Rough surface modeling of PDMS polymer through fractal dimensions

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Abstract: A good approach for rough contours or surfaces and their random behaviour is very important in some engineering problems. Fractal modeling can provide an alternative for generate rough geometries based on statistically self-similar non-Euclidean geometries. Through a frequency domain analysis, it is possible to estimate the Hurst exponent of a statistically self-similar geometry with an associated spatial frequency, and extract the fractal dimension from it; this can represent a structure as a set of self-contained unitary cells. Fractal analysis, mixed with Comsol Multiphysics, will be applied in the modeling of rough PDMS polymer surfaces for force sensing resistors, based on AC/DC, solids mechanics and MEMS modules. As a result, the fractal model can generate a statistically similar rough surface to a real surface with a given Hurst exponent. This is an important feature for the study of contact resistance in thin film insulating sensors.

Keywords: PDMS, fractal dimensions, self-similarity, Hurst exponent.

1. Introduction

The use of fractal geometry to model rough surfaces results in a versatile way to representing a self-similar structure containing a spatial cutoff frequency, based on a fractional Brownian (fBm) motion as explained by H. Peitgen *et al.*, in [1]. This representation is useful for estimating parameters such as friction or contact resistance (for thermal and electrical models), with the electrical contact resistance being the focus of this paper.

In particular, the approximation to represent a rough surface is made through the Hurst exponent (H), which describes the long-range dependence (LRD) of a spatial series [2]. This LRD represents the randomness of a surface, based on its trend and its spatial autocorrelation. Fractal dimension (D) have a close relationship with H, described by $D = N - H$ [3], where N is the dimension of the geometry (2 or 3), and H is defined between 0 and 1.

Depending on the frequency parameters set in the model, the representation for a rough surface will be proportional to a Weierstrass-Mandelbrot (WM) function [4], describing the surface as a truncated Fourier series, this implies that the surface will have a certain spatial cutoff frequency. Bearing in mind that fractal behavior could describe a 2D or 3D geometry, there may be different spatial frequencies for each axis depending on the coordinate system and the shape of the geometry.

Undoubtedly, fractal modeling helps to evaluate the effects of contact resistance on the conductive paths of force sensing resistors (FSRs) [5, 6]. This phenomenon is not insignificant in some cases and generates driffted measures that could be predicted with an appropriate model that includes this self-similar surface behavior.

2. COMSOL Implementation

2.1 Surface Generation

To generate a rough surface, it's important to consider the dimensions of the geometry; this will define whether there is more than one spatial frequency. Once the dimension has been selected and the spatial frequencies defined, H is provided; it can be estimated from the geometry to be modelled, using a rescaled range analysis (R/S). The randomness of the surface depends on a pair of random functions, for sweep the phase (θ) and the amplitude (A) of the WM function.

Note that the rough surface will be a representation of a fBm, so H will describe the covariance of a Gaussian process, this is interpreted as: for a higher H, a smoother surface. Therefore, to obtain a more or less roughness in the surface, it's necessary to sweep H between 0 and 1.

A parametric curve (2D or 3D) is defined as a surface proportional to a WM function, as follows:

$$S_{2D}(x) = \sum_{m=0}^{M} a(m)^{-D/2} Cos[2\pi(mx) + \theta(m)] \quad (1)$$

$$S_{3D}(x, y) = \sum_{l=0}^{L} \sum_{m=0}^{M} a(m, l)^{-D/2} Cos[2\pi(mx + ly) + \theta(m, l)] \quad (2)$$

Which has a spatial cutoff frequencies (M,L) and depends on random Gaussian distributions A(m,l) and (m,l), to sweep the amplitude and phase. This geometry has an exponential decay given by the fractal dimension (D), as follows:

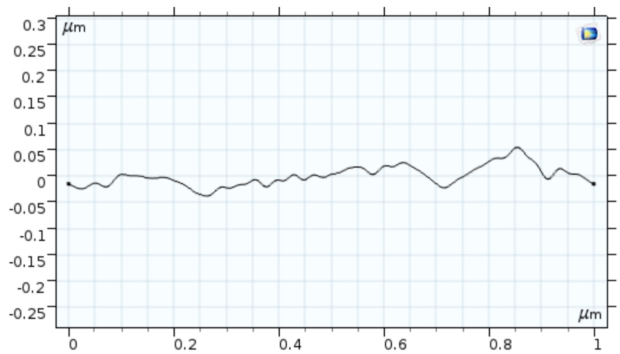

Figure 1. Parametric 2D contour with N=30, H=0.6, D=1.4.

2.2 Physics

Essentially, the geometry generated is compatible with the interactions of AC/DC, solids mechanics and MEMS modules, adapted to model the contact resistance phenomenon. This rough surface will be tested for a viscoelastic polymer creep compliance. In addition, the polymer model is doped with conductive nanoparticles; in areas where surface roughness generates contact between conductive plates and polymer nanoparticles, the boundary condition of contact impedance is considered.

3. Real Geometry

the simplest way to generate a polymer model is to base it on itself. Then, through an analysis of images from an atomic force microscope (AFM) to a nanoparticle-doped polymer sample, as shown in Figures 2,3.

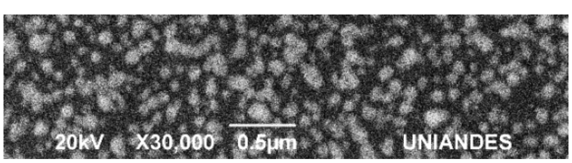

Figure 2. AFM image of nanoparticles distribution.

Fractal dimensional modeling could generate a surface reliable enough for the study of contact resistance, considering also a random deposition of nanoparticles.

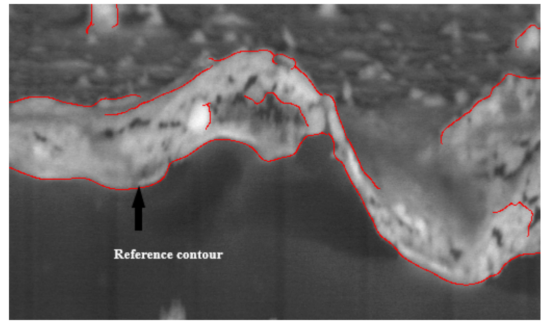

Figure 3. 2D contour, edge highlighting

4. Results and Discussion

Thus, 2D and 3D geometries obtained are intended to be applied in creep compliance and contact resistance tests. Considering the rheological behavior of the polymer, the following geometries were obtained:

4.1 2D Geometry

How it was presented in eq. (1), the two-dimensional geometry is described by D, N, M and H. With H=0.4 and M=50, this leaves some spatial characteristics with which the contour of the geometry will have a discrete cut-off frequency (M=50) and will tend to decrease since the Hurst exponent is less than 0.5. This statistically indicates that the probability of decreasing for the WM function is higher.

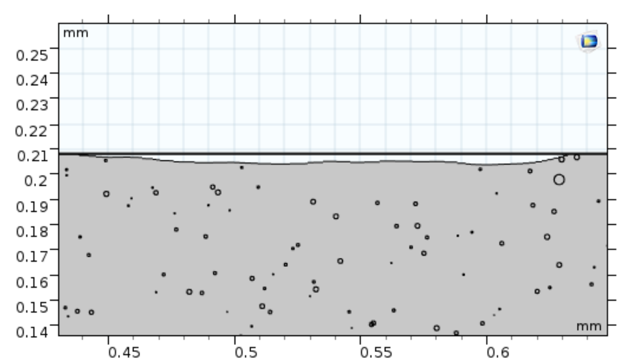

Figure 4. 2D fractal generated contour, M=50, D=1.6 and H=0.4.

The contour obtained has the necessary properties to apply the creep compliance and contact resistance tests, since being a contour that tends to decrease, valleys are obtained between the contact plates and the polymer, this makes possible a creep compliance study where the strain is not regular in all directions, and allows the study of the phenomenon of contact resistance in areas where there are nanoparticles that touch contact plates.

4.2 3D Geometry

Similar to the two-dimensional case, but now the aim is to generate a tridimensional cylindrical geometry with a highly roughened surface that matches the same considerations for creep compliance and contact resistance tests.

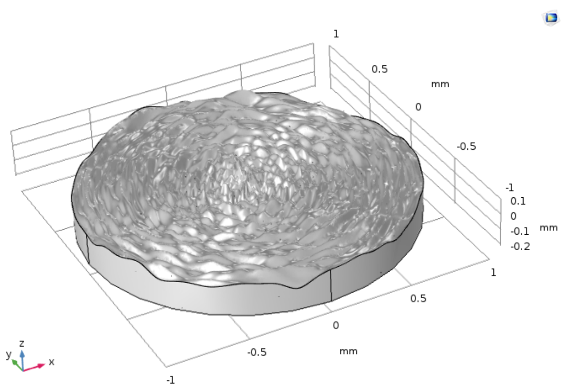

Figure 5. 3D fractal generated surface, M=L=30, D=2.95 and H=0.05.

As result, with H=0.05, could be seen that the surface tends to decay and generate a large number of valleys, suitable for the study of non-regular creep compliance and contact resistance.

4.3 Creep Compliance Test

At a pressure of 200 [kPa] applied to a polymer sample on the border, the expected irregular strain occurred for a stationary study, using the solid mechanics module.

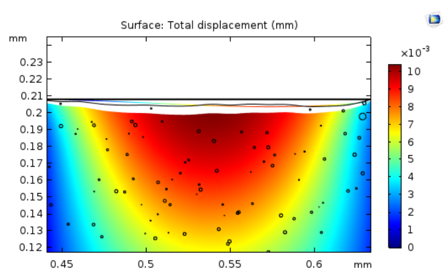

Figure 6. Creep compliance test with irregular strain.

5. Conclusions and Further Work

Using this method to generate rough surfaces with a fractal dimension defined by a Hurst exponent, results in contours or surfaces with the desired roughness level in which creep compliance and contact resistance can be studied. This process generates valleys in the geometry, which are necessary to study the changes produced in the conductive paths of a PDMS sample doped with conductive nanoparticles and subjected to a certain strain.

Furthermore, an image processing edge detection filter stage will be added to perform a previous scaled range (R/S) analysis of the contour or surface images to be modeled in order to select the most appropriate Hurst exponent for represent the geometry.

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6. Acknowlegments

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