Calculating the dissipation in fluid dampers with non-Newtonian fluid models

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September 4, 2015

Abstract

Present paper gives a comparison of the Maxwell, Upperconvected Maxwell and the Oldroyd-B model for the calculation of dissipation in high shear-rate cases. Useage of viscodampers in the automotive industry is the most common. There is a good scope of the computing this power in the case of Newtonian fluids. When a polymeric liquid is considered that part of energy that is irreversible can not be calculated as $P_{diss.} = \boldsymbol{\tau} : \mathbf{d}$. For fluids where the separation into a solvent and a polymer part is not available the deformation gradient tensor must be separated into two parts. One part consists of only the elastic deformation while the other is the non-elastic. This paper shows this separation using the three-element Maxwell and the UCM model. The steady state temperature distribution of a damper then is validated with measurement.

Keywords: fluid damper, dissipation.

1 Introduction

Present paper gives a comparison of the Upperconvected Maxwell (UCM) and the Oldroyd-B model for the calculation of dissipation in high shear-rate cases of viscodampers. When polymeric liquid is considered that part of energy that is irreversible can not be calculated as $P_{diss.} = \boldsymbol{\tau} : \mathbf{d}$, where $\mathbf{d} = \frac{1}{2}\left(\nabla\mathbf{v} + \nabla\mathbf{v}^T\right)$. The calculation is well known [1], [2],[3], [4],[5], [6], [7], [8], [9], [10]. This method is quite simple for Newtonian fluids and for the Maxwell model. For polymeric liquids, whose stress cannot be separated into a solvent and a liquid part, this computation leads to high errors if the dissipation is calculated as given in (9). The Maxwell model is widely used by engineers because of its simplicity. On the other hand this can not be used as a constitutive equation because it is not objective.For fluids where the separation into a solvent and a polymer part is not available the deformation gradient tensor must be separated into two parts.

2 Use of COMSOL Multiphysics©

To implement the UCM model into the FE model custom PDEs were used. In order not to rewrite the equation of fluid flow some parts have been eliminated and others have been added. The non-newtonian material model is attached as a source term. Due to high shear rates heat generation plays an important role in the modeling process. This coupling is easily achieved using COMSOL Multiphysics. Both CFD and heat transfer is considered and the PDE interface is also used.

1. Continuity equation

$$\nabla\mathbf{u} = 0, \tag{1}$$

where ∇ denotes the divergence operator.

2. Equation of motion

$$\frac{\partial}{\partial t}\rho\mathbf{u} = -\nabla\rho\mathbf{u}\mathbf{u} - \nabla\boldsymbol{\pi} + \rho\mathbf{g}, \tag{2}$$

where π is total stress tensor,

3. Energy equation

$$\frac{\partial}{\partial t}\rho U = -\nabla\rho U\mathbf{u} - \nabla\mathbf{q} - \boldsymbol{\pi} : \nabla\mathbf{u}, \tag{3}$$

where ρ denotes density, U is the internal energy per mass unit, $\mathbf{q}$ is the heat flux and the last term is dissipated power.

The equation of an equally linear and elastic fluid is the Maxwell model

$$\boldsymbol{\tau} + \lambda \dot{\boldsymbol{\tau}} = -2\mu_0 \mathbf{d}, \tag{4}$$

where $\lambda = \frac{\mu_0}{G}$ is the time constant (relaxation time) and μ_0 is the zero shear rate viscosity. This model is not suitable to use as a constitutive equation so the Upper Convected Maxwell (UCM) and Oldroyd-B model are introduced

$$\boldsymbol{\tau} + \lambda \overset{\triangledown}{\boldsymbol{\tau}} = -2\mu_0 \mathbf{d}, \tag{5}$$

$$\boldsymbol{\tau} + \frac{\mu_0}{G}\left(\frac{\partial \boldsymbol{\tau}}{\partial t} + \mathbf{u}.\nabla\boldsymbol{\tau} - (\nabla\mathbf{u})^T\boldsymbol{\tau} - \boldsymbol{\tau}(\nabla\mathbf{u})\right) = -2\mu_0\mathbf{d}. \tag{6}$$

The equation of the Oldroy-B fluid is

$$\boldsymbol{\tau} + \lambda \overset{\triangledown}{\boldsymbol{\tau}} = -2\mu_0\left(\mathbf{d} + \lambda_1 \overset{\triangledown}{\mathbf{d}}\right), \tag{7}$$

where λ_1 is the retardation time. The problem introduced in this paper used cylindrical coordinate system. Due to the axial symmetry the following simplification can be taken

$$\frac{\partial}{\partial \varphi} \equiv 0. \tag{8}$$

For those polymers that can be separated to a solvent and a polymer section, such that $\mu = \mu_s + \mu_p$ holds, the dissipation can be calculated as

$$P_{diss.} = \boldsymbol{\tau} : \mathbf{d}. \tag{9}$$

The original calculation leads to high errors if the dissipated power is needed.

The Maxwell model cannot be used in this way. The material law of such fluid cannot be arrenged for the stress $\boldsymbol{\tau}$ in a closed form so it cannot be inserted into Eq. (9). So the other way is to try to calculate the anoter term in Eq. (9) to calculate the plastic deformation. Different material models can be built by springs and dashpots connected parallel and/or in series. If this model contains dashpots and not only springs the calculation of the viscous dissipation needs only those deformations that occur in the dashpots. The deformation of the springs are reversible. So the rate of strain tensor should be separated. Suppose that the deformation gradient tensor $\mathbf{d}$ can be written as

$$\mathbf{d} = \dot{\boldsymbol{\gamma}}_d + \dot{\boldsymbol{\gamma}}_s, \tag{10}$$

where $\dot{\boldsymbol{\gamma}}_d$ is the rate of strain tensor of the dashpot and $\dot{\boldsymbol{\gamma}}_s$ is the rate of strain of the springs.

Namely the rate of strain tensor can be separated into the sum of two tensors. One describes the deformation of the elastic parts of the model while the other the plastic deformation. By definition, analogous to Hooke's law in strength of materials the reversible part is

$$\dot{\boldsymbol{\gamma}}_s := \frac{1}{G}\frac{\partial \boldsymbol{\tau}}{\partial t}, \tag{11}$$

$$\frac{\mathcal{D}}{\mathcal{D}t}\boldsymbol{\alpha} = \frac{D}{Dt}\boldsymbol{\alpha} + \frac{1}{2}\left(\boldsymbol{\beta}\cdot\boldsymbol{\alpha} - \boldsymbol{\alpha}\cdot\boldsymbol{\beta}\right), \tag{12}$$

where $\boldsymbol{\beta} = \nabla\mathbf{u} - (\nabla\mathbf{u})^T$ and $\boldsymbol{\alpha}$ is arbitrary tensor.

Thus for a fluid with arbitrary material constitutive equation the viscous dissipation can be calculated as

$$P_{diss.} = \boldsymbol{\tau} : \frac{1}{2}\dot{\boldsymbol{\gamma}}_d, \tag{13}$$

where

$$\dot{\boldsymbol{\gamma}}_d = \frac{1}{2}\left(\nabla\mathbf{v} + \nabla\mathbf{v}^T\right) - \frac{1}{G}\left(\frac{\partial\boldsymbol{\tau}}{\partial t} + \mathbf{v}\nabla\boldsymbol{\tau}\right). \tag{14}$$

3 Finite element implementation

3.1 The weak term

In the present study, we consider an UCM model with three relaxation times given in Table 1.

For the numerical calculations we use the standard Galerkin finite element discretisation of the Oldroyd-B equation with least squares-type weak stabilization

terms Behr et al. The stabilization terms defined as

$$\tau_{mom} = \left(\left(\frac{2||\mathbf{u}^h||}{h} \right)^2 + \left(\frac{4\mu}{\rho h} \right)^2 \right), \quad (15)$$

$$\tau_{cont} = ||\mathbf{u}^h|| h \zeta \left(Re_e \right), \quad (16)$$

$$\tau_{cons} = \left(1 + \left(\frac{2\lambda||\mathbf{u}^h||}{h} \right)^2 + \left(\lambda||\mathbf{u}^h|| \right)^2 \right)^{0.5}, \quad (17)$$

where the element Reynolds number and the smoothing function is given by

$$Re_e = \frac{\rho||\mathbf{u}^h||h}{2\mu}, \quad (18)$$

$$\zeta \left(Re_e \right) = \begin{cases} \frac{Re}{3} & 0 \leq Re \leq 3 \\ 1 & Re \geq 3 \end{cases}, \quad (19)$$

where h is the element length.

3.2 The computational domain

For the problem presented here both Dirichlet and Neumann boundary conditions are used as

$$\mathbf{u} = \mathbf{g} \quad \text{on} \quad \Gamma_g, \quad (20)$$

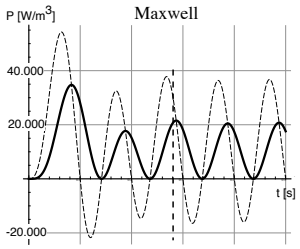
$$\mathbf{n}\boldsymbol{\tau} = \mathbf{h} \quad \text{on} \quad \Gamma_h, \quad (21)$$

where Γ_g, Γ_h are subparts of the domain.

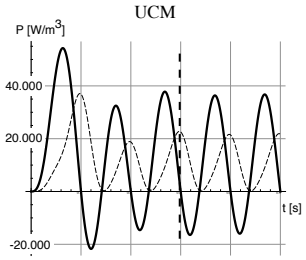
The main purpose of this calculation is to validate the calculation of the dissipation for the material models. The calculation of viscous dissipation like a Newtonian fluid leads to errors. Only the dashpots in the model gives irreversible energy. So during FEM calculations only the irreversible part of the material model is used for the dissipation. In this section the new method is compared with results from measurements. The parameters are given in Tab. 1.

Table 1: Parameters of the three element material model

parameter	value	parameter	value
G_1	8053.1 [Pa]	μ_1	360910[Pas]
G_2	33477 [Pa]	μ_2	180670[Pas]
G_3	64353[Pa]	μ_2	41649[Pas]



(a) Dissipated power for Maxwell model.



(b) Dissipated power for UCM model.

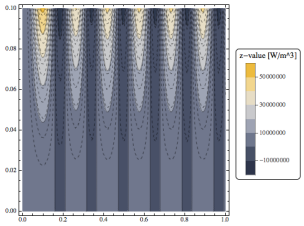
Figure 1: Comparison of the analytical calculations for the material models.

4 Results

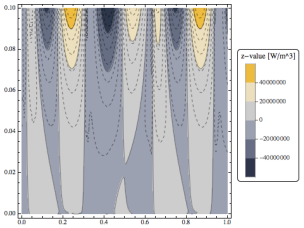
In Fig. 1a and 1b the comparison of the analytical calculations can be seen for the domain given in Fig. **??**. In Fig. 1a the dissipated power for Maxwell model calculated both as Newtonian (see dashed line) and non-Newtonian (see bold line) fluid can be seen. One can see that this method gives much lower values for the dissipated power. In Fig. 1b the dissipated power for UCM model calculated both as Newtonian and non-Newtonian fluid can be seen. Dashed line indicates the case when the dissipated power is calculated as non-Newtonian fluid. Solid line shows the case when it is calculated as Newtonian fluid. It gives a bit higher values than the Maxwell model computed with the same methodology. It can be observed that after the initial transient part the viscous dissipation reaches a steady-state.

In Fig. 2a the contours of the dissipation can be seen as a function of time (horizontal axis) and radial

distance (vertical distance) for the UCM model. In Fig. 2b the same result can be seen for the Oldroyd-B fluid. Both material models give similar results, specially at higher radius values. Due small difference in the dissipation and for simpler formula, in the finite element calculations the UCM model is applied.



(a) Dissipated power contours for UCM model.



(b) Dissipated power for Oldroyd-B model.

Figure 2: Comparison of the contours for the material models.

For the calculations the original 3D device was simplified to an axial symmetric model. The inner channel containing the fluid was also simplified. In Figure 4 the viscous dissipation can be seen. It shows that the energy increases towards the radius. In Figure 5 the dissipation can be seen as a function of time at the point marked with +. This is marked with red line. For the non-isothermal calculation the averaged value is used as an internal heat source as a function of radius

$$P_{a.}(r) = \frac{1}{T}\int_0^{t'} P_{diss.}(r)\, dt.$$

In Figure 6 the simulation results are compared to the data from measurement. The measuring points

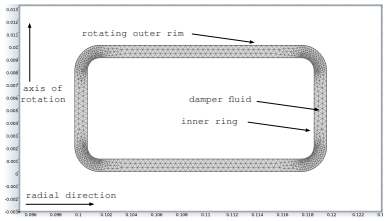


Figure 3: Schematics of a fluid damper and the finite element model.

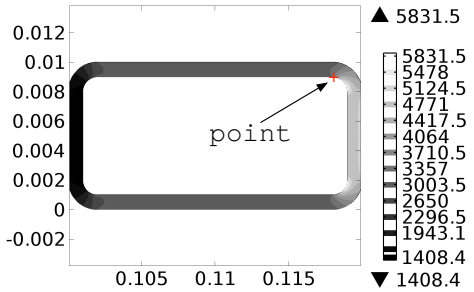


Figure 4: Power contours at a specific time. The specific point is marked with +.

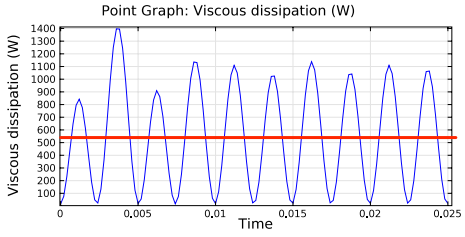


Figure 5: The viscous dissipation at the specific point.

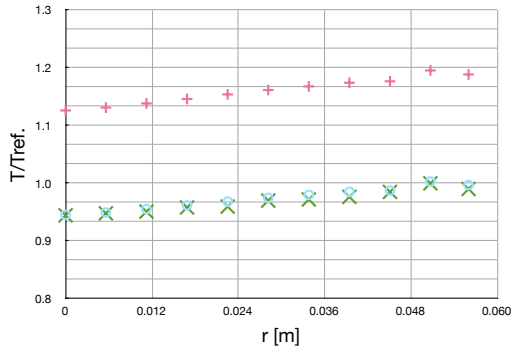


Figure 6: Temperature comparison along the outer face of the damper. The + marks the computed temperature using the Maxwell model, × marks the data from Oldroyd-B model and ◯ sign is the result if the fluid model is UCM.

of the damper are on the outer face of it. The results are correlated to the temperature values set by measurement, this is the reference $T_{ref.}$. One can see that both the UCM and Oldroyd-B models give very close results to the reference data. As expected the Maxwell model gives significantly higher values, because this is not suitable material model for computing the dissipation in damper fluids. For ease of use the UCM model is a good choice in sense of engineering calculations. Especially in the computation of heat transfer computations.

5 Conclusion

In this paper non-Newtonian material models were compared for the calculation of viscous dissipation. In COMSOL Multiphysics equation based modelling is available to use custom equations. After coefficient matching the non-linear material models can be easily implemented into the software. In the studied device more than one physics is needed to properly investigate the effects of different materials. Coupling these is just a few clicks and switching between the material models needs minimal effort.

References

[1] P. C. F.T. Pinho, Fully-developed heat transfer in annuli for viscoelastic fluids with viscous dissipation, Journal of Non-Newtonian Fluid Mechanics 138 (2006.) 7–21.

[2] G. M. K. L.E. Becker, The stability of viscoelastic creeping plane shear flows with viscous heating, Journal of Non-Newtonian Fluid Mechanics 92 (2000.) 109–133.

[3] K. R. I.J. Rao, On a new interpretation of the classical maxwell model, Mechanics Research Communications 34 (2007.) 509–514.

[4] Y. L. G.H. Wu, Creeping flow of a polymeric liquid passing over a transverse slot with viscous dissipation, International Journal of Heat and Mass Transfer 45 (2002.) 4703–4711.

[5] T. S. Ganbat Davaa, S. Momoki, Effect of viscous dissipation on fully developed heat transfer of non-newtonian fluids in plane laminar poiseuille-couette flow, International Communications in Heat and Mass Transfer 31 (2004.) 663–672.

[6] Y. S. Ryoichi Chiba, Masaaki Izumi, An analytical solution to non-axisymmetric heat transfer with viscous dissipation for non-newtonian fluids in laminar forced flow, Archive of Applied Mechanics 78 (2008.) 61–74.

[7] P. J. Oliveira, A traceless stress tensor formulation for viscoelastic uid ow, Journal of Non-Newtonian Fluid Mechanics 95 (2000.) 55–65.

[8] E. N. L. R. Byron Bird, Warren E. Stewart, Transport Phenomena, 2nd Edition, Wiley, 2006.

[9] O. H. R. Byron Bird, Dynamics of Polymeric Liquids, 2nd Edition, Vol. 1-2., Wiley-Interscience, 1987.

[10] F. A. Morrison, Understanding Rheology, 1st Edition, Oxford University Press, 2001.