Simulation of the Thermal Stability of an Optical Cavity

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Abstract: The research project under which the COMSOL simulations are performed deals with multilayer and fine thermal control of an optical reference cavity for space applications. The cavity, made of Ultra Low Expansion glass (Corning ULE), must be kept close to the zero-expansion temperature of the glass (close to room temperature). The target can only be met by active control, while leaving the cavity free of sensor and actuators. Cavity length stability should be below one part in 10^{12}. In this paper some simulations based on the heat transfer and the structural mechanics of COMSOL are used to identify the conditions under which target cavity stability is achieved.

Keywords: Thermal regulation, thermometer, thermoelectric cooler.

1. Introduction

Future ESA (European Space Agency) scientific space missions like LISA (Laser Interferometer Space Antenna) and its precursor [1], DARWIN [2], gravimetric Satellite-to-Satellite Interferometers [3], among others, require laser interferometry as driving technology. Frequency stability of laser source is a common requirement of all interferometry applications and a dimensionally stable Fabry-Pérot cavity can be used to obtain such a source on space missions. Fabry-Pérot cavity is made by a pair of highly reflecting, low-loss, spherical mirrors, inserted at the extremes of a hollow cylinder made by a low thermal expansion material, for instance glass ceramics like ULE®, having a coefficient of thermal expansion (CTE) lower than 10^{-8} K^{-1} between 5 °C and 35°C. A Pound-Drever-Hall system keeps a laser source locked at resonance frequency, which depends directly on the distance between mirror substrates, and therefore the distance stability will be closely related with laser's frequency stability.

Distance stability is obtained by insulating mirrors and cylinder from external mechanical stress and by keeping them at constant temperature as explained in [4].

2. Mechanical setup

The cavity is a cylinder made in ULE used as separator between a pair of highly reflective mirrors. It is cup-shaped as shown in Figure 1, to provide enough mechanical support to outstay the launching.

Around the cavity, two concentric aluminum shields (the inner one is shown in Figure 2) are installed and thermally regulated using flexible heaters. Finally, both aluminum shields are placed inside a vacuum chamber whose temperature is regulated using thermoelectric coolers as shown in Figure 3.

Figure 1. Dummy cavity made in plexiglass together with a capacitive sensor used to measure cavity expansion during preliminary tests.

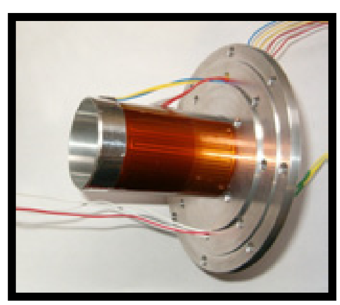

Figure 2. Aluminum shield, including the thermal ribbon sensor and the flexible heater.

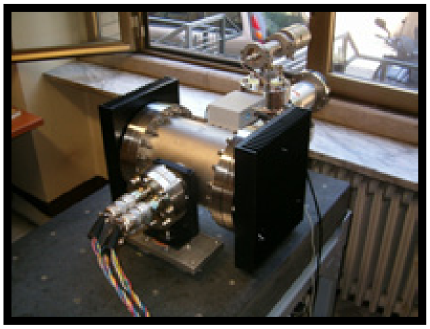

Figure 3. Vacuum chamber with TEC irradiators.

3. ULE Expansion Modeling

According to Corning Inc., ULE® CTE is zero at some temperature _ point between 5°C and 35°C called, by us, zero expansion temperature (ZET) and its maximum value inside the mentioned range will be ±30ppb. Assume the cavity temperature is homogeneous and equal to ZET, then, any variation on temperature will slightly increase bar's length. It seems reasonable just to assume CTE will linearly depend on temperature

$$CTE(\ ) = \frac{d}{d}\left(\frac{L}{\underline{L}}\right)(\ ) \approx \ (\ -\_) \quad (1)$$

where $\underline{L}$ is the length of the bar at _ and ULE CTE tolerance impose $\leq 2\times10^{-9}K^{-2}$. Cavity fractional expansion, with as a function of x would be

$$\frac{\Delta L}{\underline{L}} = \partial L = \frac{1}{2}\int_{-\underline{L}/2}^{\underline{L}/2} \underbrace{(\ (x)-\_)}_{CTE(\ )}(\ (x)-\_)dx \quad (2)$$

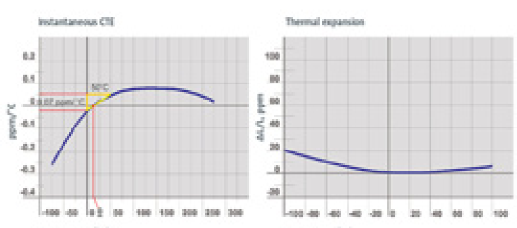

Figure 4. ULE CTE (Left) as a function of the temperature and ULE thermal expansion (right).

Since CTE is no longer assumed constant, thermal expansion is not only a function of the cavity mean temperature, making necessary to better describe cavity temperature profile. A second order Legendre polynomial $(x) = \_ + P(x,\gamma)$ is used to describe cavity's mean temperature as long with its asymmetry and concavity. Polynomial coefficients $_k(t)$ can be considered random variables with stationary mean expressed as $_k(t) = \ _k + \Delta_k(t)$, where $_k$ describes the DC component of the profile shift from ZET while $\Delta_k(t)$ its relative variations, this can be better seen as

$$(x,t) = \underbrace{\_ + P(x,\lambda)}_{_0(x)} + \underbrace{P(x,\delta(t))}_{\Delta\ (x,t)} \quad (3)$$

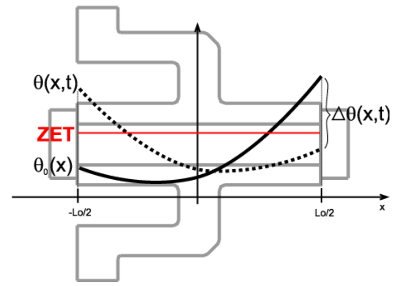

Figure 2 Cavity temperature profile.

From the time dependant expression of the temperature variations $\Delta\ (x,t)$ in (3), it is possible to find the expansion of the cavity as

$$dL(t) \approx \left(\ _0\tilde{\Delta}_0(t) + \frac{1}{3}\ _1\tilde{\Delta}(t) + \frac{1}{45}\ _2\tilde{\Delta}(t)\right) \quad (4)$$

Note how $_k$ can be seen as the sensitivity of length expansion to thermal disturbance. In fact, if first and second order terms are neglected, CTE is constant and equal to $_0$.

Active thermal control should keep

$$\mathcal{E}\{dL(t)\} = \ ^2_{\partial L}(t) \leq \tilde{\ }^2_{\max}. \quad (5)$$

If the variance of $\Delta_k(t)$ is denoted with $_k$, and $\tilde{\ }^2_{\max}$ is uniformly allocated on the three terms of (4), individual bounds can be found to be

$$\tilde{\ }_0(t) \leq \frac{1}{\sqrt{3}\ _0}[\text{mK}],\quad \tilde{\ }_1(t) \leq \frac{3}{\sqrt{3}\ _1}[\text{mK}],\quad \tilde{\ }_2(t) \leq \frac{45}{\sqrt{3}\ _2}[\text{mK}] \quad . (6)$$

Inequalities (6) are requirements to thermal control design/implementation and relevant calibrations, leading to the following design guidelines:

1) ULE® ZET must be calibrated to be used as set-point $_0(x)$ in (3).
2) Active control is needed to reach and keep set-point.
3) Active control is needed to reduce thermal gradient along cavity.

Previous guidelines don't demand for accurate absolute temperature control but only for very accurate thermal gradients control. Differential calibration method should be made with all sensors involved in the control system including those used to find the ULE® ZET.

4. Use of COMSOL Multiphysics

Axial symmetry of the instrument is exploited to simplify the simulations by using the 2D axis-symmetric mode in COMSOL.

The following conditions are assumed:

1) The vacuum chamber exchange heat with the ambient trough convective heat exchange, and with the inner shields trough conduction and irradiation.
2) The inner shields have some small portions at constant temperature, corresponding to the points in which the sensors are placed.
3) The cavity is left without sensors and exchange heat by conduction and by irradiation.
4) The cavity expansion is calculated from the temperature static solution by assuming the CTE depends on the temperature as shown in (1).

Disturbance signals affecting the temperature of the cavity are:

1) Regulation error due to control jitter and measurement noise, being the second the most significant of the two.
2) Unregulated portions of the structure are sensitive to ambient temperature due to conductive, irradiative and convective heat transfer.

The following approach is considered for the simulations: first, the sensitivity of the cavity expansion to each disturbance is simulated as static solution. Then the step response is simulated to identify the transfer function which can be combined with the spectral density of each disturbance to estimate its effects.

A description of the COMSOL model showing also a typical solution for static simuluation shown in Figure 5

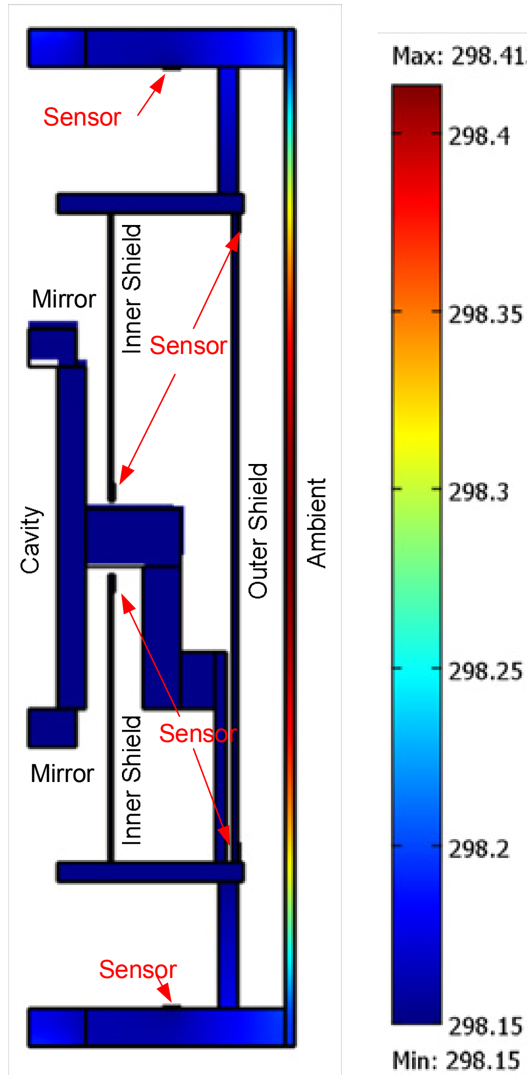

Figure 5. Simulation set-up and typical static temperature distribution.

5. Results

A parametric static simulation is performed by changing the intensity of each source of disturbance and measuring the cavity expansion as the relative displacement of the central internal point of each mirror. The displacement for regulation error of the sensor in the bottom of the outer shield, (the sensor which is thermally closer to the cavity) is plotted in Figure 6. From this values it is possible to find the sensitivity (steady state gain) converting each disturbance into fractional length deformation.

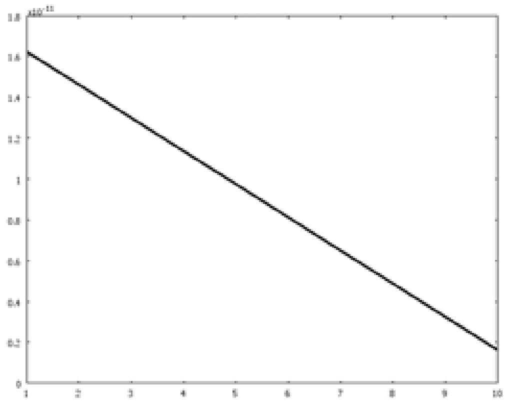

Figure 6. Thermal expansion for errors on regulation error in sensor bottom sensor of the outer shield ranging fomr 10mK to 10mK.

Disturbance [K]	Sensitivity [m/m/K]	Maximum disturbance for target [mK]
ThAmb	-6.3E-11	-15.873
ThExR	-4.1E-11	-24.3902
ThExL	-2.2E-09	-0.45455
ThInL	-3.6E-09	-0.27778
ThInR	-2.7E-09	-0.37037
ThoutL	-2.2E-09	-0.45455
ThoutR	-1.6E-08	-0.0625

Figure 7. Table relating the disturbance intensity with the cavity expansion and extrapolating the maximum disturbance at which target stability of 1 part in 10^{12}.

6. Conclusions

The sensitivity of the cavity to ambient temperature is small enough. This in fact is the objective of active control. However, sensitivity to measurement noise increases, demanding for temperature measurements as precise as 60 K (see last row in Figure 7). This is very difficult to achieve in practice. A possible solution is to mix passive and active control, by cutting the ambient temperature as much as possible with temperature sensors and heat actuators and to left the rest of the regulation to passive filter due to proper insulation of the cavity.

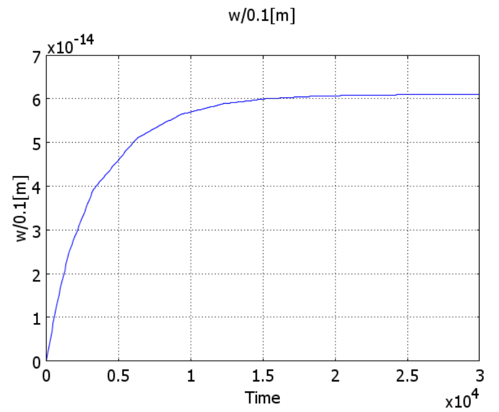

Figure 8. Step response of the cavity expansion for a step like variation of the ambient temperature.

From the transient simulation it can be seen that the system can be modeled as a first order stable dynamics system with time constant close to 3000 seconds or so. This means that any components of the disturbances which are faster than this frequency are going to be filter by the thermal inertia of the cavity.

7. References

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