Numerical Simulation of Blood Flow in a Straight Artery Under the Influence of Magnetic Field

G. C. Shit, A. Sinha, A. Mondal, Sreeparna Majee

Department of Mathematics, Jadavpur University, Kolkata – 700032, India

MOTIVATION

- To study the interaction of magnetic field with the flow of electrically conducting fluid
- Our study aims to increase of magnetic field strength in MRI machines
- Downstream Singularities in the cardiovascular system
- The MHD equations for flow of blood have been solved without neglecting the induced magnetic field.
- To characterize the unsteady entry length in functions of Reynolds number (Re) and Hartmann number (Ha).
- To determine the induced voltage in the artery with magnetic field strength

GOVERNING EQUATIONS

The basic governing equations for magnetohydrodynamic (MHD) fluid flow in dimensionless form:

$$\vec{\nabla}'\cdot\vec{V}' = 0 \qquad (1)$$

$$\frac{\alpha^2}{\mathrm{Re}}\frac{\partial\vec{V}}{\partial t} + (\vec{V}\cdot\vec{\nabla})\vec{V} = -\vec{\nabla}p + \frac{1}{Re}\nabla^2\vec{V} + \frac{Ha}{ReR_m}(\vec{\nabla}\times\vec{b})\times\hat{i} + \frac{1}{ReR_m}(\vec{\nabla}\times\vec{b})\times\vec{b} \qquad (2)$$

The magnetic induction equation:

$$\frac{\alpha^2}{\mathrm{Re}}\frac{\partial\vec{b}}{\partial t} + (\vec{V}\cdot\vec{\nabla})\vec{b} = (\vec{b}\cdot\nabla)\vec{V} + Ha\left(\hat{i}\cdot\vec{\nabla}\right)\vec{V} + \frac{1}{R_m}\nabla^2\vec{b} \qquad (3)$$

Subject to the following non-dimensional variables

$$\vec{V} = \frac{\vec{V}'}{V_0},\quad p = \frac{p'}{\rho V_0^2},\ t = \omega t',\ \vec{B} = \frac{\vec{B}'}{B_{ref}} = \frac{B_0\hat{i} + \vec{b}'}{B_{ref}},$$

$$\vec{b} = \frac{\vec{b}'}{B_{ref}},\ (x,y,z) = \left(\frac{x'}{a},\frac{y'}{a},\frac{z'}{a}\right) \qquad B_{ref} = \frac{1}{a}\sqrt{\frac{\eta}{\sigma}}$$

The dimensionless parameters that appeared is defined as

$$Ha = B_0 a\sqrt{\frac{\sigma}{\eta}} \qquad Re = \frac{a\rho V_0}{\eta} \qquad \alpha = a\sqrt{\frac{\omega}{\upsilon}}$$

$$R_m = aV_0\mu_e\sigma$$

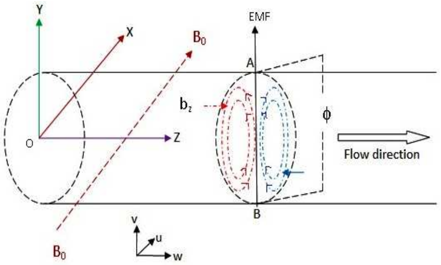

Fig. 1 Schematic diagram

BOUNDARY CONDITIONS

At the inlet : $w = \left[1 + \sin\left(t + \frac{3\pi}{2}\right)\right]$, $u = v = 0$, $(\hat{n}\cdot\vec{\nabla})\vec{b} = \vec{0}$

At the outlet: No viscous stress, $p = 0$, $(\hat{n}\cdot\vec{\nabla})\vec{b} = \vec{0}$

At the rigid wall: $u = v = w = 0$, $\vec{b} = \vec{0}$

RESULTS

Direct numerical simulations were performed with Comsol Multiphysics® software to solve the magnetohydrodynamic (MHD) partial differential equations. Comsol Multiphysics® software, which provides us to implement finite element meshing to create a three-dimensional model of blood vessel. The convergence of the numerical solutions have been verified by mesh refinement.

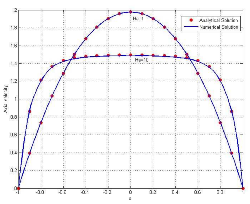

Fig. 2 Comparison of axial velocity profile with the analytical solutions

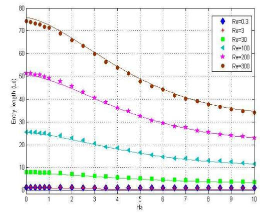

Fig. 3 Steady entrance length in terms of the Hartmann number

We have theoretically estimated the following function for the approximate entry length corresponds to our numerical as shown in the above figure.

$$Le \cong \frac{0.25\mathrm{Re}}{1 + 0.024Ha + 0.025Ha^2 - 0.0016Ha^3} \qquad (4)$$

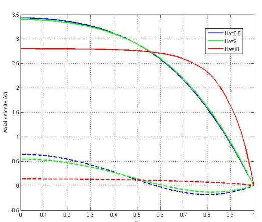

Fig. 4 Unsteady axial velocity profile During systolic and diastolic pressure

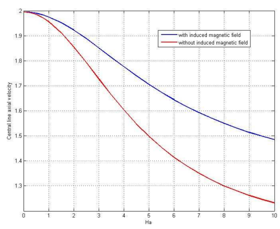

Fig. 5 Reduction in mean velocity with Hartmann number Ha

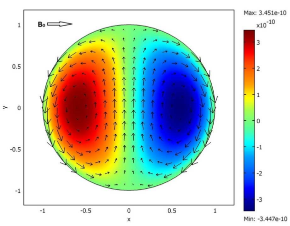

Fig. 6 Axial induced magnetic field (surface plot) and induced current Density (arrow plot)

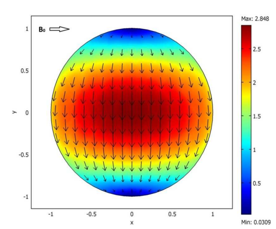

Fig. 7 Magnitude of electric field (surface plot) and direction of induced electric field (arrow plot)

CONCLUSIONS

1. The steady entrance length is found to be increase with Reynolds number Re and decrease with the increase of Hartmann number *Ha*.
2. The mean velocity at the central line decreases rapidly when the induced magnetic field neglected. Therefore without consideration of induced magnetic field over estimates the flow rate.
3. During pulsatile blood flow, the reversal flow can be strongly suppressed by applying strong magnetic field. The Womersley parameter has reducing effect on velocity in the core region and an enhancing effect in the boundary layer during its peak flow, while the trend is reversed in the case of minimal flow rate.
4. The interaction between induced currents and applied magnetic field causes reduction in flow velocity and thereby increases blood pressure in order to retain constant flow rate. The induced magnetic field forms two lobes on each side of the main current line and the induced currents re-circulates inside the vessel, due to the consideration of non-conducting vessel walls. Thus the effect of induced magnetic field should not be overcome.
5. The induced voltage increases with the applied magnetic field only.

Therefore, the study presented here bearing important physiological phenomena and provides a lot of information to the scientist/researchers/experimentalists who are interested in cardiovascular flow simulation particularly in the patient-specific models.

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