

# Buckling Behavior of 3D Randomly Oriented CNT Reinforced Nanocomposite Plate

# Outline

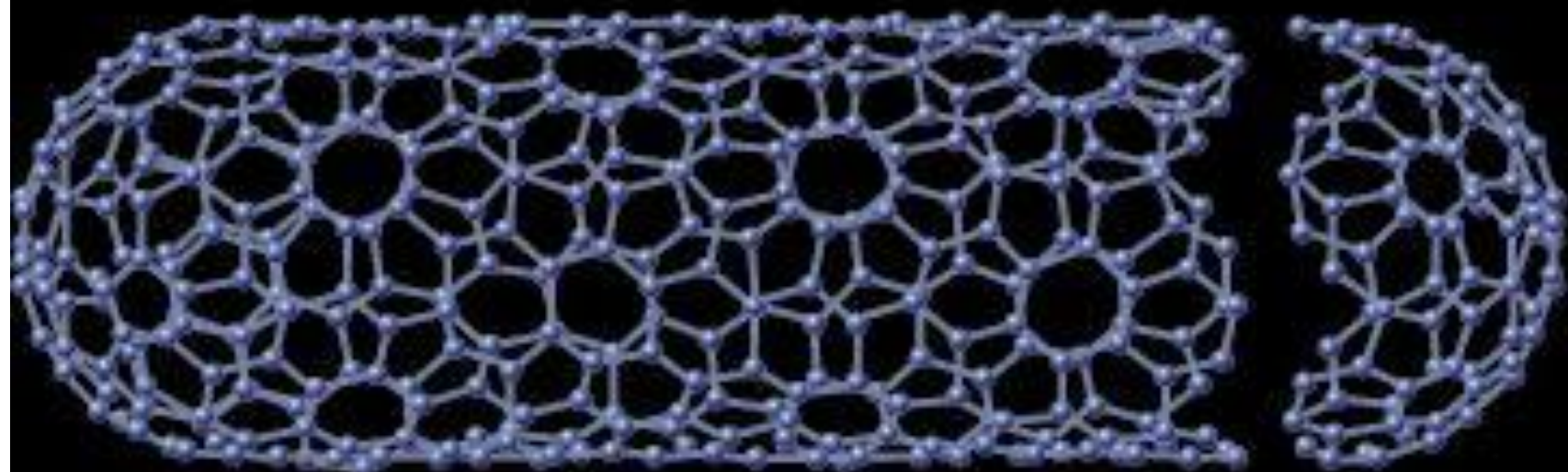
- Introduction
- Representative Volume Element (RVE)
- Periodic Boundary Conditions on RVE
- Homogenization Method
- Analytical Approach
  - Rule of Mixtures (ROM)
  - Classical Plate Theory (CPT)
- Current Study
  - Comparison of Elastic constants of CNT Composite with Rule of Mixtures (ROM)
  - Effective elastic properties of randomly oriented CNT-Mg nanocomposite
  - Buckling Load of Nanocomposite Plate
- Conclusions
- References

# Introduction

## Carbon Nano Tubes (CNT)

- Discovered in 1991 by Sumio Iijima [1], Japan
- Carbon nanotubes, are one of the most commonly mentioned building blocks of nanotechnology, with tensile strength much greater than steel, thermal conductivity better than purest diamond (almost twice), and electrical conductivity higher than copper wires. Thermally stable up to 2800<sup>0</sup>C in vacuum.
- Because of their superior electrical, thermal and mechanical properties, CNTs are potentially great reinforcing element in different matrix materials such as polymers, ceramics and metals, to enhance the overall properties of the resulting composite material.

# The Carbon Nanotube (CNT)



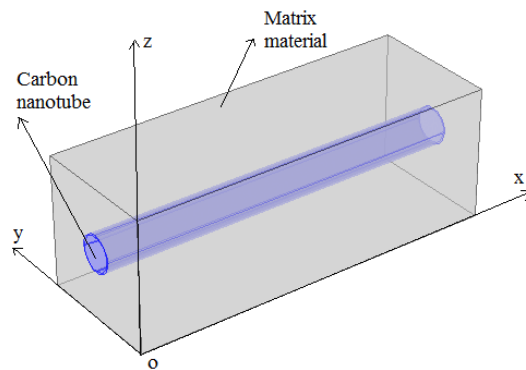
*A carbon nanotube is a structure one billionth of a meter in diameter. Because of their size, surface area, geometry and purity, carbon nanotubes are magnitudes stronger than the long-length solid fibers and metals used in manufacturing today.*

# Representative Volume Element

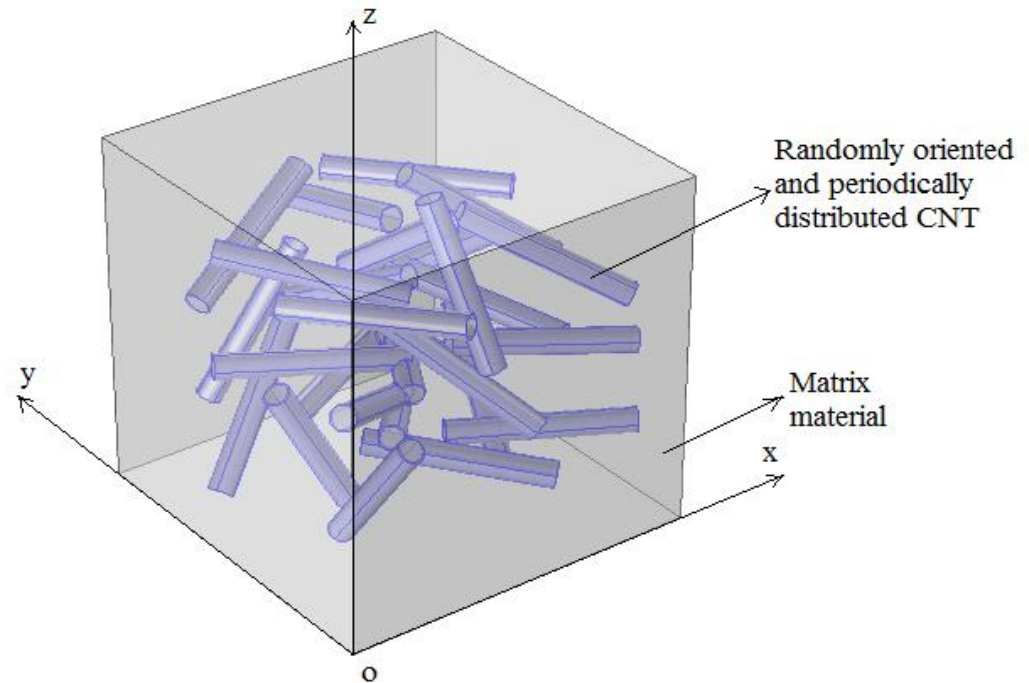
Multiscale methods are required to deal with the wide length and time scales of nanocomposites that vary from nano to macro level, by integrating the molecular dynamics and continuum mechanics approaches, which has posed many challenges to all researchers in the area.

The concept of representative volume element originally used for fiber reinforced composites [2] has also been carried forward in the characterization of nano scale materials. Liu and Chen [3] proposed three types of nanoscale representative volume element (RVE) i.e. cylindrical, square and hexagonal. It is shown that cylindrical RVE give good prediction of elastic properties of CNT nanocomposite in case of axisymmetric loading, when CNTs have different diameters, but cylindrical RVEs tends to overestimate the volume fraction of nanofillers in matrix material [4].

- Square and hexagonal RVEs are used when nano-fillers are distributed in the matrix in square and hexagonal arrays respectively.

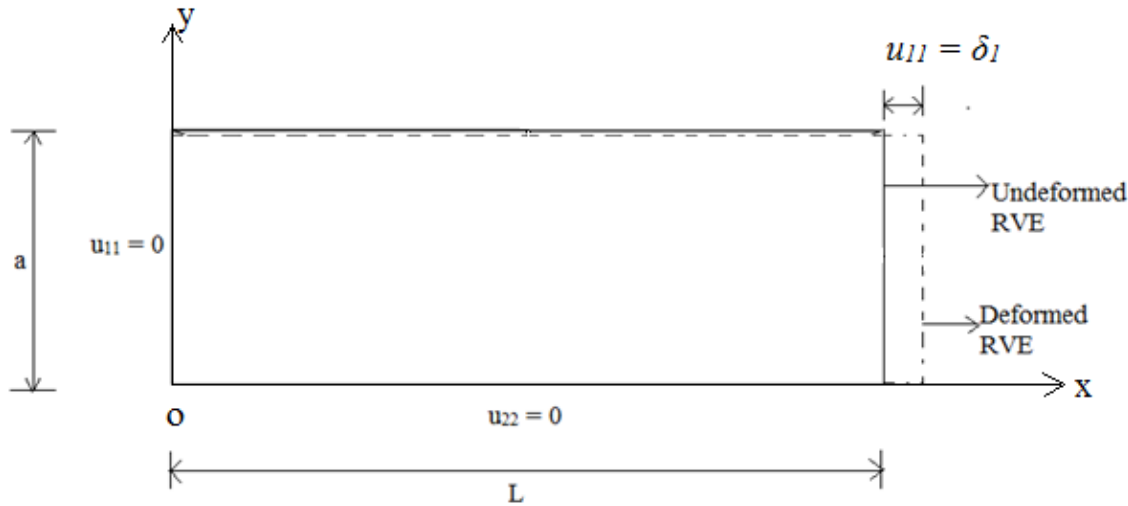


**Figure 1** Single CNT RVE for the  $v_f = 0.01$



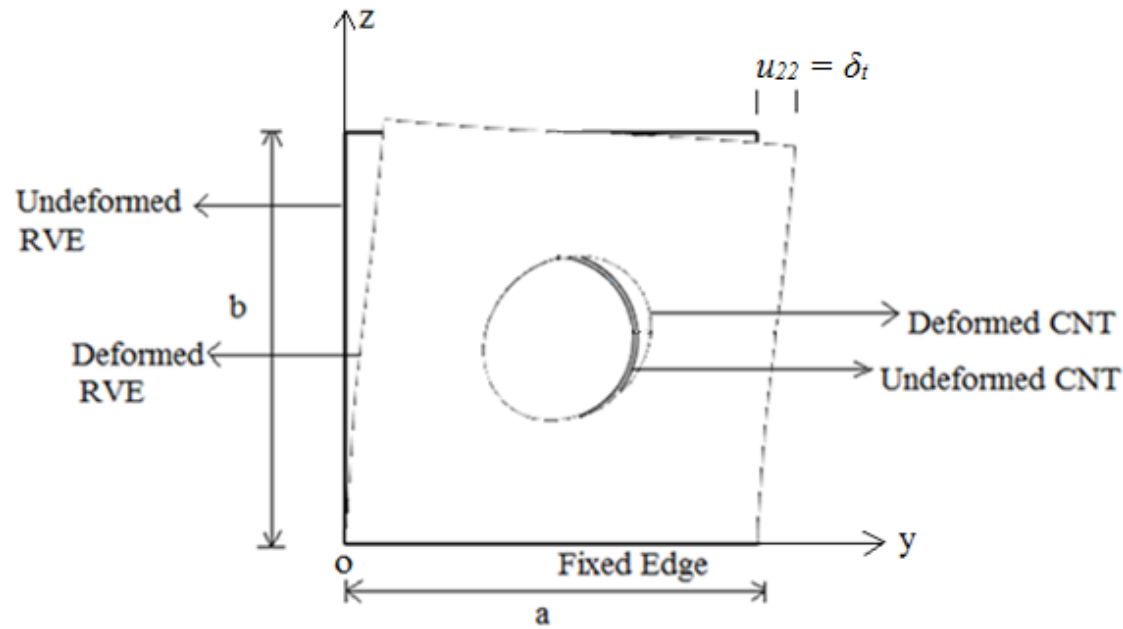
**Figure 2.** RVE consisting of randomly oriented and periodically distributed CNT for  $v_f = 0.01$

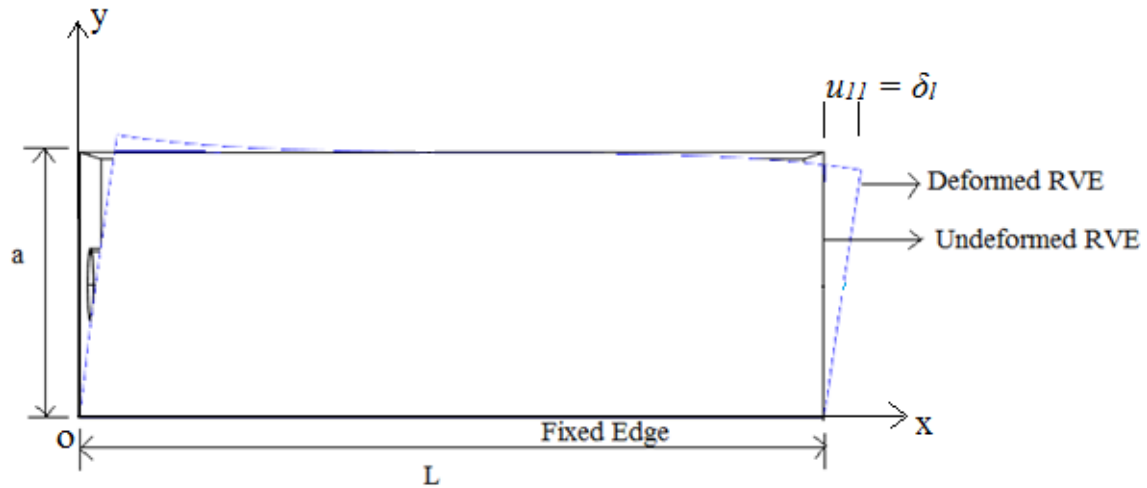
# Periodic boundary conditions on RVE



Typical RVE under normal loading in x-direction (for calculating  $E_1$ ,  $\nu_{12}$  and  $\nu_{13}$ )

RVE under transverse shear loading (for calculating  $G_{23}$ )





RVE under longitudinal shear loading (for calculating  $G_{12}$  and  $G_{13}$ )



# Homogenization Method for Evaluating Average Stress and Strain over the RVE

The effective (i.e., averaged) stiffness coefficient of nanocomposite can be calculated from,

$$\bar{\sigma}_{ij} = C_{ijkl}^e \bar{\varepsilon}_{kl}$$

Where  $C_{ijkl}^e$  refers to the effective stiffness tensor,  $\bar{\sigma}_{ij}$  and  $\bar{\varepsilon}_{kl}$  and are the volume-averaged stress and strain tensors calculated over the volume of the RVE using following integral expressions as:

$$\bar{\sigma}_{ij} = \frac{1}{V_{RVE}} \int_{V_{RVE}} \sigma_{ij}(x, y, z) dV$$

$$\bar{\varepsilon}_{kl} = \frac{1}{V_{RVE}} \int_{V_{RVE}} \varepsilon_{kl}(x, y, z) dV$$

In the present study, volume –average of the stress and the strains, for the homogenized material properties of nanocomposites is calculated using especial features of COMSOL Multiphysics.

# Analytical Approach

## 1. RULE OF MIXTURES:

Axial elastic modulus:  $E_1 = V_f E_f + (1 - V_f) E_m$

Transverse elastic modulus:  $E_2 = \frac{E_f E_m}{E_m V_f + (1 - V_f) E_f}$

In-plane shear modulus:  $G_{12} = \frac{G_f G_m}{G_m V_f + G_f (1 - V_f)}$

In-plane Poisson's ratio:  $\nu_{12} = \nu_f V_f + \nu_m (1 - V_f)$

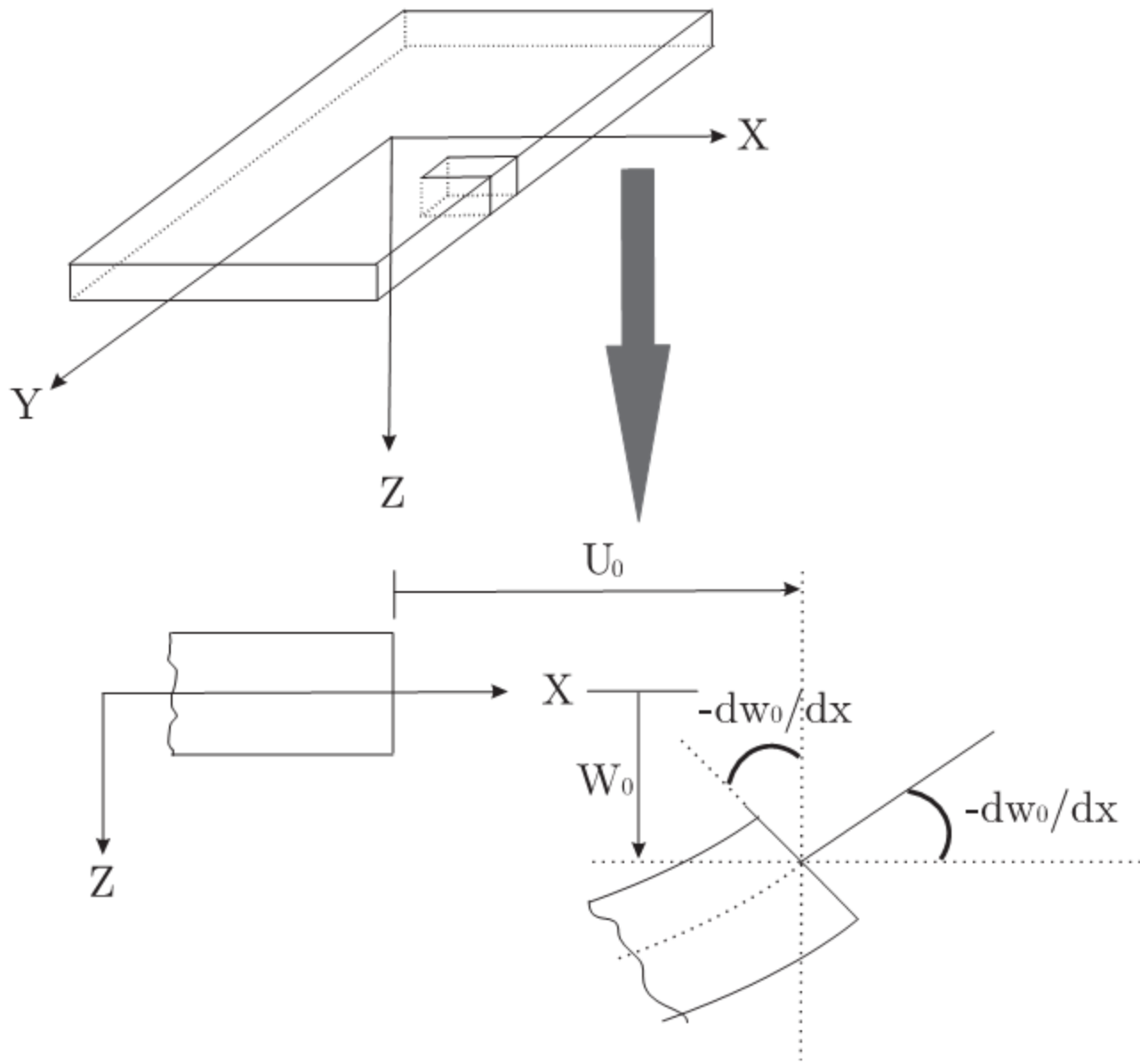
## 2. Classical Plate theory (CPT)

Increased use of nanocomposite in various structural applications necessitates the development of accurate theoretical models to predict their response. Buckling behavior of CNT reinforced plates is studied by different researchers using different methodologies [5,6]

A variety of plate theories have been proposed to study the buckling behavior of plates. The classical plate theory (CPT) provides acceptable results only for the analysis of thin plates and neglects the transverse shear effects.

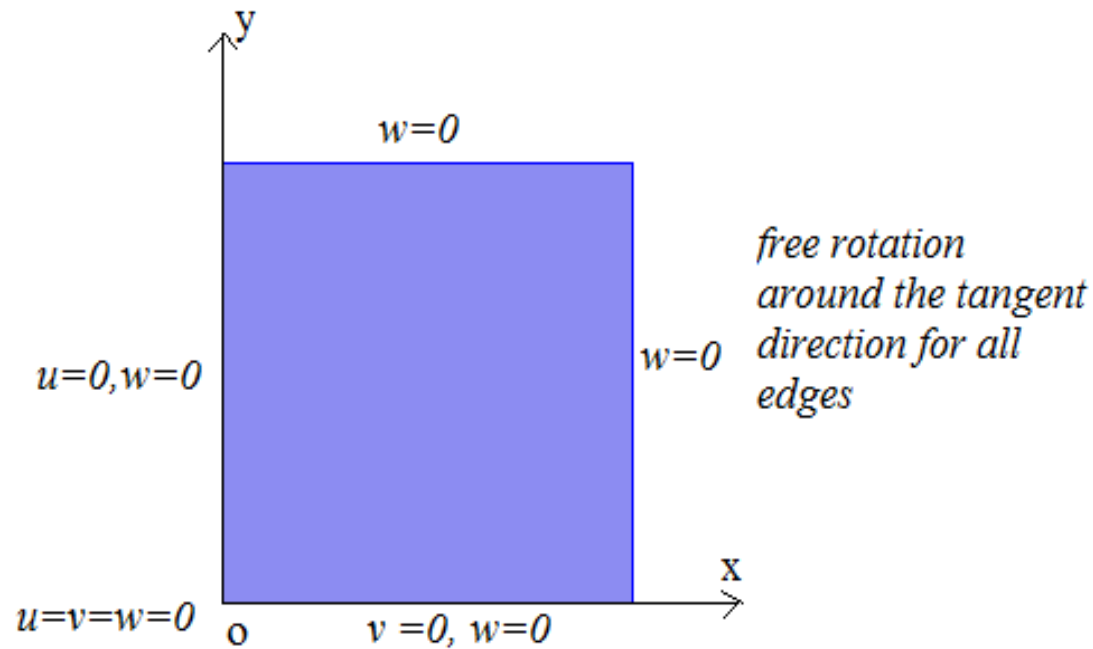
Assumptions of CPT:

1. Straight lines that are perpendicular to the midsurface (i.e., transverse normals) before deformation remain straight after the deformation.
2. The transverse normals do not experience elongation (i.e., they are inextensible).
3. The transverse normals rotate so that they remain perpendicular to the midsurface after the deformation.



Undeformed and deformed geometry of a plate according to the CPT.

# Boundary Conditions for All Edges Simply Supported Plate



# Comparison of Elastic constants of CNT nanocomposite obtained in the present study and the ROM

Elastic Constants	FEA results	ROM
$E_1$	54.5510	54.5499
$E_2=E_3$	47.2526	45.4338
$G_{12}=G_{13}$	18.4449	17.4745
$\nu_{12}$	0.2999	0.3000

# Effective elastic properties of randomly oriented CNT-Mg nanocomposite

$E_x$	$E_y$	$E_z$	$G_{xy}$	$G_{xz}$	$G_{yz}$	$\nu_{xy}$	$\nu_{xz}$	$\nu_{yz}$
48.2329	48.1131	47.8064	18.9878	18.6932	18.6764	0.2994	0.3036	0.3045

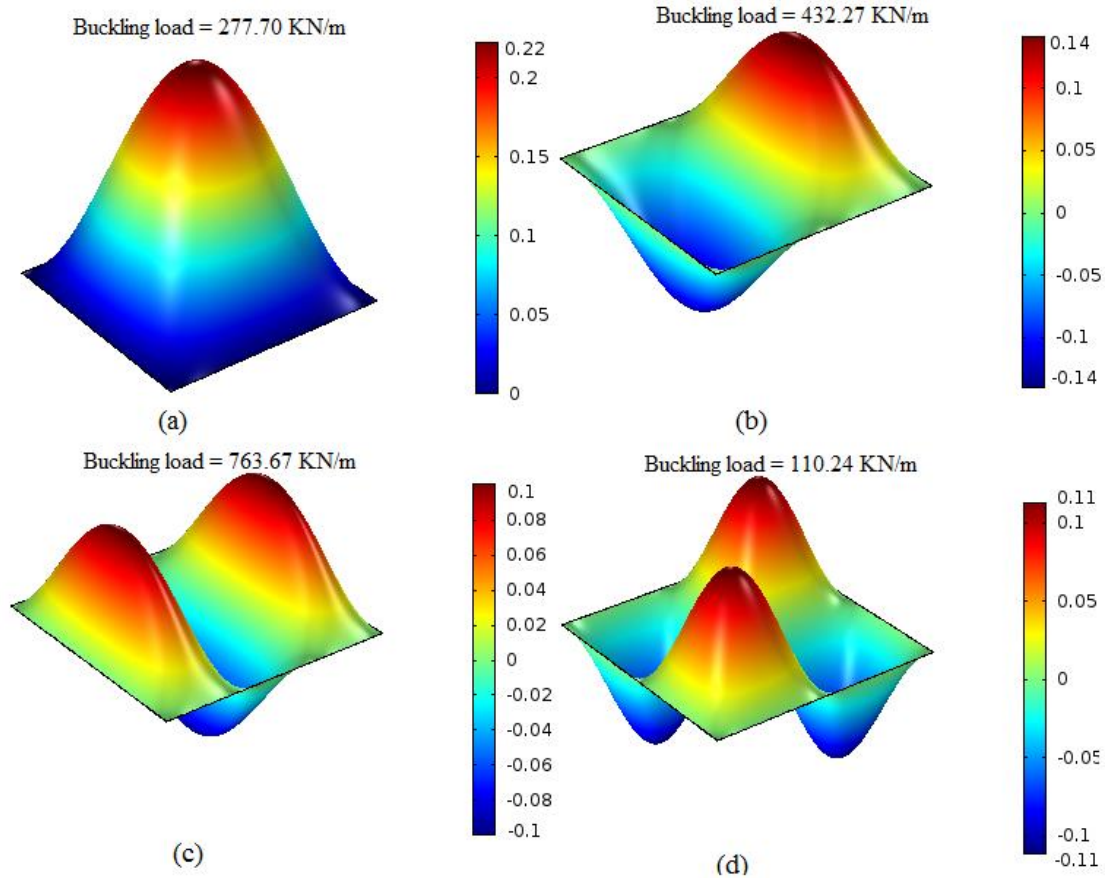
# Buckling Load of Nanocomposite Plate

Comparison of buckling strengths of pure Mg and CNT-Mg nanocomposite (having 1 % reinforcements) obtained using the COMSOL Multiphysics and the analytical formula based on CPT.

Plate Material	COMSOL Multiphysics (KN/m)	Analytical results based on CPT (KN/m)
Mg	259.64	260.2973
CNT-Mg Nanocomposite	277.70	278.4051



First four mode shapes of CNT-Mg nanocomposites plate having 1% CNT reinforcement with  $a/h = 50$ .



# Conclusions:

1. Effective elastic constants of randomly oriented and periodically distributed CNT reinforced Mg nanocomposite are predicted using cubical representative volume element subjected to different periodic boundary conditions. The obtained model shows the isotropic behavior that can mimic the actually developed isotropic nanocomposites.
2. The procedure is validated by comparing FEA result for the axial modulus with the corresponding analytical result obtained from mechanics of solid based rule of mixtures (ROM).
3. The effect of CNT reinforcement on the buckling behavior of a simply supported square plate is predicted using the *plate* physics section of COMSOL Multiphysics. Based on the present study, it can be concluded that reinforcement of CNTs enhances the stiffness properties of the metal matrix which in turn increases the buckling strength of the plate.

# References:

1. Iijima S. Helical Microtubules of Graphitic Carbon. *Nature* 1991; 354: 56–58.
2. Hyer, M.W., 1998. *Stress Analysis of Fiber-Reinforced Composite Materials*. McGraw-Hill, Boston
3. Liu, Y. , & Chen, X. (2003). Evaluations of the effective material properties of carbon nanotube-based composites using a nanoscale representative volume element. *Mechanics of Materials*, 35, 69–81.
4. Chen, X. L., & Liu, Y. J. (2004). Square representative volume elements for evaluating the effective material properties of carbon nanotube-based composites. *Computational Materials Science*, 29, 1–11.
5. Arani A. G, Maghamikia S, Mohammadimehr M, Arefmanesh A. (2011). Buckling analysis of laminated composite rectangular plates reinforced by SWCNTs using analytical and finite element methods. *J Mech Sci Technol*, 25:809–20.
6. Jafari Mehrabadi S, Sobhani Aragh B, Khoshkharesh V, Taherpour A. (2012). Mechanical buckling of nanocomposite rectangular plate reinforced by aligned and straight single-walled carbon nanotubes. *Compos Part B Eng*,43:2031–40.

Thank You

Queries ?