# Iterative Learning Control for Spatio-Temporal Repetitive Processes

**Damian Kowalów**, **Maciej Patan**

COMSOL CONFERENCE
2015 GRENOBLE

Institute of Control and Computation Engineering
University of Zielona Góra

# Outline

# Introduction

## Process repeatability

- the same tracking error, oscillations and overshot produced along each replicated trial,
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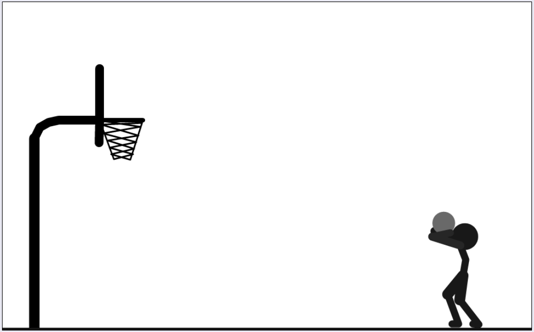
## Challenges

- compensation of random disturbances,
- general control scheme to the repetitive spatio-temporal process.

# Subject of the talk

use data from repetition of the same process controlled by PID several times to improve:

- quality of control,
- robustness with respect to model uncertainty.



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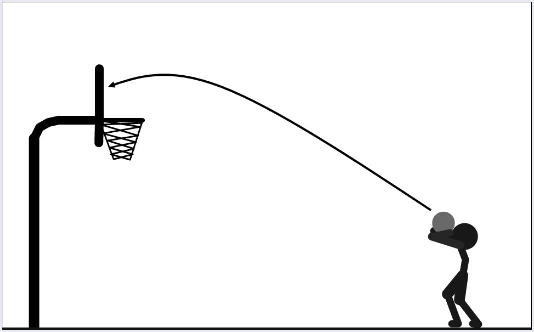

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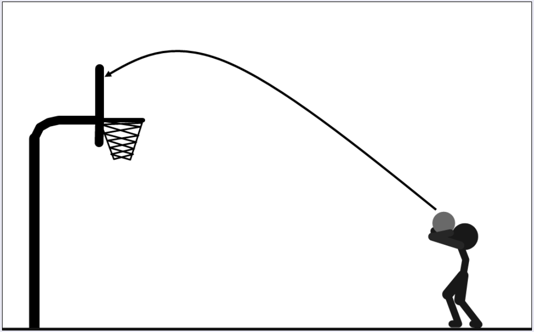

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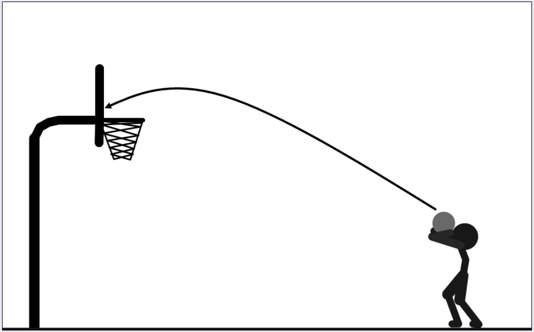

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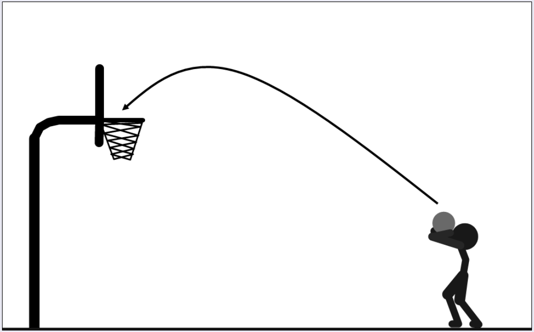


# Motivations

- self–learning methodology,
- feedforward signals for subsequent trials through iterative update,
- high performance with low cost (transient tracking error),
- objects with a lot of measurement points,
- repetitive processes,
- and many more ...

# System description

Consider $y_d(t)$ which denote a continuous reference trajectory defined over a finite time interval $T = [0, t_f]$, where $t_f < \infty$ denotes the trial length, then typical **Iterative Learning Control** law

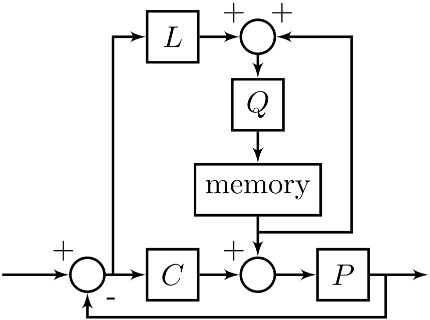
$$v_{k+1}(t) = \mu v_k(t) + \eta \dot{e}_k(t) \tag{1}$$

where

- $k \geqslant 0$ — trial or cycle number,
- $v(t)$ — the system input along the trial,
- $\mu$ — momentum coefficients,
- $\eta$ — learning coefficients,
- $y_k(t)$ — system output,
- $e_k(t) = y_d(t) - y_k(t)$ — tracking error.

Could be splits into

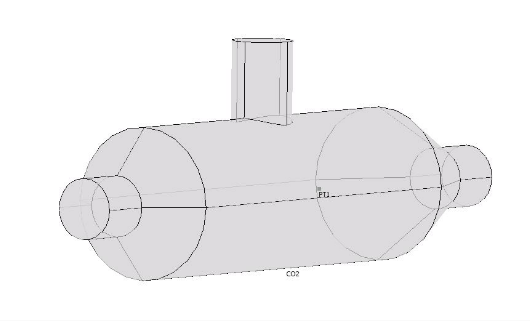
- $L$ — learning filter, inverse of process sensitivity,
- $Q$ — low pass filter,
- $P$ — object plant,



# Illustrative example

## Gas combustion chamber

- **three dimensional model**,
- inlet with constant concentration and velocity,
- inlet with constant concentration and controlled velocity,
- one outlet,
- mixing to achieve effective combustion (inside point).



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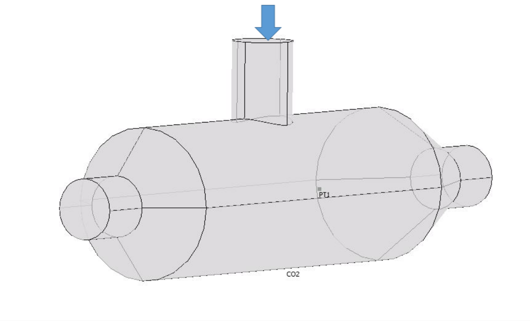

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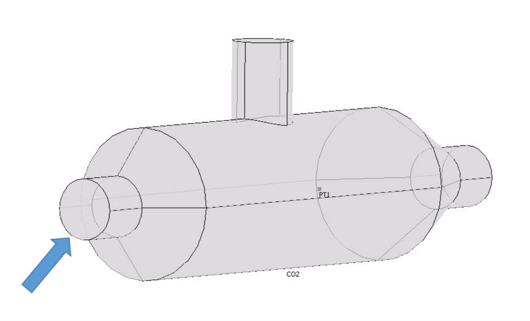

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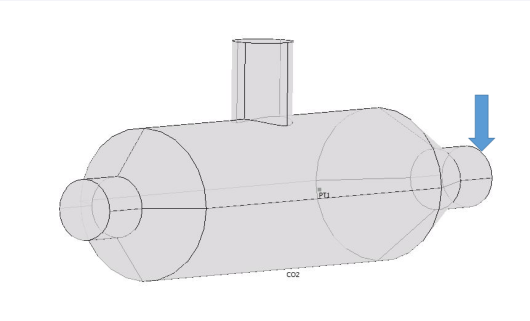

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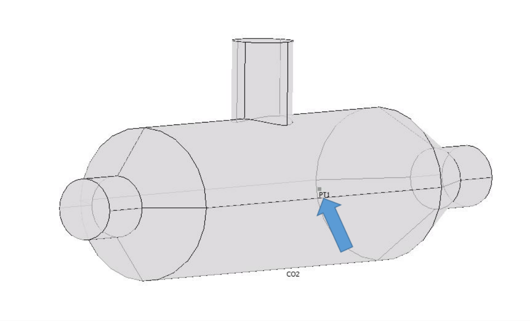


# Mathematical model of the problem

- fluid flow: Navier-Stokes equations,
- mass balance: convection and diffusion application

$$\rho \frac{\partial \mathbf{u}}{\partial t} - \nabla \cdot [\eta(\nabla \mathbf{u} + (\nabla \mathbf{u}^T)] + \rho \mathbf{u} \cdot \nabla \mathbf{u} + \mathbf{u} p = \mathbf{F}, \tag{2}$$

$$\nabla \cdot \mathbf{u} = 0, \tag{3}$$

$$\delta_{\mathrm{ts}} \frac{\partial c}{\partial t} + \nabla \cdot (-D \nabla c) = R - \mathbf{u} \cdot \nabla c, \tag{4}$$

where

- $\rho[\mathrm{kg/m^3}]$ — density,
- $\mathbf{u}[\mathrm{m/s}]$ — velocity vector,
- $\nabla$ — gradient operator
- $\mathbf{F}[\mathrm{N/m^3}]$ — volume force vector,
- $c[\mathrm{mol/m^3}]$ — concentration,
- $\eta[\mathrm{kg/m^3}]$ — dynamic viscosity,
- $p[\mathrm{Pa}]$ — pressure at output,
- $R[\mathrm{mol/(m^3)}s]$ — reaction rate,
- $D[\mathrm{m^2 s}]$ — diffusion coefficient,
- $\delta_{ts}$ — time scaling coefficient.

# Boundary conditions

For mass balance:

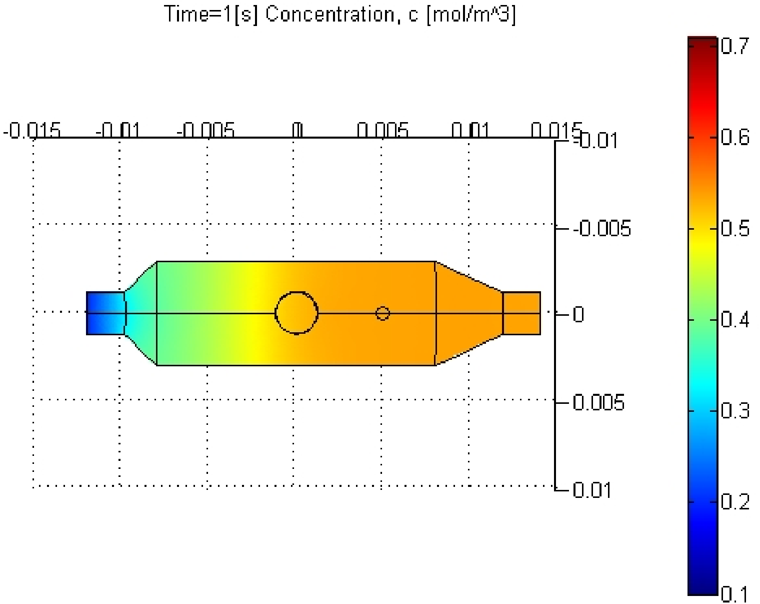
- $c_{\mathrm{t}}[\mathrm{mol/m^3}]$ — concentration at upper input,
- $c_{\mathrm{c}}[\mathrm{mol/m^3}]$ — concentration at controlled input,
- $\mathbf{n}\cdot(-D\nabla c) = 0$ — output boundary condition,
- $\mathbf{N}\cdot\mathbf{n} = 0$ — for walls where molar flux $\mathbf{N}[\mathrm{mol/m^2 \cdot s}]$

For fluid flow :

- $\mathbf{u} = (0, -u_{\mathrm{t}}, 0)$ — constant inlet,
- $\mathbf{u} = (u_{\mathrm{c}}, 0, 0)$ — controlled inlet,
- $p_0 = 0$ — pressure at output,
- $\mathbf{n}\cdot\mathbf{n} = 0$ — inlet sections,
- $\mathbf{u} = 0$ — walls.

transport of the reactants at the outlet and dispersal in the main direction of the convective flow was neglected.

# Simulations results – concentration level



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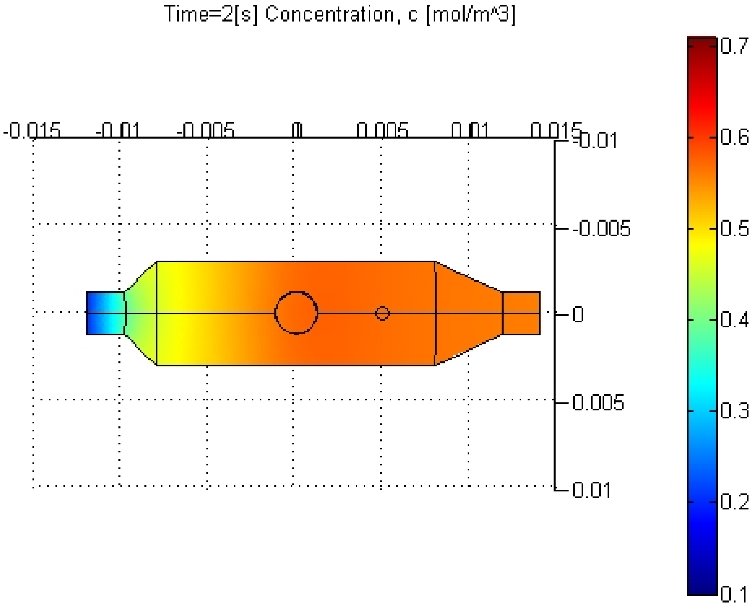

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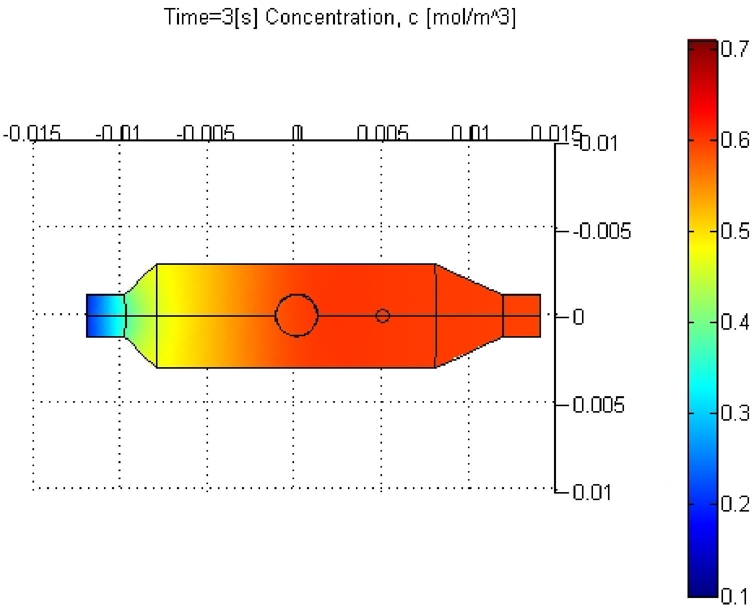

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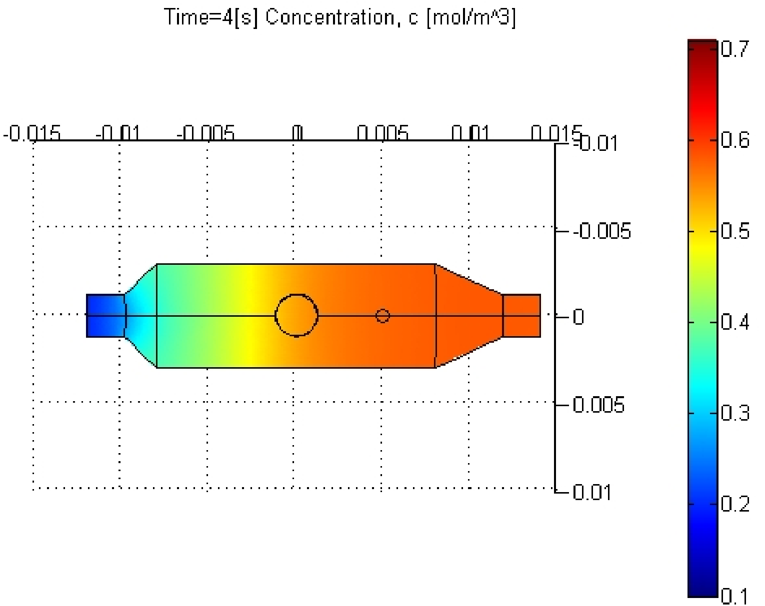

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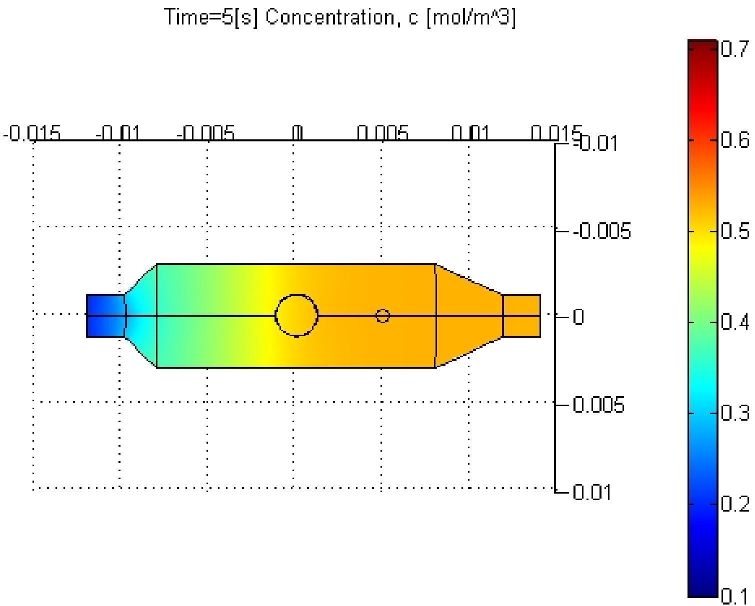

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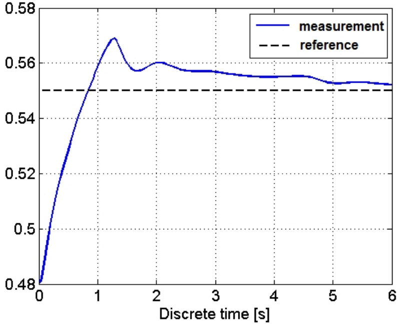
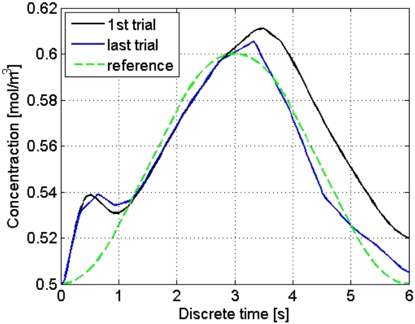



Simple PID and constant reference tracking




Iterative Learning Control in $1^{\text{st}}$ and last trial

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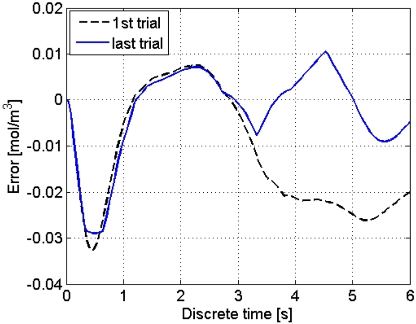
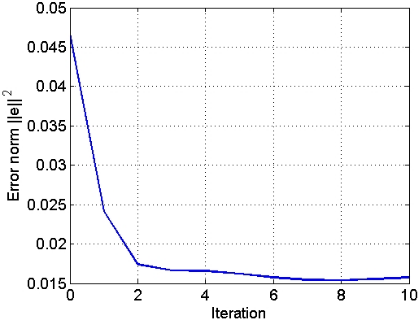



Error reached in $1^{\mathrm{st}}$ and last trial of ILC




Error norm in each trial of ILC

# Conclusion

- Summary of the contributions provided by this work to the state-of-the-art:
  - iterative learning control for distributed parameter system was presented as an promising approach for the improvement of control quality
  - control scheme was illustrated on the application to the fluid dynamics with the mass transport as an example of real chemical process.
- Further work:
  - more general methodology for combining sequential design and ILC techniques in order to increase the control quality for processes,
  - extensions to more wider range of systems,