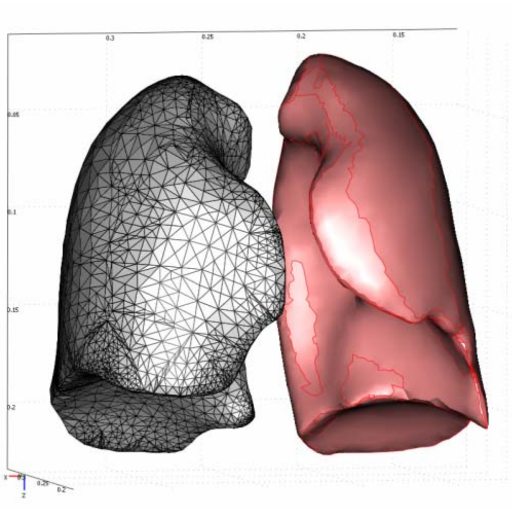
Presented at the COMSOL Conference 2008 Hannover

Modeling of Respiratory Lung Motion as a Contact Problem of Elasticity Theory



R. Werner, J. Ehrhardt, H. Handels

Department of Medical Informatics,
University Medical Center Hamburg-Eppendorf,
Hamburg, Germany

Motivation / Clinical Background

- Breathing-induced motion of lung tumors and organs is a significant source of error in radiotherapy of lung tumors
- Enlarging safety margins increases radiation dose delivered to healthy tissues

Motivation / Clinical Background

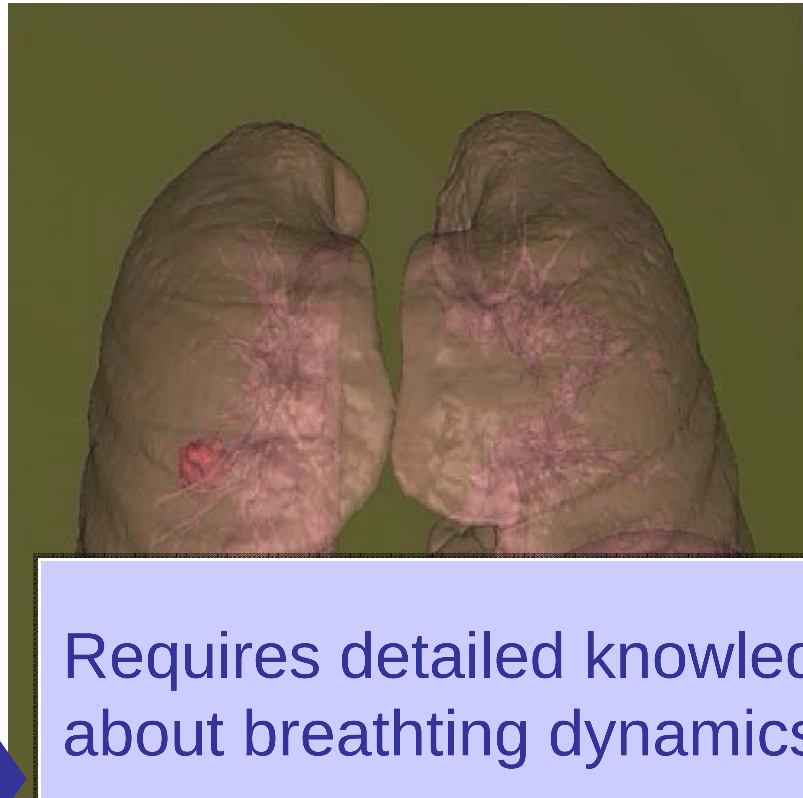
- Breathing-induced motion of lung tumors and organs is a significant source of error in radiotherapy of lung tumors
- Enlarging safety margins increases radiation dose delivered to healthy tissues
- **Searched for:**

  Methods to compensate for respiratory motion during radiation delivery

Motivation / Clinical Background

- Breathing-induced motion of lung tumors and organs is a significant source of error in radiotherapy of lung tumors
- Enlarging safety margins increases radiation dose delivered to healthy tissues
- **Searched for:**

  Methods to compensate for respiratory motion during radiation delivery



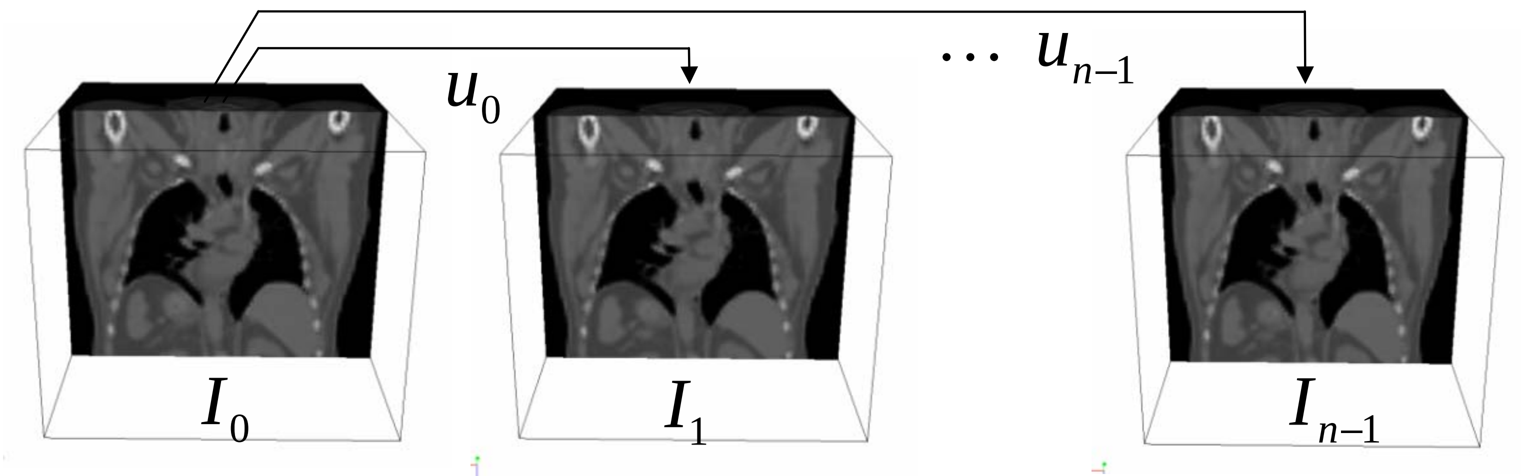
Requires detailed knowledge about breathting dynamics

⇒**Motion analysis**

⇒**Motion modeling**

Motivation / Clinical Background

Key aspect in modeling and motion analysis:
Estimation of motion fields in 4D (=3D+t) Data

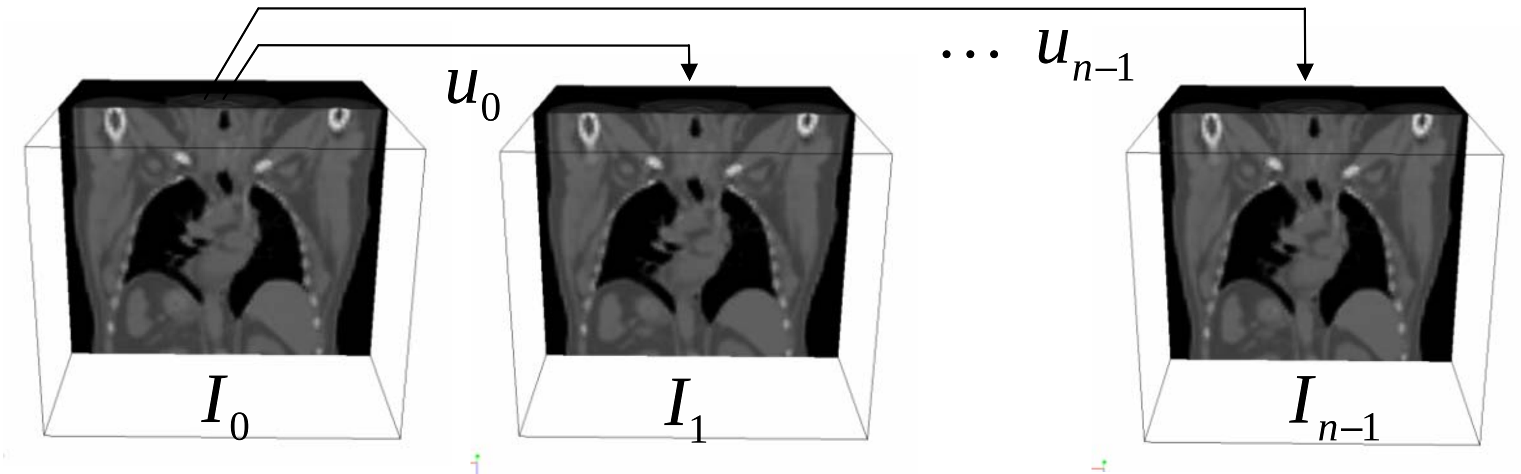


$I_i \subset R^3$ image data at different breathing phases

$u_i : I_i \to I_{i+1}$ motion field estimators

Motivation / Clinical Background

Key aspect in modeling and motion analysis:
Estimation of motion fields in 4D (=3D+t) Data

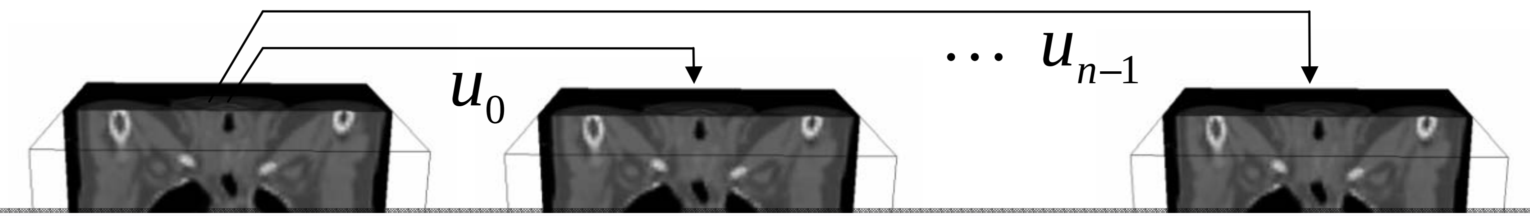


$I_i \subset R^3$	image data at different breathing phases
$u_i : I_i \rightarrow I_{i+1}$	motion field estimators

- Application 1: to quantify respiratory motion (Werner et al., Meth Inform Med 2007)
- Application 2: dose accumulation (Keall et al., Med Phys **33**, 2007)

Motivation / Clinical Background

Key aspect in modeling and motion analysis:
Estimation of motion fields in 4D (=3D+t) Data



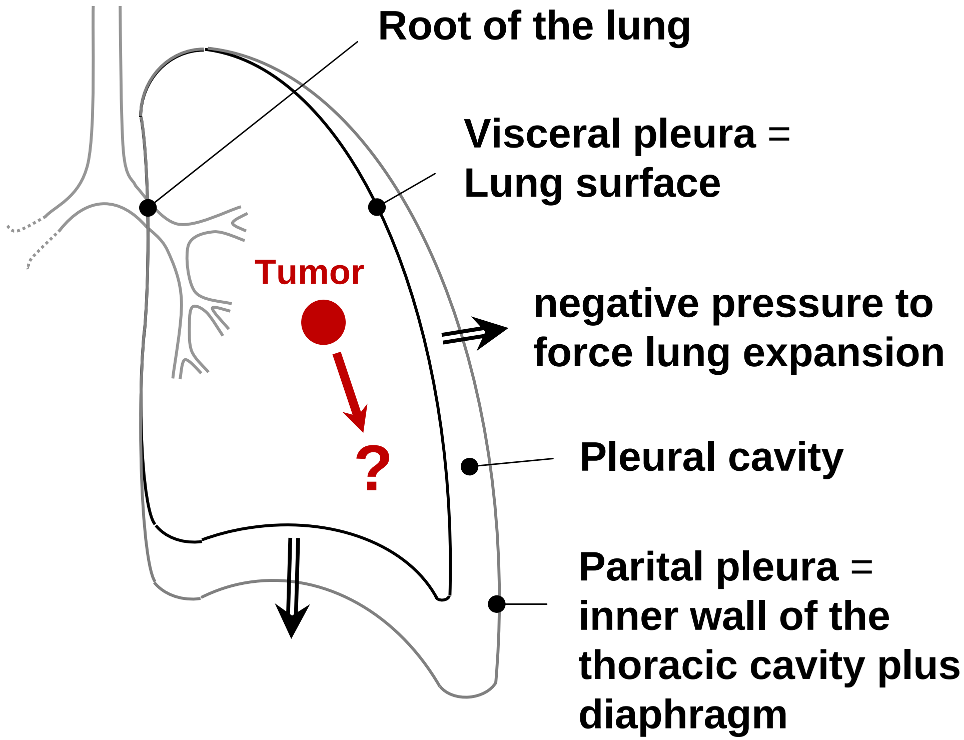
How to integrate (a-priori) knowledge about breathing anatomy and physiology into the modeling process?

- Application 1: to quantify respiratory motion (Werner et al., Meth Inform Med 2007)
- Application 2: dose accumulation (Keall et al., Med Phys **33**, 2007)

Methods: Modeling Approach

Aspect to model: Process of lung ventilation

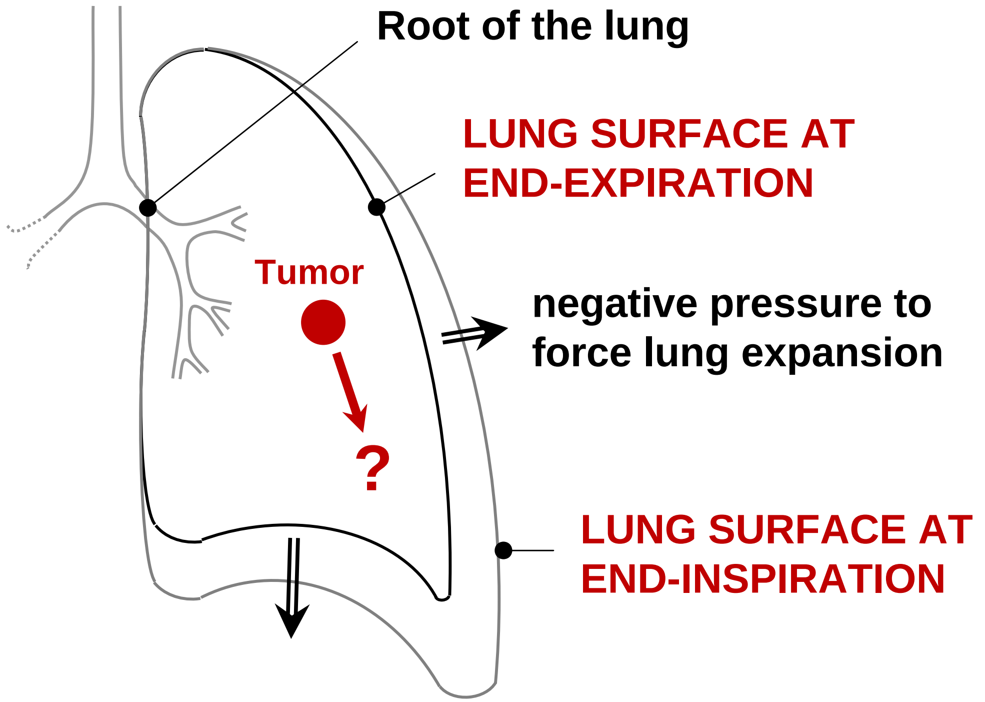
Contraction of breathing muscles
(diaphragm, intercostals)
⇓
Expansion of the thoracic cavity
⇓
Intrapleural pressure increases
⇓
Pressure states a surface force,
applied to the lung surface
⇓
Lung expands, following the
expansion of the thoracic cavity



Methods: Modeling Approach

Modeling idea: Lung ventilation as a contact problem

(Zhang et al., Medical Physics **31**, 2004; Sarrut et al., IEEE Transactions on Medical Imaging **26**, 2007)



Starting Point:

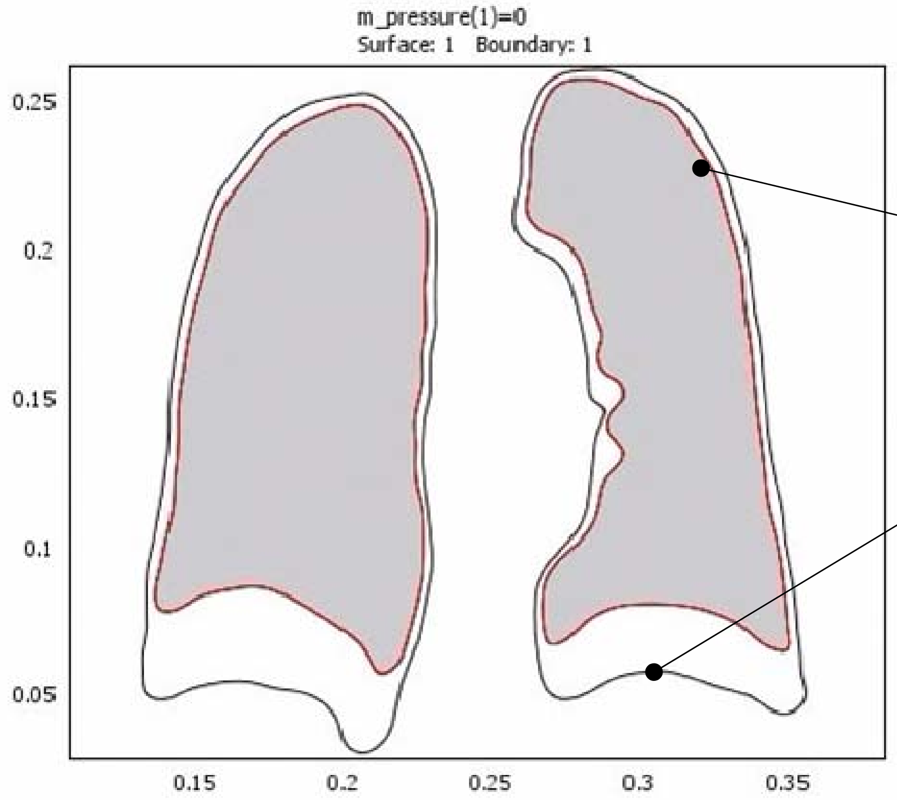
- Lung geometry at end-expiration (EE geometry)
- Lung geometry at end-inspiration (EI geometry)

Modeling Process:

- Apply pressure to expand EE geometry
- Increase the pressure until EE and EI geometries nearly match
- Searched for: Displacement field for the deformed state

Methods: Modeling Approach

Modeling idea: Lung ventilation as a contact problem



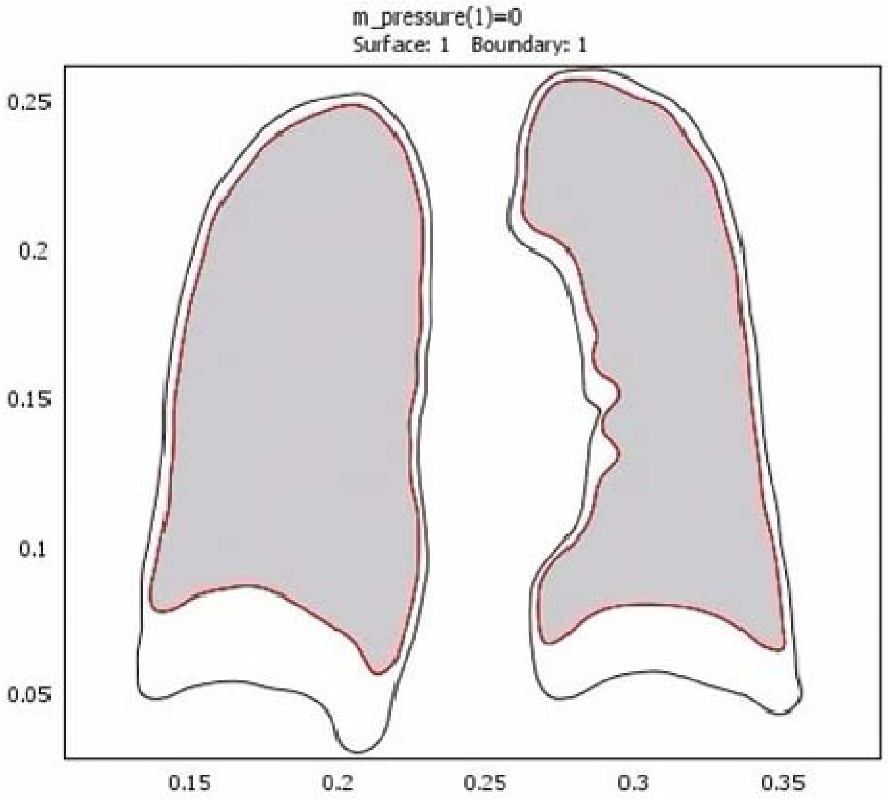
Initial situation

Lung boundary at end expiration
(red: deformed boundary)

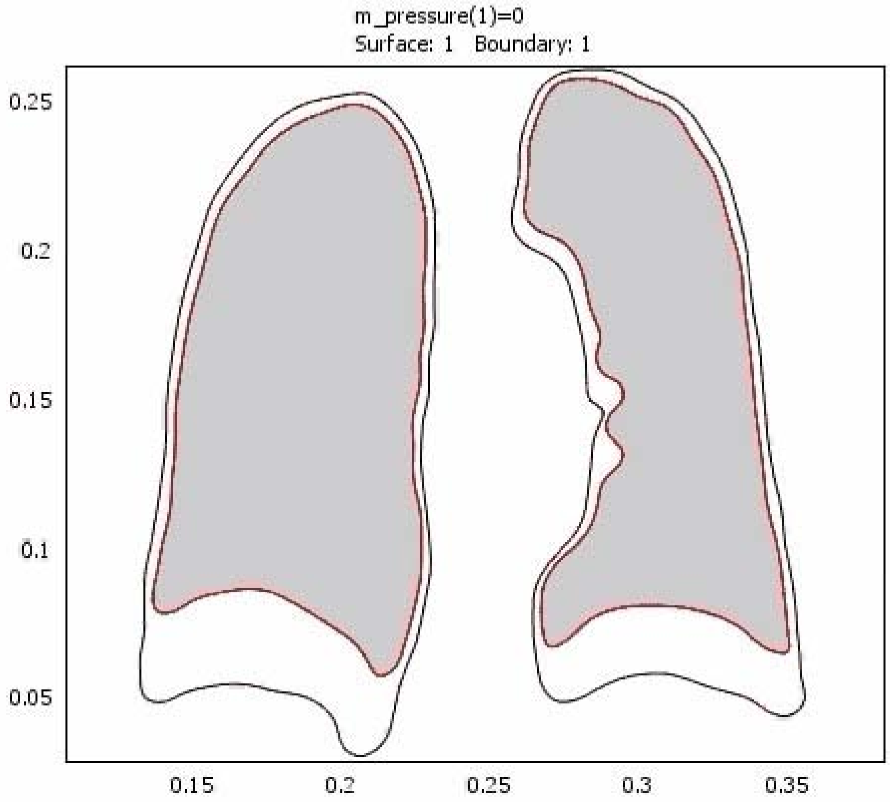
Lung boundary at end inspiration
(= limiting boundary)

Methods: Modeling Approach

Modeling idea: Lung ventilation as a contact problem



Initial situation



Modeling process

Methods: Modeling Approach

Modeling idea: Lung ventilation as a contact problem

- Searched for: $u : R^3 \to R^3$
- Equilibrium: $\nabla \cdot \sigma = 0$ with $\sigma = \frac{1}{\det F} FSF^T$

σ : Cauchy stress tensor
S : 2. Piola-Kirchhoff stress tensor
F = ∇u + I : Deformation gradient

- Constitutive equation (generalized Hook‘s law):

$$S = C(E, \nu)\varepsilon \quad \text{with} \quad \varepsilon = \frac{1}{2}\left(\nabla u + \nabla u^T + \nabla u^T \nabla u\right)$$

S : 2. Piola-Kirchhoff stress tensor
ε : Green-Lagrange strain tensor
C : elasticity tensor
E : Young‘s modulus
v : Poisson‘s ratio

Methods: Modeling Approach

Modeling idea: Lung ventilation as a contact problem

- Dirichlet BC's:

$u = 0$ for the limiting geometry and at the root of the lung

- von Neumann BC's:

$$\sigma n = p_{\text{int}}\, n + p_{\text{contact}}\, n$$

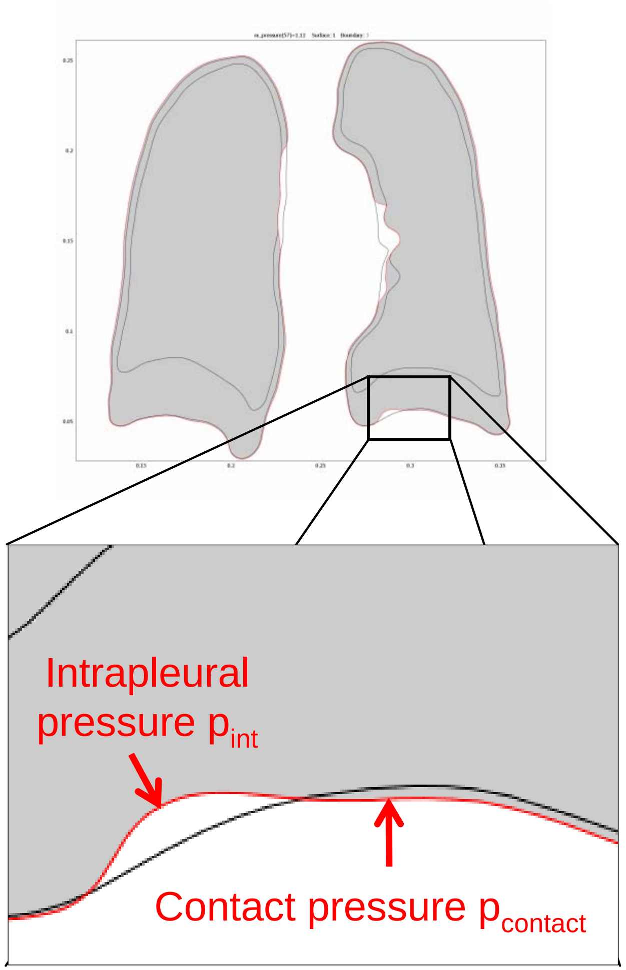
p_{int}: intrapleural pressure
n: surface normal
$p_{contact}$: contact pressure

- Signorini conditions:

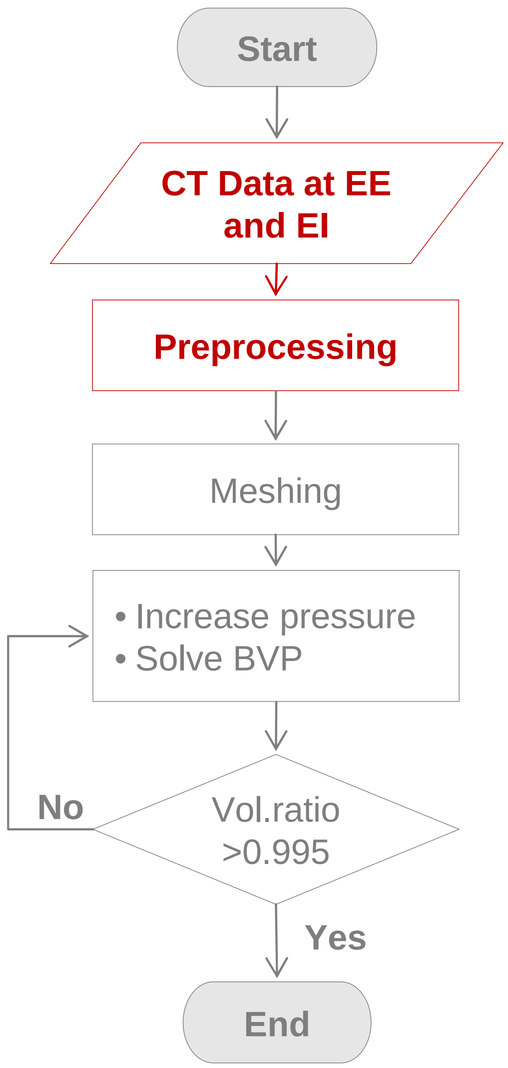
$$g \geq 0 \wedge p_{\text{contact}} \leq 0 \wedge p_{\text{contact}} \cdot g = 0$$

g: gap between EI and deformed EE geometries

Contact conditions

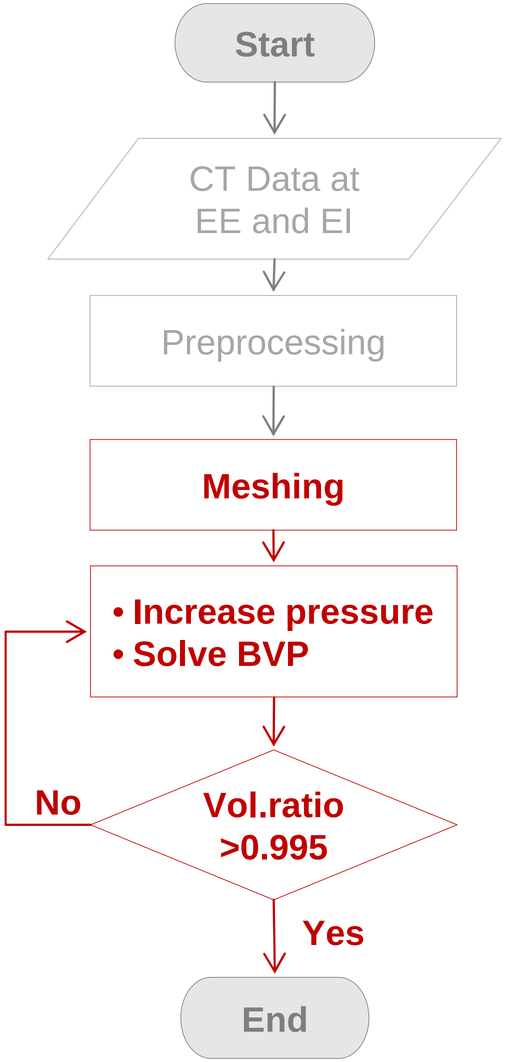


Methods: Implementation



- **Data base: 4D CT data**
  (Ehrhardt, Werner et al., Medical Physics 34, 2007)
  - 12 Lung tumor patients
    - 6 data sets with small tumors (Ø < 3 cm)
    - 3 with “middle sized” tumors (3 cm < Ø < 5 cm)
    - 3 with big tumors (Ø > 5 cm)
  - Spatial resolution: 1.0 x 1.0 x 1.5 mm
  - 14 breathing phases each patient
- **Preprocessing:**
  - Lung segmentation at EE and EI
  - Generation of lung surface models (Marching Cubes, Laplace smoothing)
  - Import of surface models to COMSOL via STL

Methods: Implementation

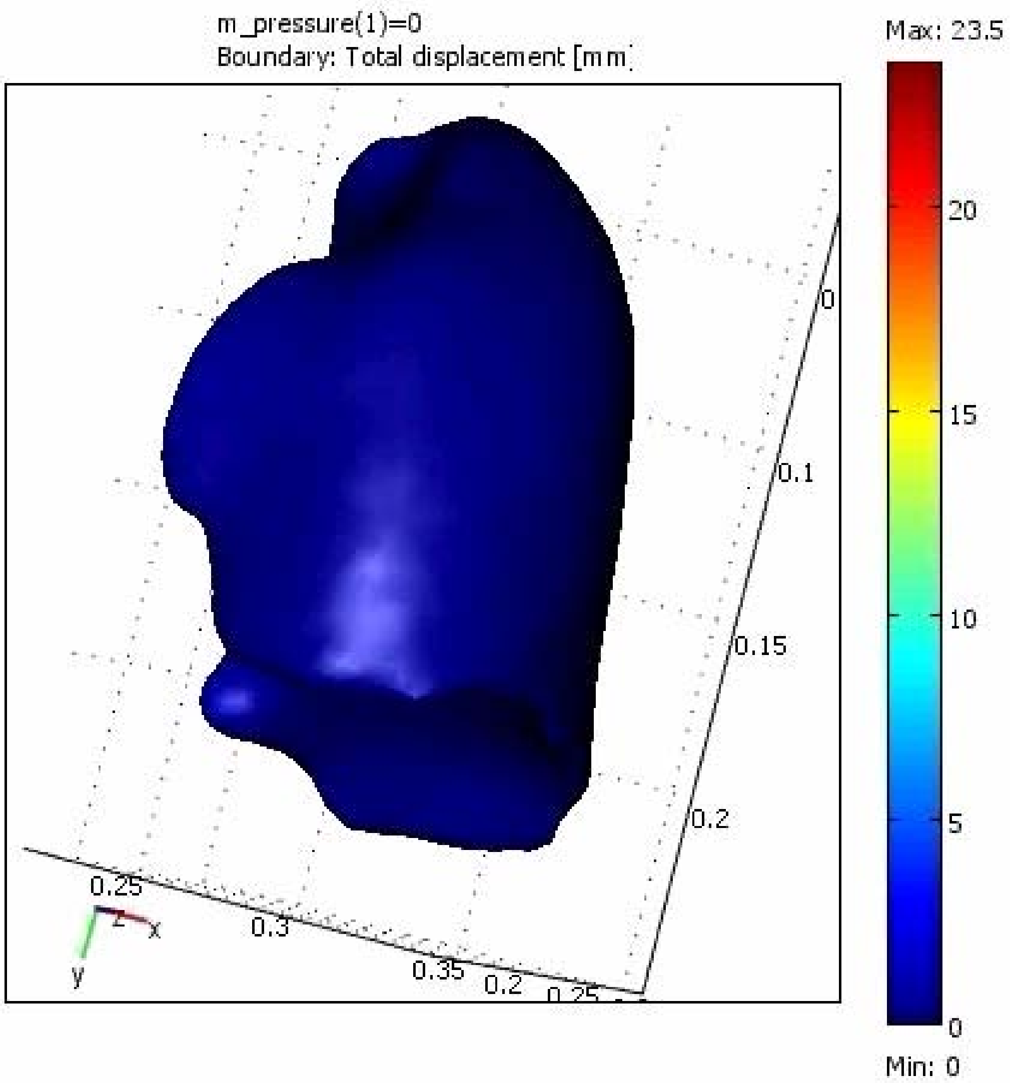


- **Meshing:**
  - Tetrahedral meshes, by COMSOL mesher
- **Solving process:**
  - Increase pressure gradually until:

$$\frac{\text{Volume of deformed EE geometry}}{\text{Volume of limiting geometry}} > 0.995$$

  - Each pressure value: Solve contact problem by the Augmented Lagrange algorithm
    - » Iterative process, solving the problem in a segregated way
    - » Part of the Structural Mechanics Module

Results



Modeling process: Expansion of the initial lung geometry

Color coded:

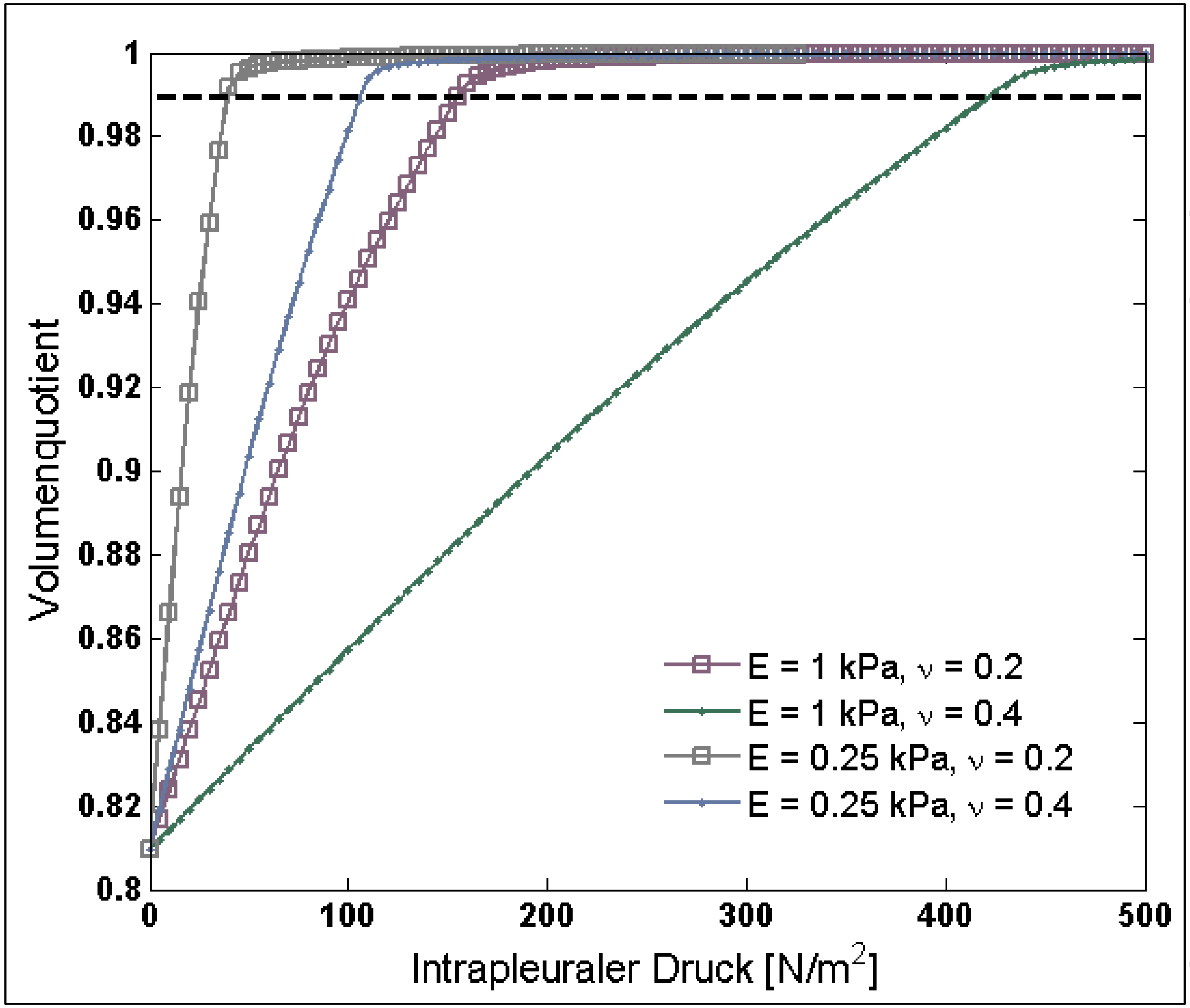
distance of surface points with respect to their initial positions (in mm)

Details:

Approx. 60 000 tetrahedrons
Solving time: approx. 0.5 h

Results

A) Influence of the elastic constants:



Literature values:

	E [kPa]	v
West et al., J Appl Physiol **32**, 1972	0.25	0.3
De Wilde et al., J Appl Physiol **52**, 1981	0.73	0.3
Zhang et al., Med Phys **31**, 2004	4	0.35
Sundaram et al., Med Image Anal **9**, 2005	0.1	0.2
Villard et al, MediVis 2005	0.823	0.3

Results

A) Influence of the elastic constants:

1) Pressure to fullfill a volume ratio of >0.995 depends on E and ν values

2) BUT:
Differences in displacement vectors are small (for the given application!): < 0.5 mm in magnitude

⇒ Choice of E and ν values of minor impact!

Literature values:

	E [kPa]	ν
West et al., J Appl Physiol **32**, 1972	0.25	0.3
De Wilde et al., J Appl Physiol **52**, 1981	0.73	0.3
Zhang et al., Med Phys **31**, 2004	4	0.35
Sundaram et al., Med Image Anal **9**, 2005	0.1	0.2
Villard et al, MediVis 2005	0.823	0.3

Results

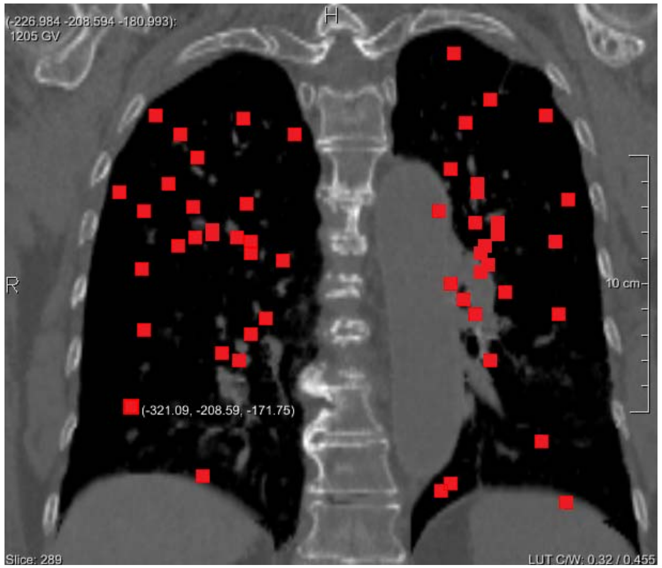
B) Modeling accuracy

<u>Evaluation concept</u>:

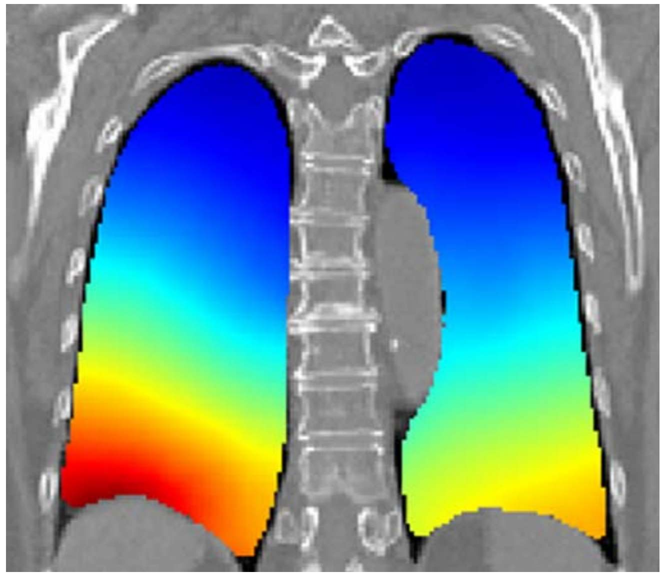
- Evaluation based on landmarks identified by an medical expert
- 35 to 45 landmarks each lung and each breathing phase (here: only EE and EI)
- Quality measure of model based predicted motion field:

$$R_k = \delta p_k^{[predicted]} - \delta p_k^{[actual]}$$

[„Registration residual“]



Landmark locations



Motion field estimator

Results

B) Modeling accuracy

Results averaged over all patients:

- Mean landmark motion observed: **6.6 ± 5.2 mm**
- Intraobserver variability: **0.9 ± 0.8 mm**
- No systematic prediction error: $\mathbf{R_{CC}} \cong \mathbf{R_{AP}} \cong \mathbf{R_{ML}} \cong 0$
- Registration residual magnitude: **3.3 ± 2.1 mm**

Results

B) Modeling accuracy

Results averaged over all patients:

- Mean landmark motion observed: **6.6 ± 5.2 mm**
- Intraobserver variability: **0.9 ± 0.8 mm**
- No systematic prediction error: $R_{CC} \cong R_{AP} \cong R_{ML} \cong 0$
- Registration residual magnitude: **3.3 ± 2.1 mm**

Literature values (non-linear registration),
e.g. Vik et al. [Philips Medical Systems], SPIE Medical Imaging 2008:

» 2.5 ± 2.2 mm (surface-tracking)
» 2.9 ± 3.1 mm (B-Spline based reg.)
» 3.3 ± 3.1 mm (non-parametric reg.)

Results

B) Modeling accuracy

Influence of lung tumors on modeling accuracy:

Lan[illegible]	Registration residual magnitude $\|\|\mathbf{R}\|\|$ in mm			
	[illegible]	[illegible]	[illegible]	[illegible]or > 65.4 cm^3
in th[illegible]	[illegible]	[illegible]	[illegible]	4.2 ± 2.6
near the pleura	2.9 ± 1.8	3.3 ± 1.7	4.4 ± 2.2	4.2 ± 2.3
close to the tumor	—	3.0 ± 1.5	4.1 ± 2.2	6.3 ± 3.1
mean	2.8 ± 1.6	3.1 ± 1.6	4.2 ± 2.4	4.6 ± 2.7

⇒ Modeling accuracy decreases with increasing tumor size!

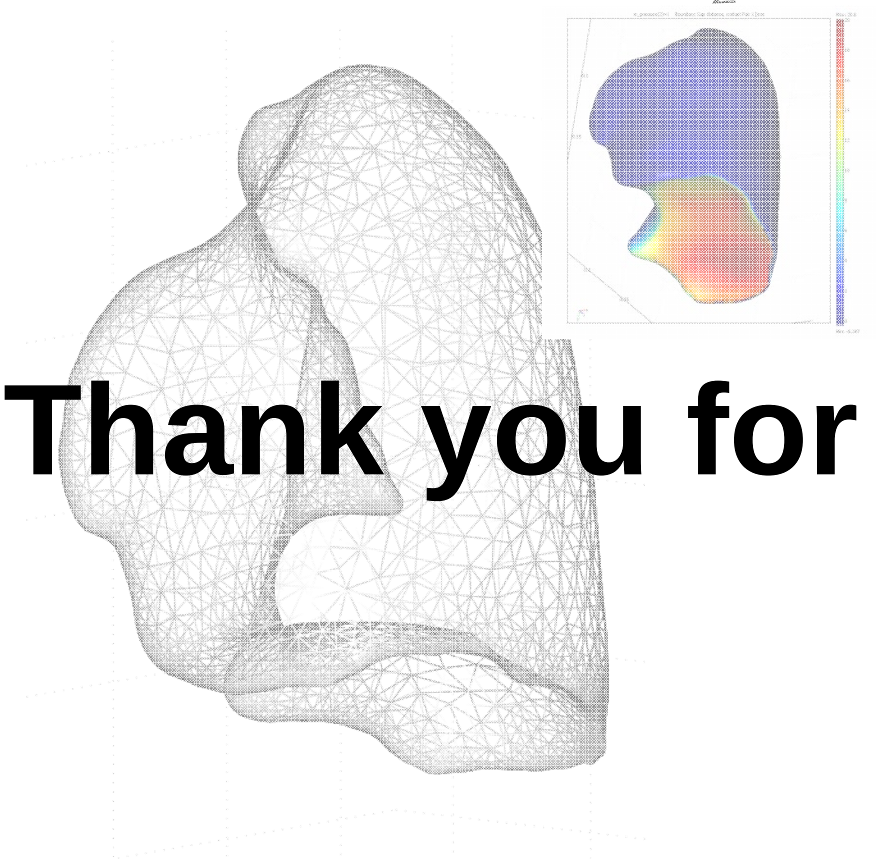
Summary & Discussion

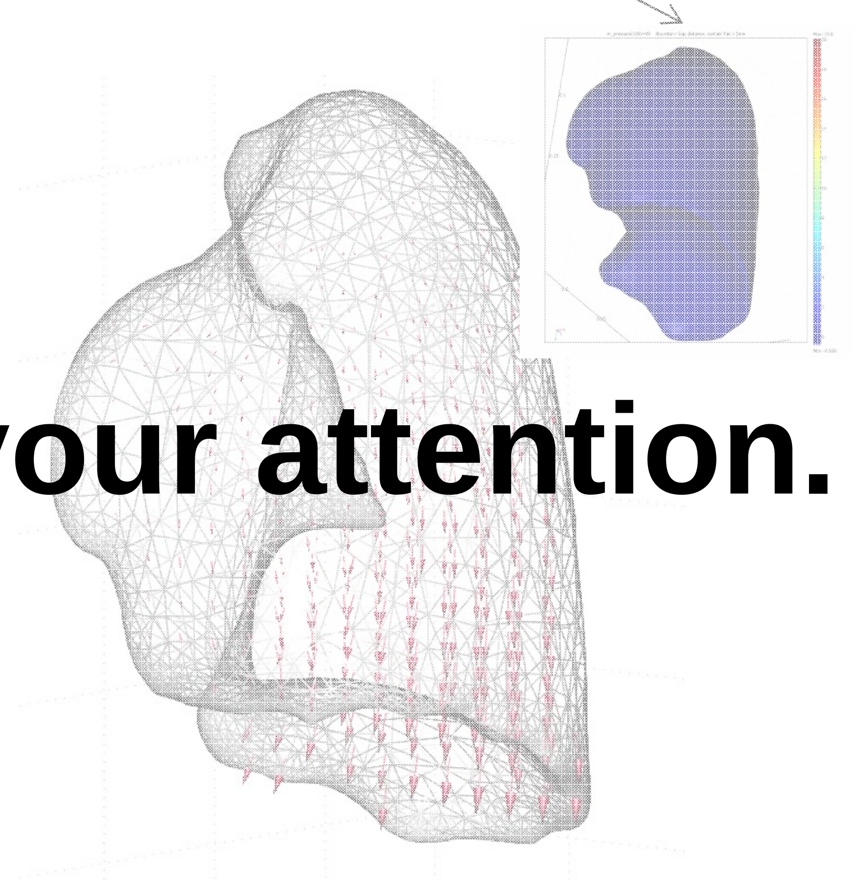
- Biophysical approach to modeling respiratory lung motion
  - Lung ventilation as a contact problem of linear elasticity theory
- Implementation using FEM / COMSOL SME Module
- Evaluation of model accuracy by means of 4D CT data
  - Registration residual of approx. 3 mm

⇒ Modeling approach promising, but …

- Assumption of lung tissue to be homogenous oversimplifying!
  - Explicit modeling of inner lung structures
  - Import of “complex” structures to COMSOL?

Gap beetween initial and final geometry (red: > 20 mm)





Thank you for your attention.

Left lung at EE (i.e. the initial, the undeformed geometry)

Deformed left lung (i.e. lung at the final situation, the predicted lung shape at EI)