Design and Optimization of Multilayer Ideal Cloak

Jyothi. M.H. [1], Bipin. D.V. [*2], and Balamati. Choudhury[3]
Centre for Electromagnetics, CSIR-National Aerospace Laboratories
Bangalore, India, 560017
[1]jyothiece.komal@gmail.com, [2]dvbipin007@gmail.com, [3]balamati@nal.res.in

Abstract: The development in metamaterial science and technology has created many exhilarating applications from microwave to optical frequency region in which invisibility cloaking is one of the exciting application. In recent years, the RCS reduction characteristics through cloaking structures have been explored by the aerospace researchers, as a novel concept to achieve stealth platform. This paper deals with the design of an ideal spherical cloaking structure in accordance with the transformation optics theory. The performance of the designed cloak is analyzed by comparing the RCS of PEC sphere without cloaking and with multilayer cloaking shells. It has been observed that multilayer spherical cloak shows reduced RCS in C-band with respect to the PEC sphere.

Keywords: Invisibility cloaking**,** RCS, Transformation optics theory, PEC

1. Introduction

An ideal cloak is a volumetric shell which covers an object to make it invisible by bending the electromagnetic ray incident on it. From past few years, the concept of cloaking has gained much attention of the researchers because of its exciting application in different frequency spectrum from microwaves, terahertz till optics. Among all the related research, cloak with spherical and circular-cylindrical shell geometrics have been of more interest. In principle, objects of any shape can be hidden in these two kinds of cloaking shapes [1]. The term invisibility in aerospace refers to hiding an object from detecting device like radar to achieve a low observable platform. Apart from other RCS reduction techniques such as shaping, radar absorber structures etc invisibility cloaking concept can be a stealth technology that can provide optimal stealth [2-4].

The cloaking concept was first demonstrated in 2006 through transparent gradient-index structure, in which multilayer metamaterial was designed to obtain the electromagnetic invisibility [5]. The design of the invisibility cloak is an inverse problem, where the electromagnetic wave propagation direction will be known and the material properties will be obtained according to that, by using the coordinate transformation theory.

Coordinate transformation theory explains the bending of electromagnetic wave trajectory around the cloaked object which causes spatial variation in the constitutive parameters like relative permittivity and relative permeability of the electromagnetic structure. Maxwell's equations are form-invariant to coordinate transformations [6], hence the permittivity and permeability tensor components are affected by the transformation and become both spatially variant and anisotropic. By applying these complex material properties to the cloaking shell, it will control the flow of incident electromagnetic wave and bend it around the object so that cloaking shell will neither scatter nor absorb the incident wave.

Also an alternative method was introduced to design the cloaking device by using conformal mapping in an isotropic media [7,8]. The propagation of wave in this method is based on Hamilton's correlation between the ray trajectory in the media and movement of the particles, analysed by principles of classical mechanics. But this phenomenon can be applied only if the refractive index does not robustly change over scales when compared with wavelength of light [9].

This paper focuses on the design and simulation of ideal spherical cloaking shell. Further, the RCS of a PEC sphere and reduction using multilayer cloaking has been carried out. In Section 2, coordinate transformation theory has been used to derive the permittivity and permeability tensors. Using the derived material tensor parameters, ideal cloak has been simulated and the bending of electromagnetic waves has been shown for normal as well as different angle of incidence. Section 3 describes RCS estimation of a PEC sphere in Mie region and reduction of the same through four layers of cloaking shell has been shown which is the optimized one. The cloaking effect has been shown through bending of electric field, simulated using FEM based COMSOL Multiphysics 5.2. Even the RCS estimation of PEC sphere with and without cloaking shell has been predicted using the same software.

2. Design of Ideal Spherical cloak

Transformation optics is the basic concept of cloaking design. Coordinate transformation in conjunction with transformation optics is necessary to progress in the design steps. As the intention of this paper is to design an ideal spherical cloak, the relation between spherical *(r, θ, φ)* and cartesian coordinate system *(x, y, z)* is given as,

$$x = r \times \sin\theta \times \cos\varphi \tag{1}$$

$$y = r \times \sin\theta \times \sin\varphi \tag{2}$$

$$z = r \times \cos\theta \tag{3}$$

Where r is the radius of the sphere, θ is the azimuth angle varying from 0° to 180° and φ is the elevation angle that varies from 0° to 360°.

Transformation optics theory defines a spatial transformation that maps a spherical region $0<r<b$ in the original coordinate (r, θ, φ) into an annular region $a<r'<b$ in the new coordinate (r', θ', φ') as

$$r = a + r' \times \left(\frac{b-a}{b}\right) \tag{4}$$

$$\theta = \theta' \tag{5}$$

$$\varphi = \varphi' \tag{6}$$

Where a and b are inner and outer radius respectively.

Using this transformation, the elements of the permittivity and permeability tensors [10] can be determined by solving the following equation;

$$\varepsilon^{ij} = \begin{pmatrix} \left(\frac{b-a}{b}\right) \times r'^2 \times \sin\theta' & 0 & 0 \\ 0 & \left(\frac{b}{b-a}\right) \times \sin\theta' & 0 \\ 0 & 0 & \left(\frac{b}{b-a}\right) \times \frac{1}{\sin\theta'} \end{pmatrix} * \frac{1}{r^2 \times \sin\theta} \tag{7}$$

Above equation can be simplified as:

$$\left|\varepsilon^{ij}\right| = \left(\frac{b-a}{b}\right)^3 \times \left(\frac{r-a}{r}\right)^2 \tag{8}$$

This transformation leads to the expression of permittivity and permeability tensor components of the spherical cloak, given by

$$\varepsilon_r = \mu_r = \left(\frac{b-a}{b}\right) \tag{9}$$

$$\varepsilon_\theta = \mu_\theta = \left(\frac{b-a}{b}\right) \times \left(\frac{r-a}{r}\right) \tag{10}$$

$$\varepsilon_\varphi = \mu_\varphi = \left(\frac{b-a}{b}\right) \times \left(\frac{r-a}{r}\right) \tag{11}$$

Eq. (9-11) shows that, the entire tensor component has gradients as a function of radius, leading to complex metamaterial design.

A PEC sphere with a radius of a=3cm and spherical cloaking shell of radius b=11cm has been considered. The permittivity and permeability properties of the cloaking shell are obtained from the transformation theory, Eq. (9-11).

RF and wave optics module of COMSOL Multiphysics 5.2 is used to simulate and validate the performance of the spherical cloak. This approach describes the propagation of electromagnetic waves based on numerical integration of a set of Hamilton's equations obtained by taking the geometric limit of Maxwell's equations in anisotropic and inhomogeneous media. The simulated steady state electric field and power flow pattern are shown in Figure 1. It can be observed from Figure 1 that the incident electric field bends around the spherical cloak and gives a perfect invisibility.

To study the cloaking effect for different angle of incidence the analysis of steady state electric field has been carried out by projecting EM waves from random directions. Figure 2 shows the cloaking effect at different angle of incidence viz. 45^0, 80^0, 200^0 and 310^0.

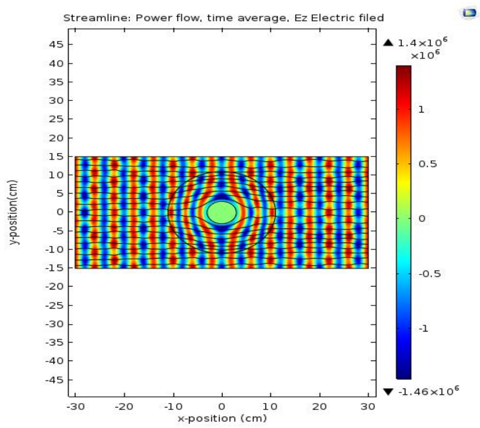
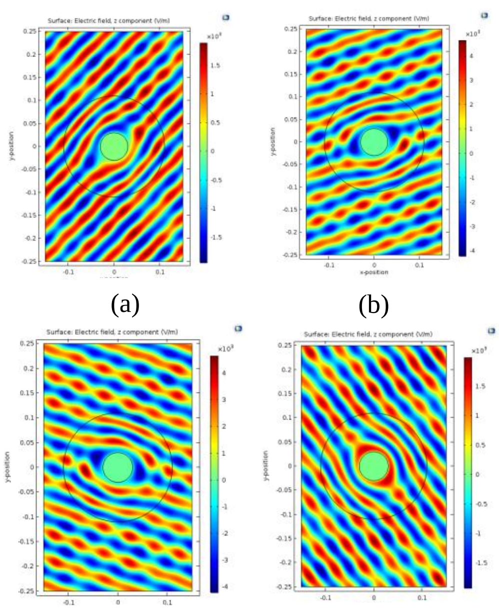


Figure 1. 2D spherical cloak, Electric filed patterns, with stream lines indicating the direction of power flow.



(a) (b)

(c) (d)

Figure 2. 2D spherical cloak with EM wave incident at (a) 45˚ (b) 80˚ (c) 200˚ (d) 310˚.

3. RCS Calculation of Multilayer Cloak Structure

Radar cross section (RCS) deals with the detectability of an object and it serves as one of the quantitative measures for calculating the performance of an ideal cloak illuminated by a plane wave in free space. The RCS estimation of perfect electric conductor (PEC) sphere and multilayered spherical cloak is performed using the RF module of COMSOL Multiphysics 5.2. The computational view of PEC sphere for the RCS estimation is shown in Figure 3.

The simulation setup involves a PEC sphere of radius 3 cm placed at the center of two concentric spherical shells and the complete model is divided symmetrically to apply PEC and perfect magnetic conductor (PMC) boundary condition to the symmetric planes. Inner and outermost shell is considered as air domain and perfectly matched layer (PML) domain respectively. Then, an incident plane wave at an operating frequency of 3 GHz with electric field orientation along z-direction is applied. The RCS plot of a PEC sphere obtained by applying the above mentioned boundary condition is shown in the Figure 4.

The PEC sphere is now enclosed with the four layers of cloaking shell having the radius of 5 cm, 7 cm, 9 cm and 11 cm as shown in Figure 5. Each layer is having a thickness of 0.7 cm with the air gap provided between them. The permittivity (ε) and permeability (μ) components for the cloaking shells are applied based on the derived tensors obtained from the transformation theory, Eq. (9-11).

The RCS of designed ideal sphere without cloaking shell and with multilayer cloaking shell has been calculated and compared. The RCS comparison plot in Mie region is shown in Figure 6, it can be seen that as the number of layers increases there is a RCS reduction in C band and hence the operating frequency can be chosen in this band to get a better performance. From Figure 6 it is also clear that, with 4 layers of cloaking shell, RCS value was found to be reduced as compared with other three cloaking layers.

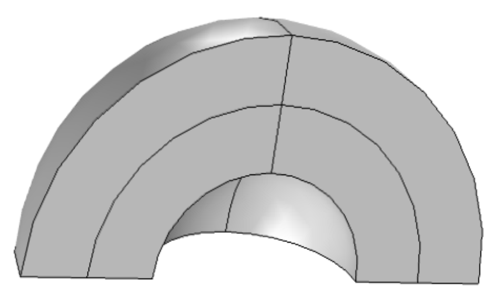


Figure 3. Computational domain for estimating the RCS of a PEC sphere in free space.

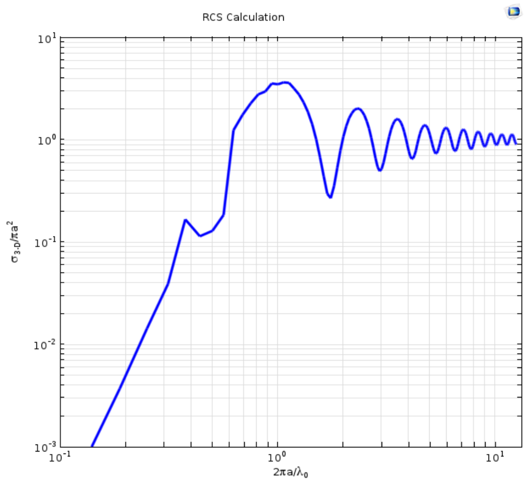


Figure 4. Simulated Normalized RCS plot of the PEC sphere.

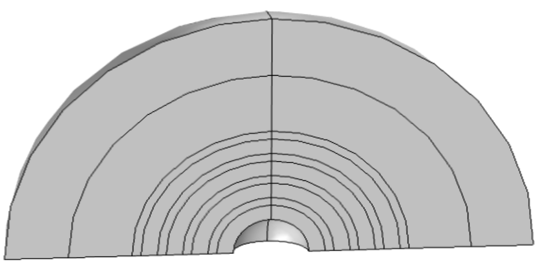


Figure 5. Computational domain for RCS calculation of four layer cloaking shell in free space.

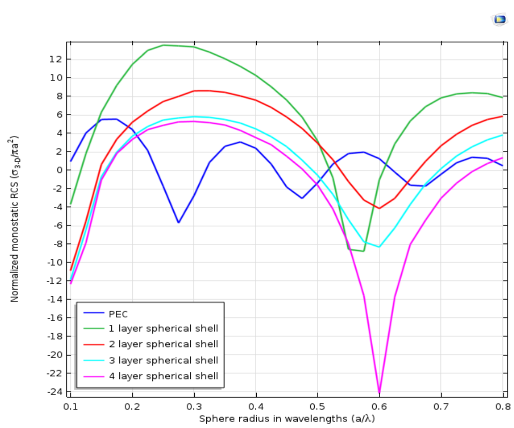


Figure 6. Normalized RCS plot for cloaked and uncloaked PEC sphere

4. Conclusion

In this paper, the design of an ideal spherical cloak has been carried out using RF and wave optics module in COMSOL Multiphysics 5.2. Electric field distribution for the cloaked structure is shown and analyzed and it is seen that the projected EM waves along z-direction is bending around the object and emerging out without any scattering and absorption. Physical insight for calculating the RCS of the multilayer spherical cloak has been provided. The RCS plot of the sphere without cloak and with multilayer cloaking shell has been analyzed, it can be justified that as the number of layers increases the RCS is reduced and gives a better RCS reduction in C band.

5. References

1. L. Yu, J. Zhang, H. Chen and B. Wu, "Full-wave analysis of prolate spheroidal and hyperboloidal cloaks," *Journal of Physics*, **Vol. 41**, pp. 1-6,(2008).
2. L. Tsang, J. A. Kong, and K. Ding, *Scattering of Electromagnetic waves:Theory and Applications*, Wiley Interscience, New York, ISBN: 978-0-471-38799-2, 426P,(2000).
3. J. A. Kong, *Electromagnetic waves Theory*, EMW Publishing, Cambridge, MA, ISBN: 0-9668143-9-8, (2005).
4. B. A. Kemp, T. M. Grzegorczyk, and J. A. Kong, "Optical Momentum Transfer to Absorbing Mie Particles," *Phys. Rev. Lett*. **Vol. 97**, 133902, (2006).
5. D. Schurig, J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, "Metamaterial electromagnetic cloak at microwave frequencies," *Science*, **Vol. 314**, pp. 977-980, (2006).
6. J. B. Pendry, D. Schurig, and D. R. Smith, "Controlling electromagnetic fields," *Science*, **Vol. 312**, pp. 1780-1782, (2006).
7. U. Leonhardt, "Optical conformal mapping." *Science* 312.5781 1777-1780, (2006).
8. U. Leonhardt, "Notes on conformal invisibility devices", *physics.optics*, 8, 118, (2006).
9. M. Born, Wolf E., *Principles of optics: Electromagnetics theory of propagation, interference and diffraction of light*, CUP, Cambridge, ISBN 0521642221, (2000).
10. B. Choudhury, P. V. Reddy, R. M. Jha, *Permittivity and Permeability Tensors for Cloaking Applications*, Springer, New York, ISBN: 978-981-287-805-2, 71P. (2016).