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3D Simulation of the Thermal Response Test in a U-tube Borehole Heat Exchanger

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Abstract:
Simulated Thermal Response Test data are analyzed in order to evaluate the effect of the tri-dimensionality model's feature in determining the proper value of the soil thermal conductivity and borehole thermal resistance, necessary for the geothermal energy storage systems' design in real conditions.
The 3D system's simulation during the Thermal Response Test is realized by adopting the finite element method implemented within the Comsol Multiphysics® environment. The comparison of the numerical results with the analytical solution of the Line Source Model problem, enables to discuss the capability of the Thermal Response Test with regards to the characterization of borehole energy storage systems in real conditions.

Keywords: Geothermics, Borehole Heat Exchanger, Line Source Model, Thermal response Test.

1. Introduction

On account of environmental and energetic emergency, the usage of high efficiency energy systems, like ground coupled heat pumps (GCHP), offers an interesting solution. As a matter of fact the soil represents one of the most attractive heat sources thanks to its temperature that is year round roughly constant [1]. The proper design of the GCHP requires the knowledge of the soil thermal properties. For this aim an estimation method, known as Thermal Response Test (TRT), is often adopted, especially in vertically oriented loops. It allows in-situ-determination of the thermal properties necessary to the design of these systems; this approach is based on the comparison between the analytical solution of an unsteady heat conduction problem within the soil, and the average fluid temperature experimentally acquired, directly on the borehole heat exchanger [2]. It was first adopted by Morgensen [3], who approximated the borehole storage system's behavior by the so called Line Source Model. This simple model consists of the unsteady heat conduction problem in a semi-infinite isotropic homogeneous constant properties medium in which a line heat source is present. The comparison between TRT data and the Line Source Model solution allows to set out two important properties for the GCHP design, i.e. the soil thermal conductivity and the borehole thermal resistance. More complex models have been suggested, like by Kavanaugh and Rafferty with the Cylinder Source Model [4], or by Bandos et al. [5], who take into account the finite dimension of the heat source. While the Cylinder Source Model tends to overestimate the thermal conductivity, the Line Source Model is considered more reliable [6,7]. More general estimation approaches, based on the direct numerical solution of the partial differential equations governing the phenomenon has been suggested, too [8,9,10].
An important weakness of the Thermal Response Test based on the Line Source Model with regards to the design of underground thermal energy storages is due to the fact that it relies on the assumption of problem's axial-symmetry, while it has been observed that the system's behavior during the TRT is clearly more complex [11]. Moreover, it assumes the heat flow in the U-tube as constant and uniform along the axial coordinate, while the heat exchange varies with increasing depth.
In the present paper, simulated Thermal Response Test data are analyzed in order to evaluate the effect of the tri-dimensionality model's feature in determining the proper value of the soil thermal conductivity and borehole thermal resistance, necessary for the borehole energy storage systems' characterization in real conditions.
The 3D simulation of the Geothermal Energy Storage System is realized by adopting the finite element method implemented within the Comsol Multiphysics® environment. It consists of a 3D

model that couples the tri-dimensional transient conduction heat transfer problem within the soil with the one-dimensional convective problem within the carrier fluid in the U-tube borehole.

The comparison of the numerical results with the analytical solution of the Line Source Model problem, enables to discuss the capability of the Thermal Response Test based on the Line Source Model, with regards to the characterization of borehole energy storage systems in real conditions.

1.1 Thermal Response Test

Soil thermal properties, i.e. thermal conductivity and borehole thermal resistance, represent the main design data in thermotechnical applications that exploit the ground heating storage.

When the heat storage system is vertically oriented, that is the case here considered, the common method to estimate the soil properties is named Thermal Response Test. Essentially a carrier fluid (usually water) is circulated through a BHE, which consist of a High Density Polyethylene (HDPE) U-tube in close thermal contact with the soil.

The time-dependent behaviour of the outlet fluid temperature represents the experimental data which, by taking into account also the test conditions, enables to determine the soil and eventually the BHE's thermal properties. An essential role in the estimation procedure is played by the choice of the reference model adopted to recover the above-stated unknown quantities; this procedure is based on the comparison of the experimental data with the solution of the equations describing the model's behaviour.

1.2 The Line Source Model

By assuming that the soil is a uniform and isotropic medium and also that the temperature difference between the inlet and outlet section of the heat exchanger remains constant over time, the thermal unsteady problem in the system can be approximately solved by considering the geothermal heat exchanger as a linear heat source, which suddenly releases a finite, uniform and constant quantity of energy in a homogeneous medium unlimited in the radial direction and having uniform initial temperature. For this model, known as the Line Source Model, the analytical solution is [12]:

$$T(r,t) = T_0 + \frac{Q}{4\pi\lambda H} E_1\left(\frac{r^2}{4\alpha t}\right) \tag{1}$$

where E_1 is the integral exponential function.

For sufficiently large values of the parameter $4\alpha t/r^2$, Equation (1) can be approximated by:

$$T(r,t) \cong T_0 + \frac{Q}{4\pi\lambda H}\left(\ln\left(\frac{4\alpha t}{r^2}\right) - \gamma\right) \tag{2}$$

By defining the thermal resistance per unit length between the working fluid and the cylindrical surface at $r=r_b$, as follows

$$R_b = \frac{[T_f(t) - T(r_b,t)]H}{Q} \tag{3}$$

where T_f is the mean fluid temperature, it can be concluded that:

$$T_f(t) \cong T_0 + \frac{Q}{4\pi\lambda H}\left(\ln\left(\frac{4\alpha t}{r_b^2}\right) - \gamma\right) + \frac{R_b Q}{H} \tag{4}$$

Equation (4) is the mathematical model which the Thermal Response Test usually refers to, in order to estimate the thermal equivalent conductivity of the soil and the borehole thermal resistance, too.

It should be noted that the use of Equation (4), implicitly assumes the following simplifications:

-the thermal properties of the heat exchanger and soil are the same;

-the pipe in which the working fluid flows is placed on the symmetry axis of the system and it has a negligible diameter;

-the fluid temperature doesn't change along the axial direction.

Accordingly, the model is expected to give a poor approximation of the real system behaviour in the early regime, when the capacitive effects of the heat exchanger are particularly relevant.

The estimation procedure is based on the comparison, under a least square approach, between the temperature of working fluid, experimentally acquired and evaluated as the arithmetic mean between the inlet and outlet fluid temperature, and equation (4). While the equivalent thermal conductivity of the soil can be easily derived, the borehole thermal resistance R_b is recovered by assuming that the soil thermal diffusivity is known. In particular, this last assumption is based on the hypothesis that the thermal capacity per unit volume of the ground is known. In fact, in the practical application of the Thermal Response Test, for the soil volumetric thermal capacity it is generally adopted a value which is assumed typical of most type of soils. However, it should be noted that the variation of the soil thermal capacity per unit volume may be of the same order of magnitude of the variation shown by the thermal conductivity.

Regarding this, the results reported in [13] partially confirm the propriety of this approach in relation to the design of geothermal borehole energy storage systems. In particular they show that the thermal capacity per unit volume, and therefore the thermal diffusivity, has a minor effect on the estimation of the soil thermal conductivity and of the heat flux exchanged per unit length only under the hypothesis that the ground energy storage surrounding the borehole extends indefinitely in the radial direction. In situation when the medium in which the borehole is immersed cannot be considered semi-infinite, the thermal diffusivity has instead a significant effect on the heat flux exchanged by the heat exchanger.

As the definition of resistance suggests, the Borehole Thermal Resistance is a steady-state parameter that depends on borehole geometry and thermal properties of the borehole filling; however, it is actually derived from transient measurement. Another important aspect that the TRT recovers in the estimation of the Borehole thermal resistance is the contact resistance between the soil and the borehole wall, that strictly depends on soil type, water presence, and also on In Situ installation methods.

The present analysis is in particular focused on the validation of the Line Source Model estimation approach based on the Thermal Response Test to real situation in which both the U-tube and the whole borehole play a role in the heat exchange, by thus requiring a 3D approach. To this aim, the governing partial differential equations, have been implemented and solved within Comsol Multiphysics® environment through a 3D simulation of the phenomenon by considering two schematic cases, corresponding to different soil thermal conductivities.

2. The 3D Geothermal Energy Storage System's Modeling within Comsol Multyphysics®

In order to evaluate the effective heat rates exchange during a real TRT, a tri-dimensional geothermal energy storage system has been simulated, that couples the tri-dimensional transient conduction heat transfer problem within the soil, the borehole filling material and the HDPE tubes, with the one-dimensional convective problem within the carrier fluid.

The scheme of the geothermal heat exchanger and of the coupled energy storage system considered in the present analysis, is schematically shown in Figure 1a. It consists of a U-tube pipe having inner radius r_1=16.3 mm with downward flow in the right-tube section and upward flow in the left-tube section, immersed in a filling material. The HDPE tubes, having a 3.7 mm thick wall, have been modelled too, while for the sake of simplicity the wall thickness of the borehole was disregarded.

The system shows a symmetry plane and it is considered practically unlimited in the radial direction (R→∞), in order to respect the Line Source Condition, while in axial direction it is limited by two adiabatic surfaces placed at z=0 (soil surface) and z=H. (simulated depth of the heat exchanger).

The Geothermal Energy Storage System modeled in the COMSOL Multyphysics® environment is reported in Figure 1b:

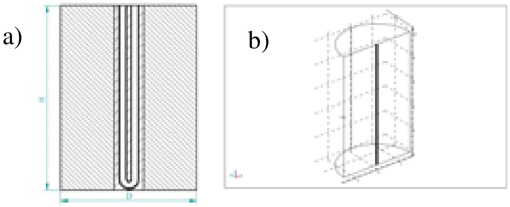

Figure 1 – Geothermal Energy Storage System's Geometry.

2.1 Numerical Simulation within Comsol Multiphysics®

Transient heat transfer conduction is governed by the Fourier equation, which is:

$$\rho c_p \frac{\partial T}{\partial t} = \mathrm{div}(\lambda \nabla \mathrm{T}) \tag{5}$$

For the thermal problem schematized in Figure 1, Equartion (5) has been solved in each domain (soil, fill and tubes) with the initial condition:

$$T(r,z,0) = T_0 \tag{6}$$

The coupling between the tube wall and the carrier fluid has been realised setting the Robin boundary condition described by:

$$-\lambda \left.\frac{\partial T}{\partial r}\right|_{r=r_t} = h_o\left[T_{fluid}(z,t) - T\,(r_t,z,t)\right] \tag{7}$$

being h_o the convective heat transfer coefficient, derived from the Dittus-Boelter correlation, associated to the working fluid flowing in the tube section with temperature T_{fluid} and r_t the HDPE tube inner radius. Equation (7) holds for both the downward (T_i) and upward (T_f) stream.

By assuming that the convection problem in both the tubes of the heat exchanger is one-dimensional, the energy equation for the right-tube downward fluid flow is:

$$A\ \rho_f c_{pf}\left(\frac{\partial T_i}{\partial t} + u\ \frac{\partial T_i}{\partial z}\right) = h_o\left[T\ (r_t,z,t) - T_i(z,t)\right] \tag{8}$$

with the initial condition:

$$T_i(z,0) = T_0 \tag{9}$$

being T_i the right-tube downward fluid temperature and u the fluid mean velocity in the axial direction and A the tube's cross section area.

The corresponding equation for the left-tube upward fluid flow is:

$$A\ \rho_f c_{pf}\left(\frac{\partial T_f}{\partial t} - u\ \frac{\partial T_f}{\partial z}\right) = h_o\left[T\ (r_t,z,t) - T_f(z,t)\right] \tag{10}$$

with the initial condition:

$$T_f(z,0) = T_0 \tag{11}$$

being T_f the left-tube upward fluid temperature.

The U-connection at the bottom of the pipe between the downward and upward fluid is here modelled by imposing for z=H that the mean temperature of the upward fluid equals the mean temperature of the downward fluid:

$$T_i(H,t) = T_f(H,t) \tag{12}$$

The condition of constant power supplied to the working fluid is implemented by the periodic edge condition:

$$T_i(0,t) = T_f(0,t) + \Delta T \tag{13}$$

with ΔT constant over the whole temporal domain.

The continuity condition of both temperature and heat flux at the interface between solid domains of different thermal proprieties, completes the statement of the problem.

The above equations have been solved by means of the finite element method implemented within the Comsol Multiphysics® environment.

Equations (8-11), expressing the energy equation in the fluid domain, have been implemented by means of the *weak form formulation* according to the scheme outlined in [13].

The whole geometry has been discretized by means of 41080 prism elements obtained by first discretizing the domain radial section with triangular elements, and then extruding the mesh along the axial direction, as shown in Figure 2a-b:

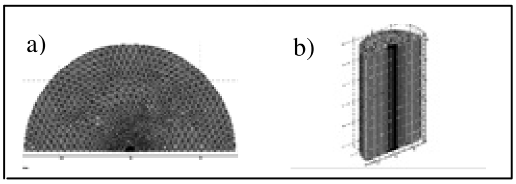

Figure 2 – a) Radial meshing**;**
b) Axial meshing.

The adopted mesh has proved to be sufficient to reach the convergence of the results, although for the sake of simplicity the mesh's sensitivity analysis is not reported here.

3. Results

The effect of the tri-dimensionality model's feature in determining the proper value of both the thermal conductivity and borehole thermal resistance trough the Line Source estimation Method has been considered here. The main features of the tested cases and the material thermal properties are reported in Table 1:

Table 1 – Main input data for the tested cases

Working Fluid Mass Flowrate	0.1 kg/s
Fluid density	1000 kg/m^3
Fluid specific heat	4186 J/(kg K)
Inlet-Oulet Fluid temperature difference	3.6 K
Convective Coefficient (Dittus-Boelter)	1960 W/(m^{2}K)
Soil density	1000 kg/m^3
Soil specific heat	2000 J/(kg K)
Soil thermal conductivity	**CASE A**: 2 W/(m K) **CASE B**: 3 W/(m K)
Fill density	1000 kg/m^3
Fill specific heat	1000 J/(kg K)
Fill thermal conductivity	0.9 W/(m K)
HDPE density	950 kg/m^3
HDPE specific heat	1900 J/(kg K)
HDPE thermal conductivity	0.48 W/(m K)

Typical values corresponding to a bentonite-water mixture have been assumed here as the filling material's thermal properties.

The simulation has been carried out for 72 hours, which corresponds to the standard duration of the Thermal Response Test.

The average fluid temperature is reported, for the case A, in Figure 3; it shows that after a sufficiently long time, a linear trend with time in a semi-logarithmic scale is reached, as predicted by Equation (4). In the same figure the best-fit line according to the Line Source Model and obtained by considering a fitting period of 48 h, staring from t=24 h, is reported too.

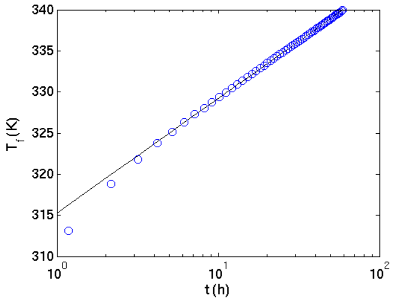

Figure 3 – Average fluid temperature versus time for case A.

The fluid temperature trend confirms not to be affected, after a sufficiently long time, by the borehole configuration.

The temporal borehole temperature distribution at the inlet section (z=0), is reported in Figure 4:

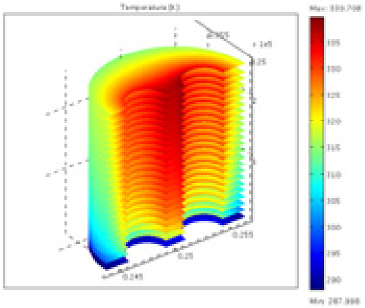

Figure 4 – Temporal borehole temperature distribution at z=0.

The soil thermal conductivity, estimated according to Equation (4) by considering a fitting period of 15h is reported in figures 5-6 versus the initial fitting time, for each soil type considered in the present analysis.

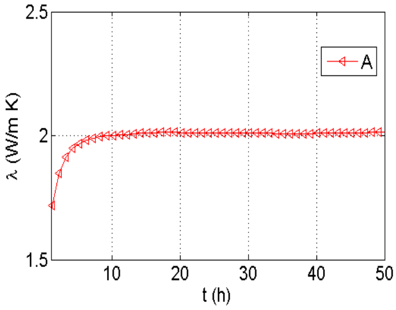

Figure 5 – Estimated thermal conductivity for case A.

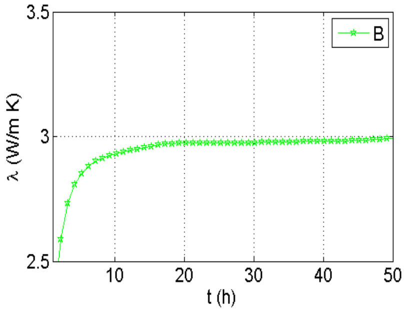

Figure 6 – Estimated thermal conductivity for case B.

As expected, the Line Source Model satisfactory predicts the thermal conductivity of the soil, recovering the parameter proper value already in the first 30 hours.

Regarding the borehole thermal resistance, the values of both cases A and B obtained with the Line Source estimation Method are shown in Figure 7; they are recovered accordingly with Equation (4), once the thermal conductivity has been obtained, by considering a fitting period of 15 h.

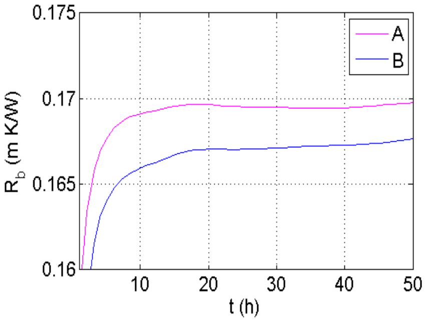

Figure 7 – Estimated borehole thermal resistance.

As outlined above, this parameter should not be affected by the soil properties, so we expected the same value in both the analyzed cases; on the contrary, it has been observed that different values of the soil thermal conductivity affect the recovered borehole thermal resistance. However the two cases differ each other only for 1,5% for a variation of 50% of the soil thermal conductivity; moreover, they differ from the value numerically calculated in steady state regime through a 2D simulation of the borehole for less than 5% (R_b=0.1625). These differences can be assigned to the Line Source Model simplifications described in the introduction.

Furthermore, it can be observed that R_b has been recovered assuming the heat capacity per unit volume known, while in real estimations from experimental tests it is an unknown parameter.

In order to evaluate the effect of this approximation, a heat capacity per unit volume value 50% higher than the simulated one has been adopted in the Equation (4) to estimate the borehole thermal resistance in the case A. Results suggest that this assumption makes the recovered parameter 12% higher than the correct one.

Finally, it has to be noted that this analysis has been carried out by comparing numerical outcomes, without taking into account the experimental noise, that probably makes the estimation procedure more difficult.

4. Conclusions

The governing equation of the conduction/convection heat transfer phenomena describing the behavior of a geothermal energy storage system have been solved within Comsol Multiphysics® environment. In particular the analysis has been focused on the discussion of the application to the Thermal Response Test of the estimation procedure based on the Line Source Model, generally adopted to predict the soil thermal conductivity and the borehole thermal resistance, in the real conditions of non-linear and non-uniform heat source. The analysis confirms that the Line Source Model applied to the Thermal Response Test represents a sufficiently accurate approach also in the U-tube configuration. However, the 3D approach appears necessary when other more complex geometric configurations have to be considerate.

5. Nomenclature

A	Tube cross section	m^2
c_p	Specific heat at constant pressure	J/kg°C
H	Heat exchanger length	m
Q	Heat flux	W
r	Radial coordinate	m
R	Radius of the energy storage	m
R_b	Thermal resistance	°Cm/W
t	Time	s
T	Temperature	°C, K
u	Fluid velocity	m/s
z	Axial coordinate	m

α	Thermal diffusivity	m^2/s
γ	Eulero costant ≅ 0.57721	
λ	Thermal conductivity	W/m°C
ρ	Density	kg/m^3

Subscripts

b	Geothermal heat exchanger
f	Upward fluid
i	Downward fluid
t	Tube
0	Initial value

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