Design for Reliability and Robustness through probabilistic Methods in COMSOL Multiphysics with OptiY

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2nd European COMSOL Conference

November 4-6, 2008 in Hannover

Design Challenges

Variability, Uncertainty and Randomness

- A nominal Value of Design Parameter
- A arbitrary Stochastic Distribution

Causes

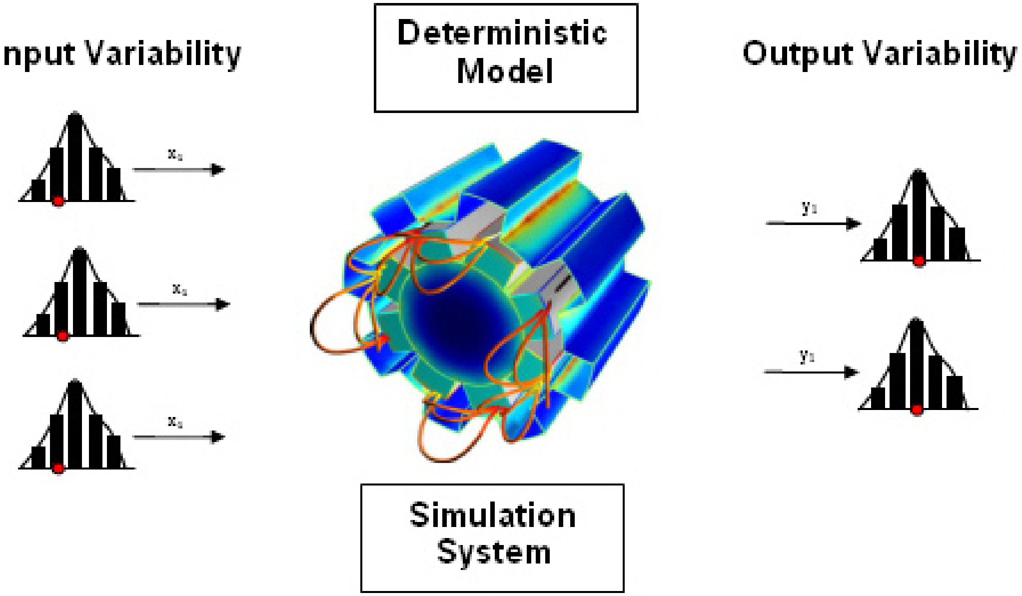
- Manufacturing Inaccuracy
- Material Property Scattering
- Environment Influences
- Process Uncertainty
- Human Factors etc.

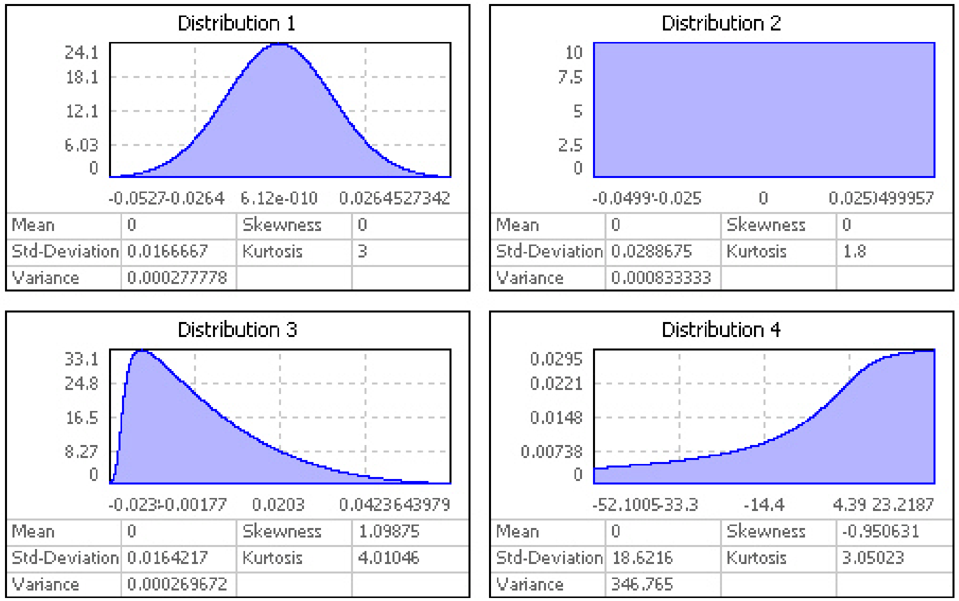
Problems

- Rejection at Manufacturing
- Low Reliability, Bad Quality
- Customers Warranty

Industry

- Prototypes (time and cost effort)
- Design of Experiment





Survey of Probabilistic Methods

- Monte-Carlo-Sampling (Stochastic Simulation)
- Response Surface Methodology
- Analytical Approach (Moment Methods)

Computing Output Distributions from Input Distributions based on the Deterministic Model (Input-Output-Relationship)

Exclusive Tools for Reliability and Robustness Investigation of a Product or a Process at the early Design Stage

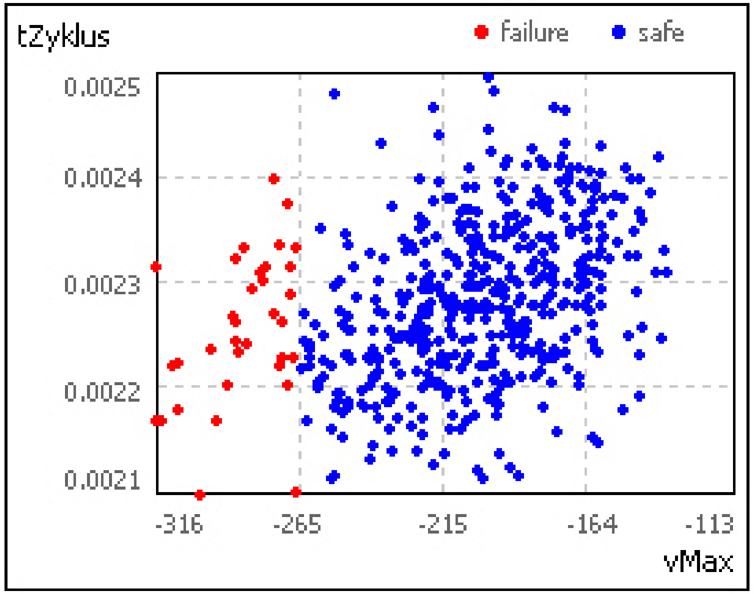
Monte-Carlo-Sampling

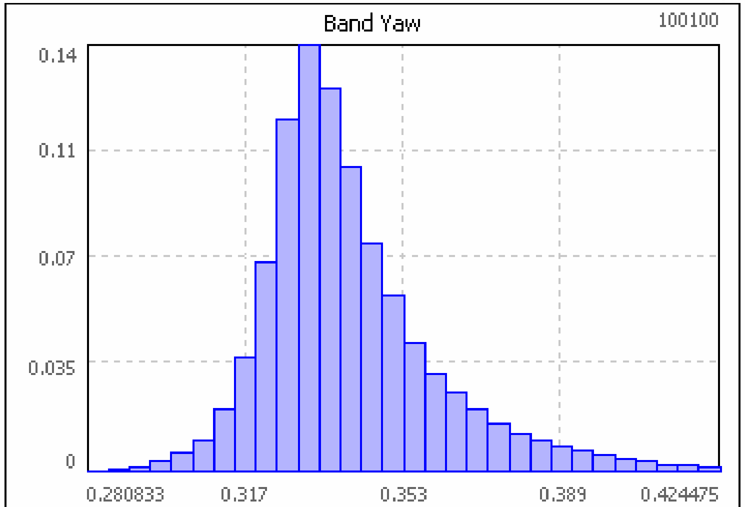
Principle (State of the art)

- Generate input distributions using random number generator
- Different types of random number generators: Plane Monte Carlo, Latin Hypercube, Sobol etc.
- Model calculation for each sample of the sample size
- Statistical evaluation of outputs after all model calculations

Advantages and Disadvantages

- Relationship between input and output is not required to deduce (black box model)
- Easy to implement and to use
- Converges very slowly at large sample size
- Thousands of model calculations required for accurate output distributions (long computing time)
- Results are stochastic and instable because of random numbers
- Not recommended for robust design optimization





Mean	0.342224	Skewness	2.49897
Std-Deviation	0.0258578	Kurtosis	18.3542
Variance	0.000668626		

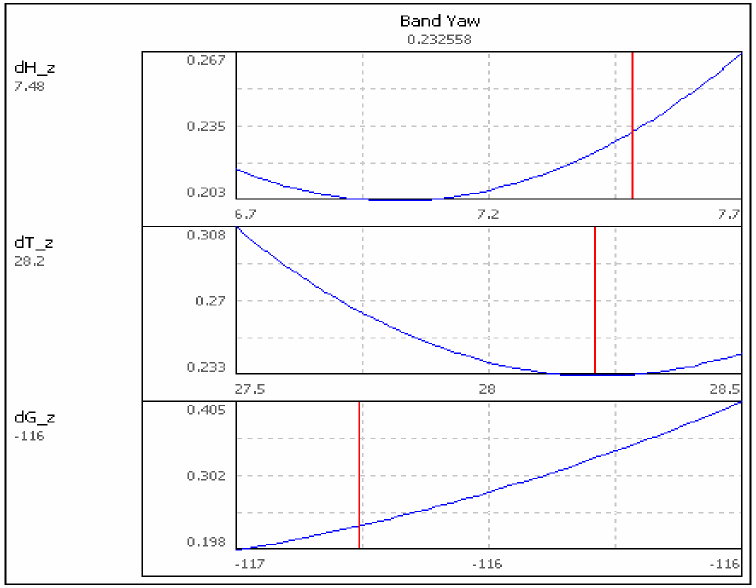
Response Surface Methodology

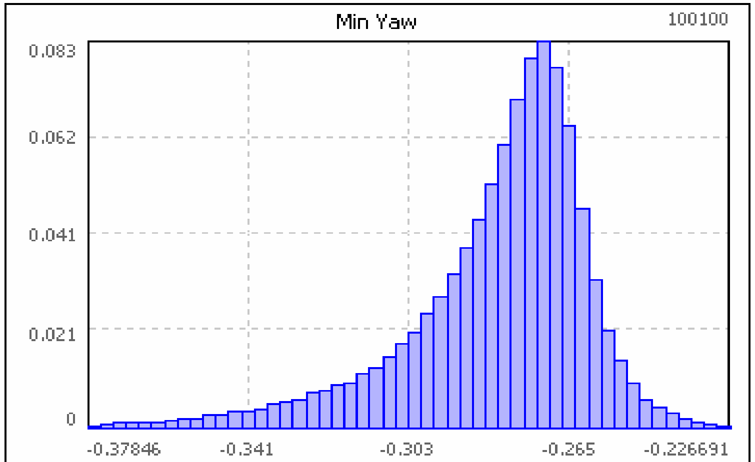
Principle (State of the art)

- Use a surrogate model (so-called meta-model) replacing the true input-output-relationship (model)
- Different meta-model types: Polynomial, Kriging, Radial Basis Function, Neural Network etc.
- Support points for meta-model using Design of Experiment: Monte Carlo Sampling, Factorial Design, Central Composite Design etc. (model calculations for each support point)
- Fit the meta-model with model calculations using Least Square Method.
- Output Distributions through a virtual sample based on the meta-model (very fast)

Advantages and Disadvantages

- High accuracy through a high-order approximation
- Fast computing time
- Results are also stochastic and instable because of random numbers
- Conditionally applicable for robust design optimization





Mean	-0.283753	Skewness	-2.47152
Std-Deviation	0.0282271	Kurtosis	17.1004
Variance	0.000796767		

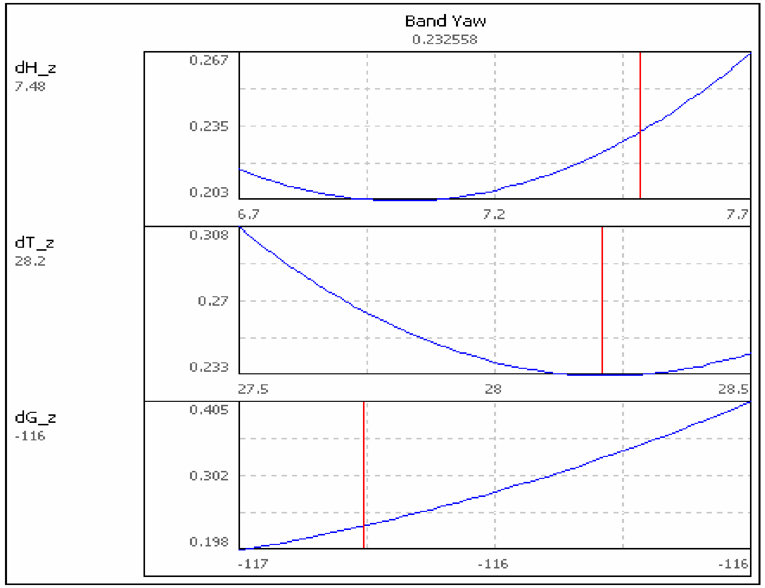
Analytical Approach (Moment-Methods)

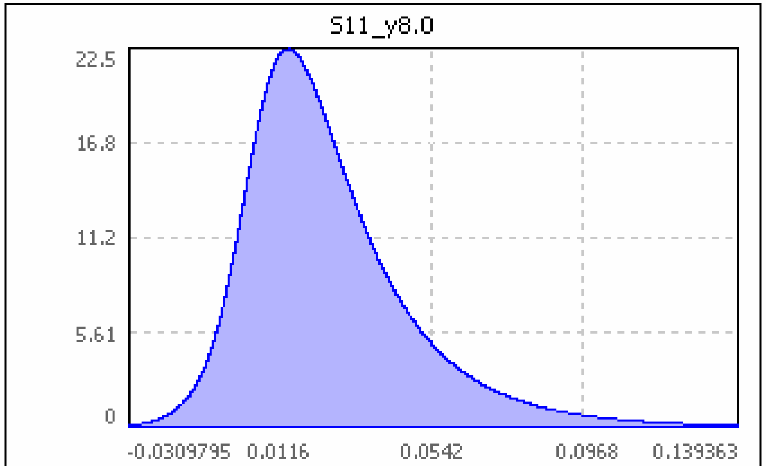
Principle (New)

- Using also meta-model replacing the true input-output-relationship
- Meta-model type is Polynomial (first and second order)
- Partial derivatives of model calculations are calculated for fitting of the meta-model
- Computing the statistical moments of output distributions from moments of input distributions
- Fitting output distributions using well-known table of moments

Advantages and Disadvantages

- Only second-order Approximation of the meta-model is possible (sufficient in the most cases of simulation)
- Fast computing time
- Results are analytically calculated and therefore very stable
- Very applicable for robust design optimization





Mean	0.0258336	Skewness	2.5269
Std-Deviation	0.0261531	Kurtosis	12.8524
Variance	0.000683986		

Design for Reliability and Robustness Based on the Output Distributions

Robustness Evaluation

- A good Product Quality is characterized by a small Change of Outputs due to the same Input Variability
- Robust Design evaluate the Variance of the Output Distributions:
  - A small Variance of Output = Robust Design
  - A large Variance of Output = Non-Robust Design

Reliability Analysis

- Investigate the Violation of Constraint boundaries due to Input Variability
- Calculate the Rejection at a Mass Manufacturing Process based on the Probability Density Function (PDF)
- Design Goal: minimal Rejection (better Quality)

Sensitivity Study

- Identify the important Influence Design Parameters
- Disregard insignificant Design Parameters
- Investigate Interactions between Input Parameters

Thin Film Resonator as MEMS-Example

Structure

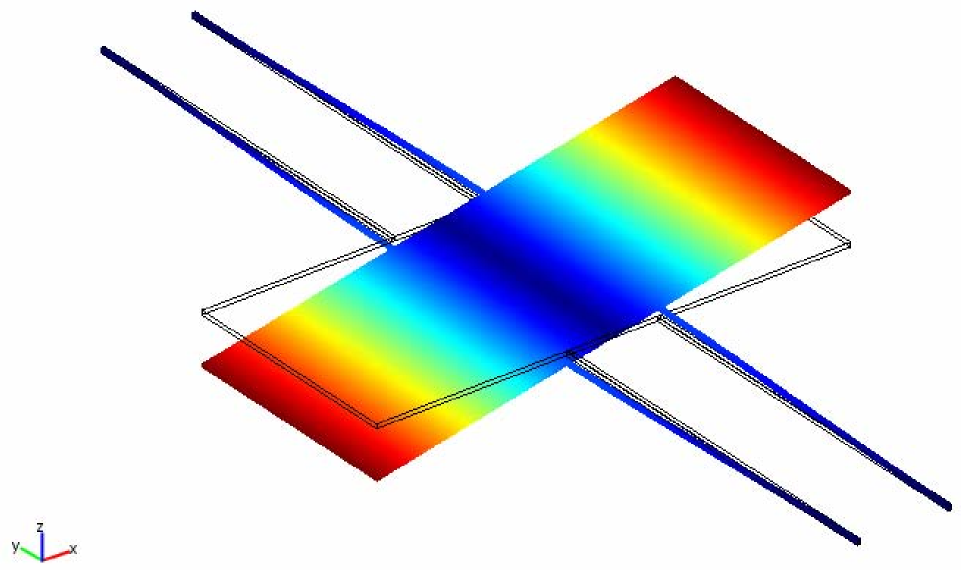
- A Shuttle
- 4 straight Cantilevers fixed outer
- Residual Stress

Failure Mechanism

- Problem-specific
- Characterized through the first resonant Frequency (f >18 kHz)

Design Problems (MEMS-Typical)

- The scattering of the design and process parameters is relatively large, compared to the dimensions of the system
- The functionality is highly influenced by the tolerances
- The materials is poorly characterized



Deterministic Simulation

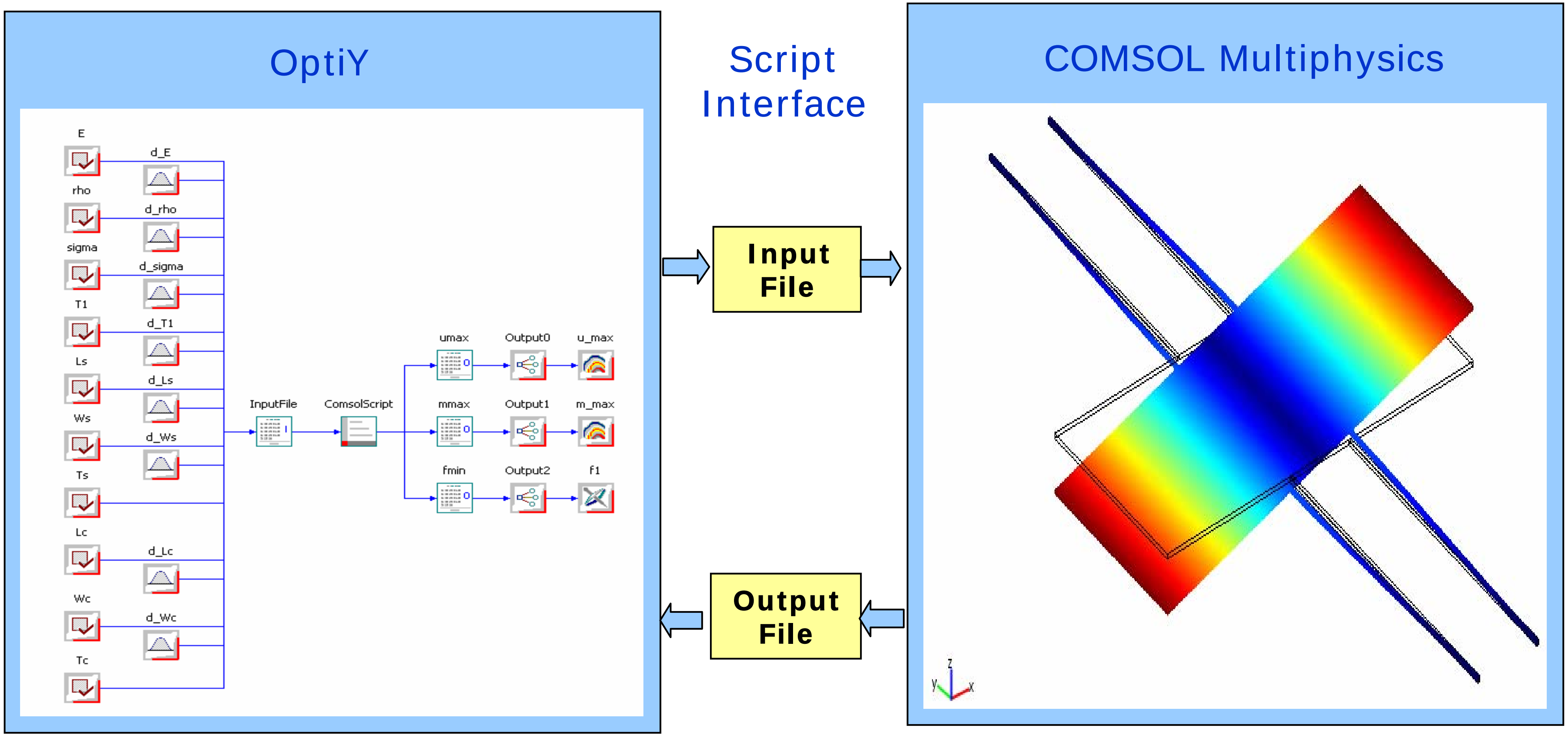
in COMSOL Multiphysics

First Resonant Frequency = 19 kHz

Design Parameters of Thin Film Resonator

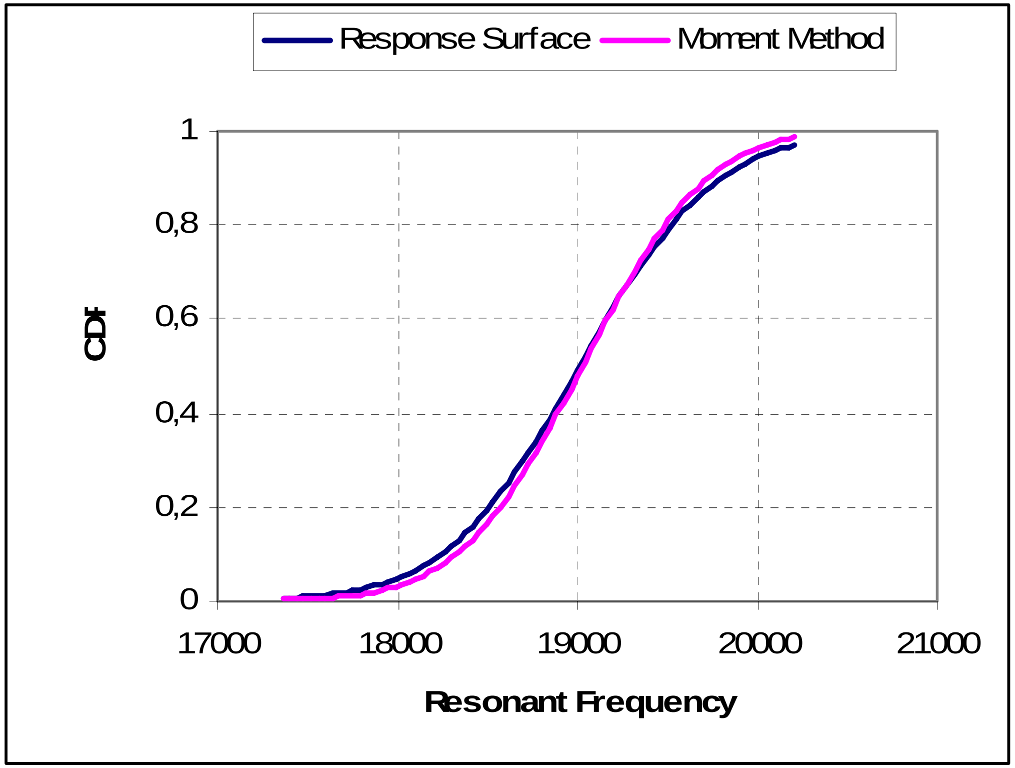
Design Parameter	Nominal Value	Tolerance Value	Distribution
Young's modulus **E** (GPa)	155	15.5	Uniform
Density **rho** (kg/m^3)	2330	200	Uniform
Residual stress **sigma** (MPa)	50	5	Uniform
Deposition Temperature **T1** (K)	678.15	200	Uniform
Shuttle Length **Ls** (µm)	250	0.5	Normal
Shuttle Width **Ws** (µm)	120	0.5	Normal
Cantilevers Length **Lc** (µm)	200	0.5	Normal
Cantilevers Width **Wc** (µm)	2	0.5	Normal

Coupling: OptiY - COMSOL Multiphysics



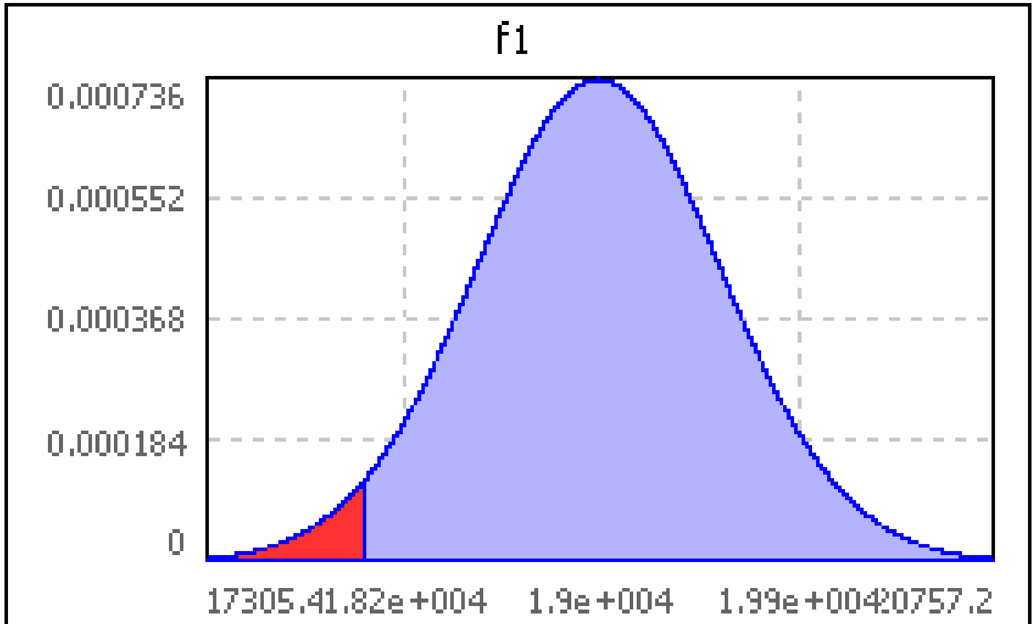
Compare: Response Surface vs. Moment Method

- Preliminary Investigation: Only linear Input-Output-Relationship between the First Resonant Frequency and the Material and Geometry Parameters
- Use the First-Order-Polynomial Meta-Model Without Interaction between inputs to obtain also a high accuracy
- 9 Deterministic Model Calculations for 8 stochastic Parameters
- Response Surface Methodology and Moment Method yield the same Results (Cummulative Distribution Function of the First Resonant Frequency)
- Demonstration for a high Accuracy and Correctness of both implemented Methods in OptiY



Probabilistic Analysis

- Inputs: 8 Distributions (Uniform and Normal)
- Output: Normal Distribution of the First Resonant Frequency
- Mean = 19 kHz
- Standard Deviation = 0.54 kHz (Robustness)
- Range between 17-20 kHz
- 3% Rejection at a Mass Manufacturing Process for the Requirement F1 > 18 kHz (Reliability)



Mean	19031.3	Skewness	0
Std-Deviation	545.475	Kurtosis	2.91772
Variance	297543	Partial Failure P	0.0301181

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Sensitivity Analysis

- Main Effect is the quotient of the Variance of Output Y caused by a single Input Variability Xi to the Variance caused by the Variability of all Inputs **X**:

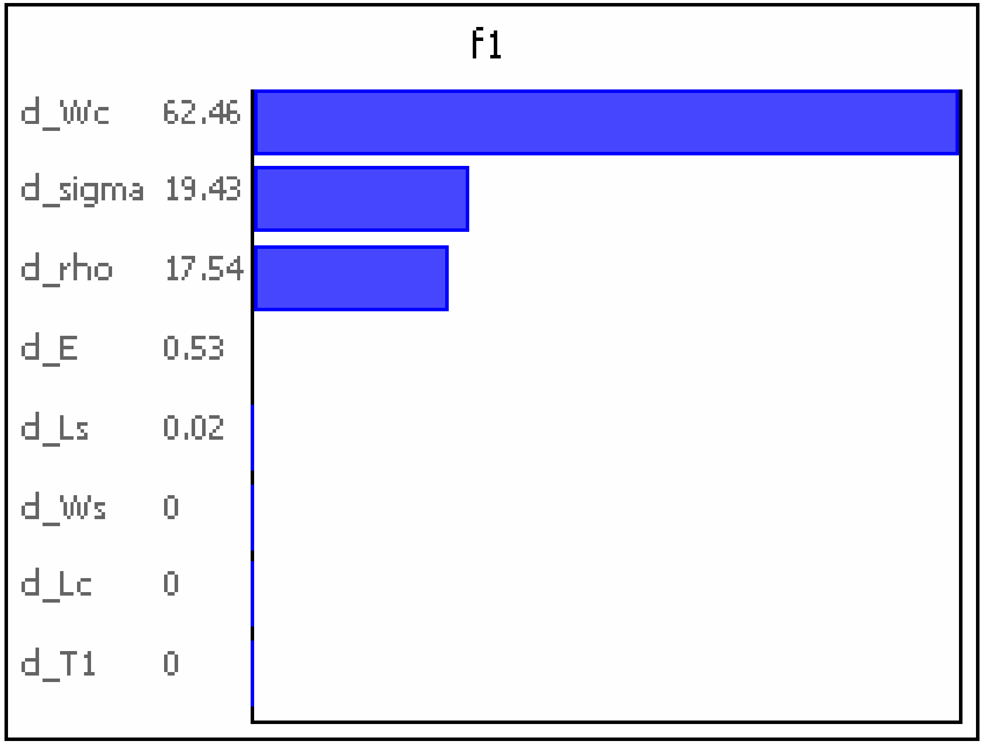
$$S_H = Var(Y|Xi)\ /\ Var(Y|X)$$

- Total Effect composes the Main Effect and the Interactions between Inputs:

$$S_T = S_H + Var(Y|Xi,Xj) / Var(Y|X)$$

- For Thin Film Resonator only the Main Effect considered because of using linear Input-Output-Relationship without Interaction (First-Order Polynomial)

- 3 Important Influence Parameters on the First Resonant Frequency: Shuttle Width, Residual Stress and Material Density



Conclusions

- Variability, Uncertainty and Randomness play an important part in the Design Process for Reliability and Robustness
- Probabilistic Methods are exclusive Tools for Reliability and Robustness Investigation of a Product or a Process at the early Design Stage
- Response Surface Methodology and Moment Methods are fast and best applicable for this Process
- Demonstration on the Thin Film Resonator shows the lack of deterministic Simulation today. Probabilistic Simulation brings the virtual Components closer to Reality
- **OptiY** for probabilistic Simulation is easy to connect to **COMSOL Multiphysics** for deterministic Simulation using the Script Interface