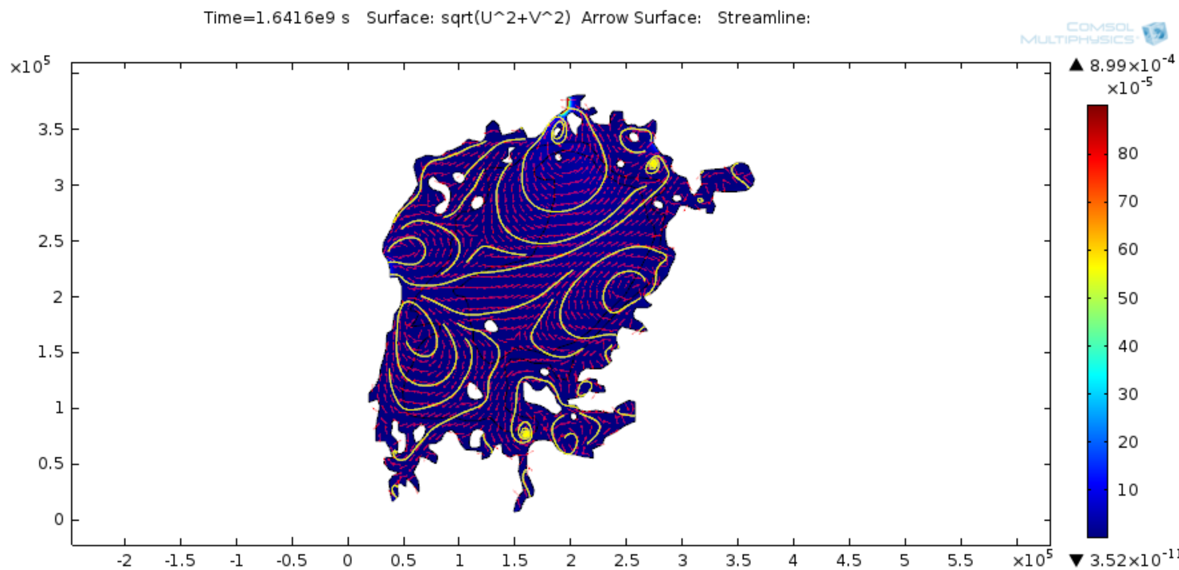




Lake Victoria 2D model in COMSOL

- In 2D lake model has been applied inflow, outflow, precipitation and evaporation data.
- Lake Victoria has 23 several inflow basin.
- Kagera river is the main upstream river basin of the lake.
- The only outlet is the Victoria Nile which exits the lake near jinja. There have a heavy fluid pressure.
- Fluid velocity is high near the lake upstream inflow.
- Where streamline is created vorticity heavy fluid pressure have in there.





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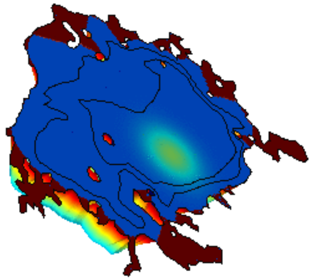
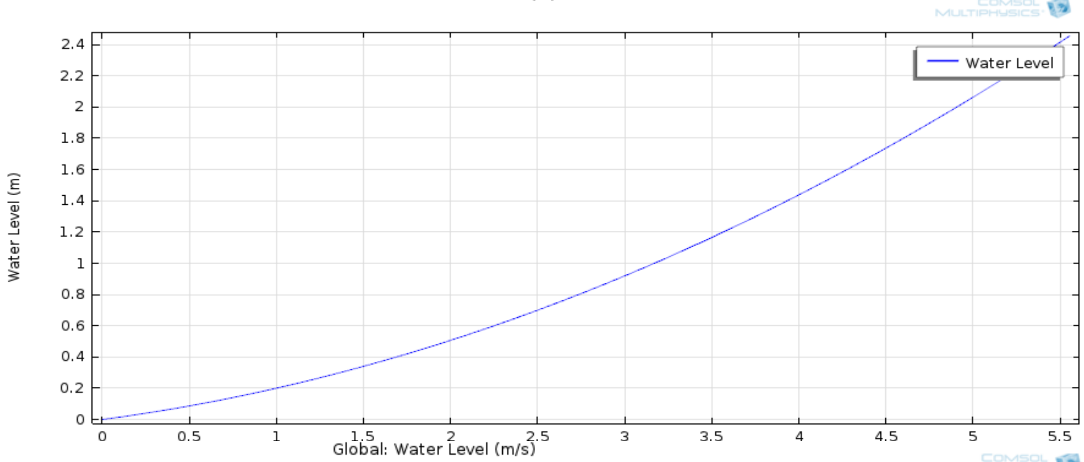
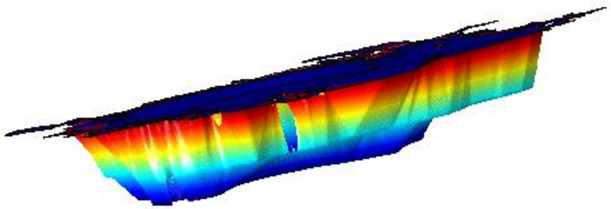
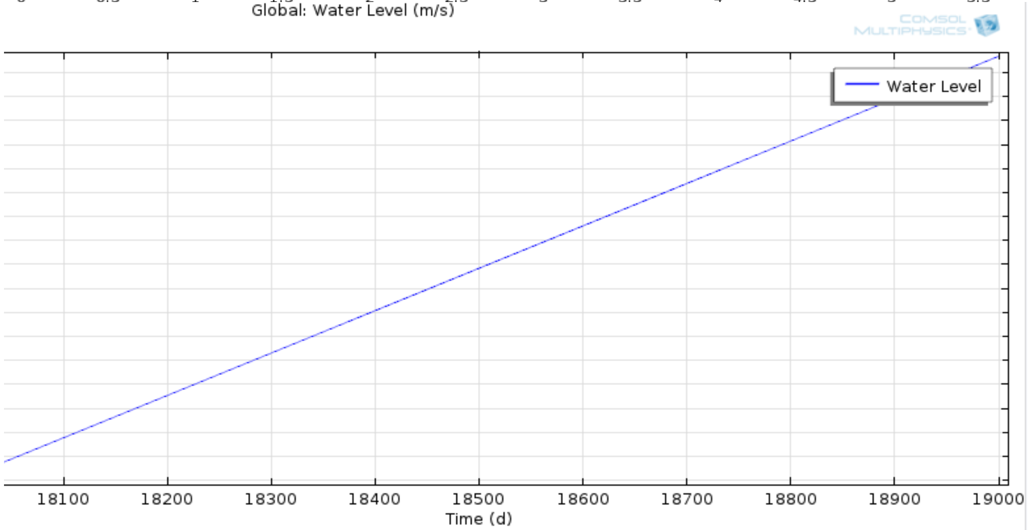
Water balance and water level Analysis

Several years water balence is much lower than couple of hours water level.



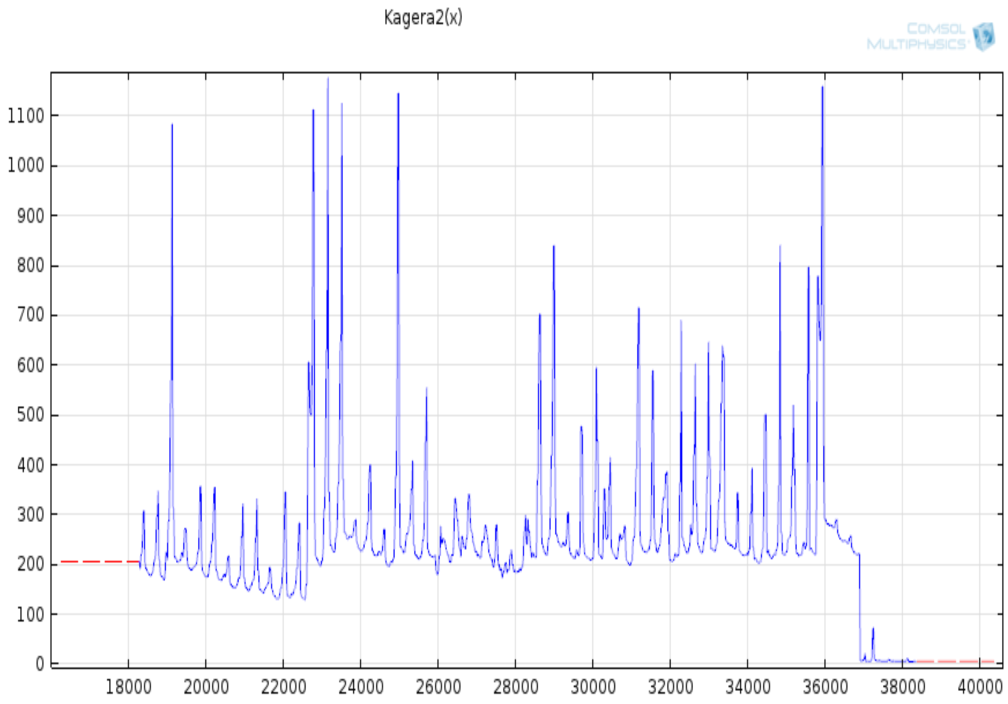


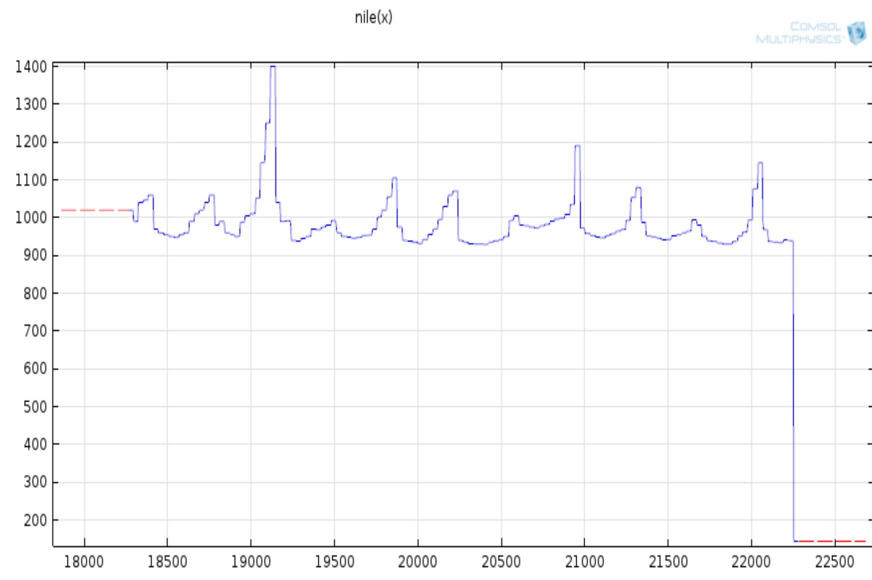




Main river in- and outflow

More than 50 years data on Kagera rivers inflow and Blue Nile outflow obtained from Makerere U., Uganda





Further Work

Much data remains to collect and coordinate

- Modern, accurate depth data from Makerere U.
- Precipitation and evaporation data from literature
- Water level observations at 20 stations
- - Check model prediction of water level over 10 years
- - Temperature and density variation analysis for whole lake.
- - Solute transport of pollution from Kagera river.

Thank you
for your attention