COMSOL CONFERENCE 2018 LAUSANNE

Determination of Constitutive Properties using DIC-Displacement Data and U-FEM

A. A. Alshaya, R. I. Bourisli, J. M. Considine

Kuwait University – Kuwait

Forest Products Laboratory, Madison

OBJECTIVES AND MOTIVATION

- Objective and Motivation
  - Determines constitutive properties of a paperboard laminated composite.

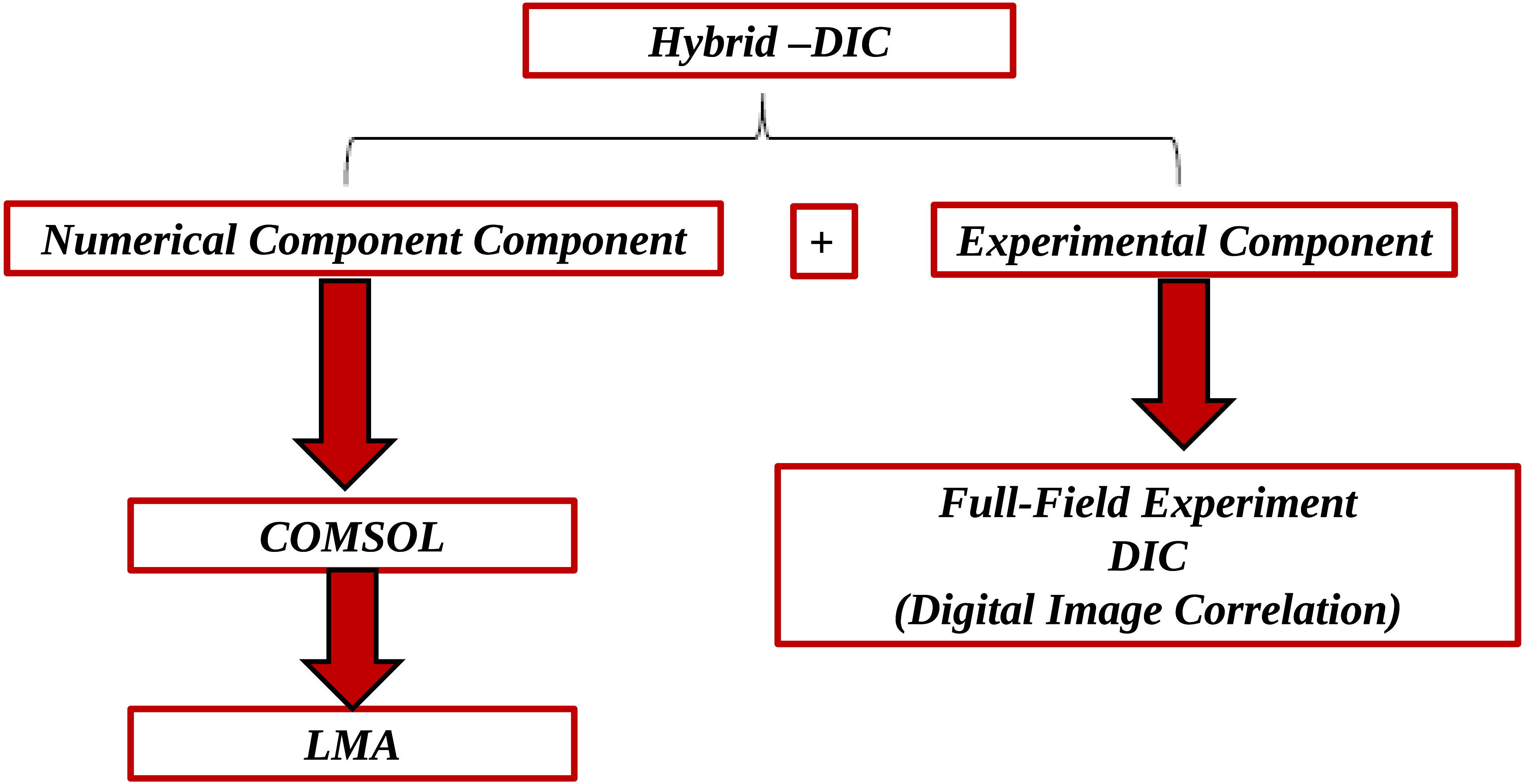
- Hybrid Approach
  - Experimental Components
    - ✓ Digital Image Correlation (DIC)
  - Numerical Component
    - ✓ COMSOL LiveLink
    - ✓ Levenberg-Marquardt Algorithm (LMA)

- Applications and Method Validation
  - Composite plate with circular notch - Orthotropic

Perforated Sprockets

Engineering structures

Hybrid – Approach



Problem Definition and governing Equation

The governing equations for anisotropic elasticity consist of
1- Equilibrium equations,
2- Strain-displacement relations (small deformations)
3- Stress-strain laws for linear anisotropic elastic solids,

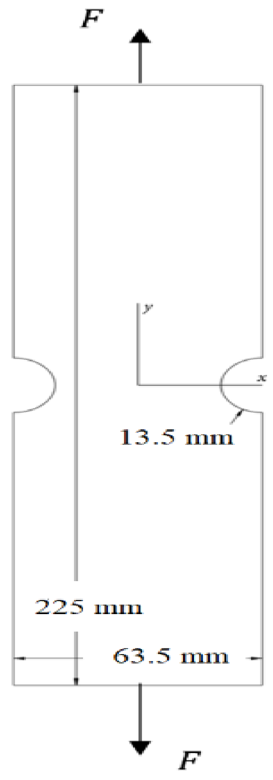
$$\frac{\partial \sigma_{ij}}{\partial x_j} + f_i = 0, \qquad i = 1,2,3$$

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \qquad i,j = 1,2,3$$

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}, \qquad i,j,k,l = 1,2,3$$

Where:
$\sigma_{ij} = \sigma$ is stress tensor,
$\varepsilon_{ij} = \varepsilon$ is strain tensor,
$u_i = u$ is displacement vector,
C_{ijkl} is the 4th-order stiffness tensor,
f_i is the body forces.

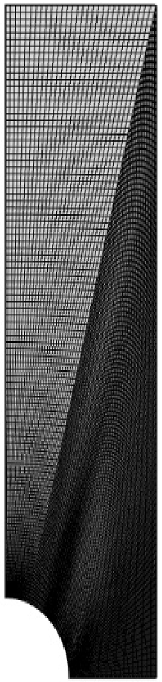
15 equations with 15 unknown functions $u_i, \varepsilon_{ij}, \sigma_{ij}$, $i, j = 1,2,3$, in terms of three spatial coordinate variables x_i, $i = 1,2,3$.



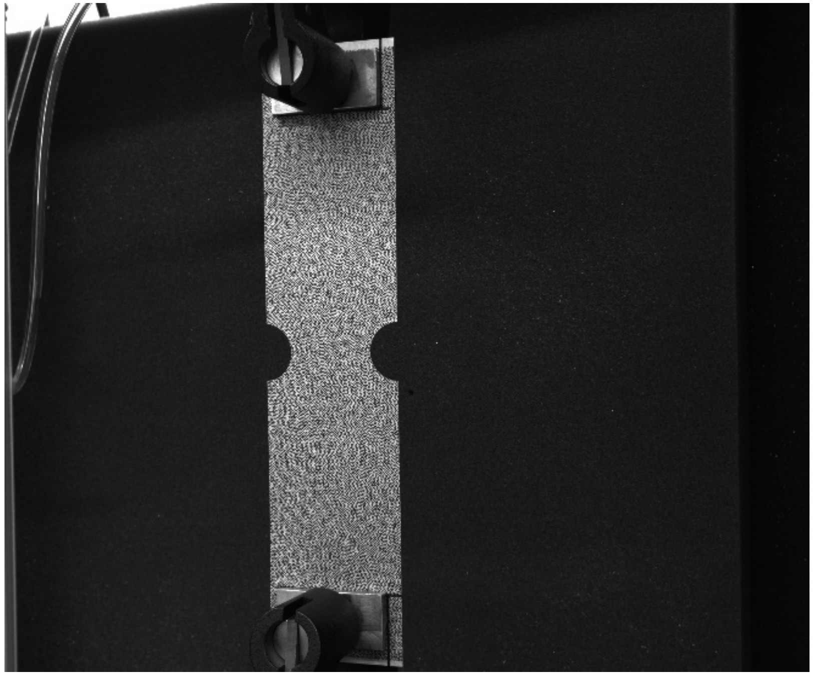
Vertically-loaded finite paperboard composite plate with circular side notches.

Plate Geometry

Material Properties		Plate Geometry	
$E_x = E_2$ (GPa)	2.12	Notch radius, R (mm)	13.5
$E_y = E_1$ (GPa)	4.52	Plate Length, L (mm)	225
$G_{xy} = G_{12}$ (GPa)	1.27	Plate Width, W (mm)	63.5
$\nu_{xy} = \nu_{21}$	0.3838	Plate thickness, t (mm)	0.31



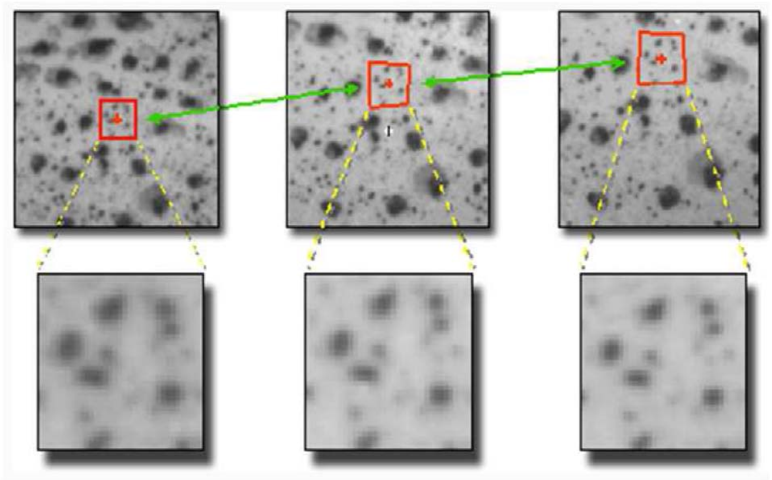
Vertically-loaded finite paperboard composite plate with circular side notches.



Digital Image Correlation

- Contemporary, non-contacting, non-destructive experimental method.
- Provides displacement/strain info
- Speckle pattern
- Rapid data acquisition

Tracking motion of speckle pattern by comparing gray scale value at a point (subset) in deformed and undeformed configuration



DIC method to track gray value pattern in small subsets [1]

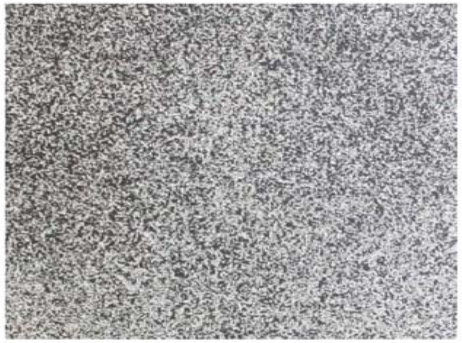
- Achievable DIC resolution depends on
  - ➢ camera resolution,
  - ➢ lens optical quality,
  - ➢ speckle size and quality.

[1] Correlated Solutions [online]: http://www.correlatedsolutions.com/index.php/principle-of-digital-image-correlation

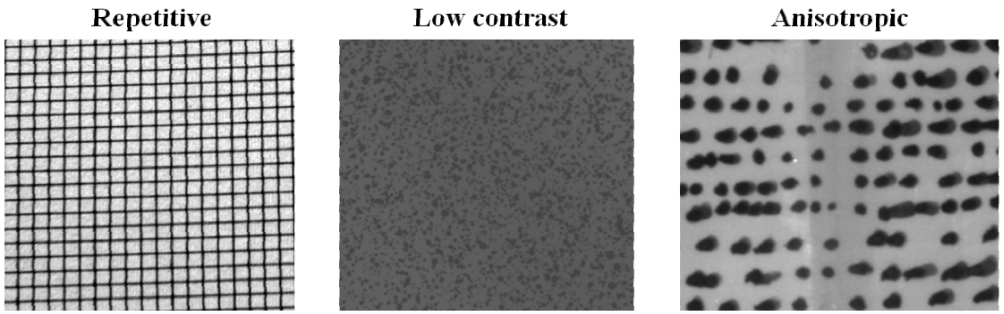
DIC: Test Preparation

- Speckle pattern has to be applied to the structure (Black and White Ultra-Flat paint)

Record images of specimen in its loaded and unloaded conditions using Vic-Snap software (by Correlated Solutions, Inc.).



- Speckle pattern must be non-repetitive (random), isotropic (not exhibit a bias in any orientation) and high contrast (show dark blacks and bright whites)

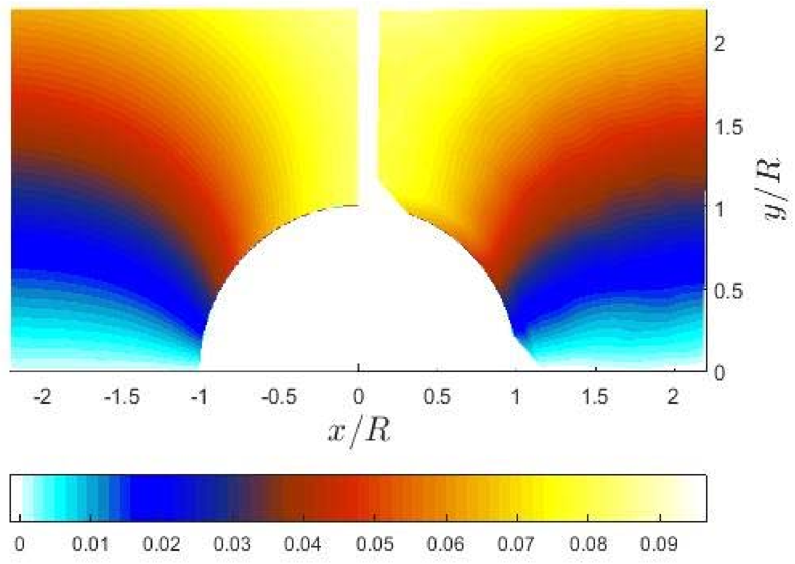


Technical Information

Parameter	Setting
Technique	Stereo Image Correlation
Cameras	The Grasshopper (Point Grey Research), Model GRAS-50S5M-C
Imaging sensor	Sony ICX625 CCD, 2/3'', 3.45 μm
Lens	CM120 BK 15 COMPACT-0901 (focal ratio: 1.9 and focal length: 35 mm)
Sensor/digitization	2448×2048 at 15 FPS
Lightening	Ambient white light
Pixel to inch conversion	1 pixel = 0.01 inch
Software	MatchID software.
Subset, step	21, 10
Strain Resolution	0.005% (50 microstrain)

DIC Data

- DIC MatchID software provided approximately 8,160 values of u and v
- Plate is geometrically and mechanically symmetrical about the vertical y-axes
- Zero vertical displacement was shifted to be at the horizontal middle of the plate.
- Averaging v-displacement data (cancel any asymmetry)



Inverse Method Procedure

- Levenberg-Marquardt Algorithm

$$f(\hat{v}_{FEM}, P) = \|r\|, \qquad \text{where } r = \hat{v}_{DIC} - \hat{v}_{FEM}$$

where $\hat{v}_{FEM}$ and $\hat{v}_{DIC}$ are vector containing nodal v^* displacement determined by FEM (COMSOL) and DIC, respectively

P is a vector containing the constitutive parameters, $E_1, E_2, \nu_{12}, G_{12}$.

$\|r\|$ is the norm of r

$$P_{i+1} = P_i - \left(J^T J + \lambda \cdot \operatorname{diag}(J^T J)\right)^{-1} J^T r$$

where i is iteration number

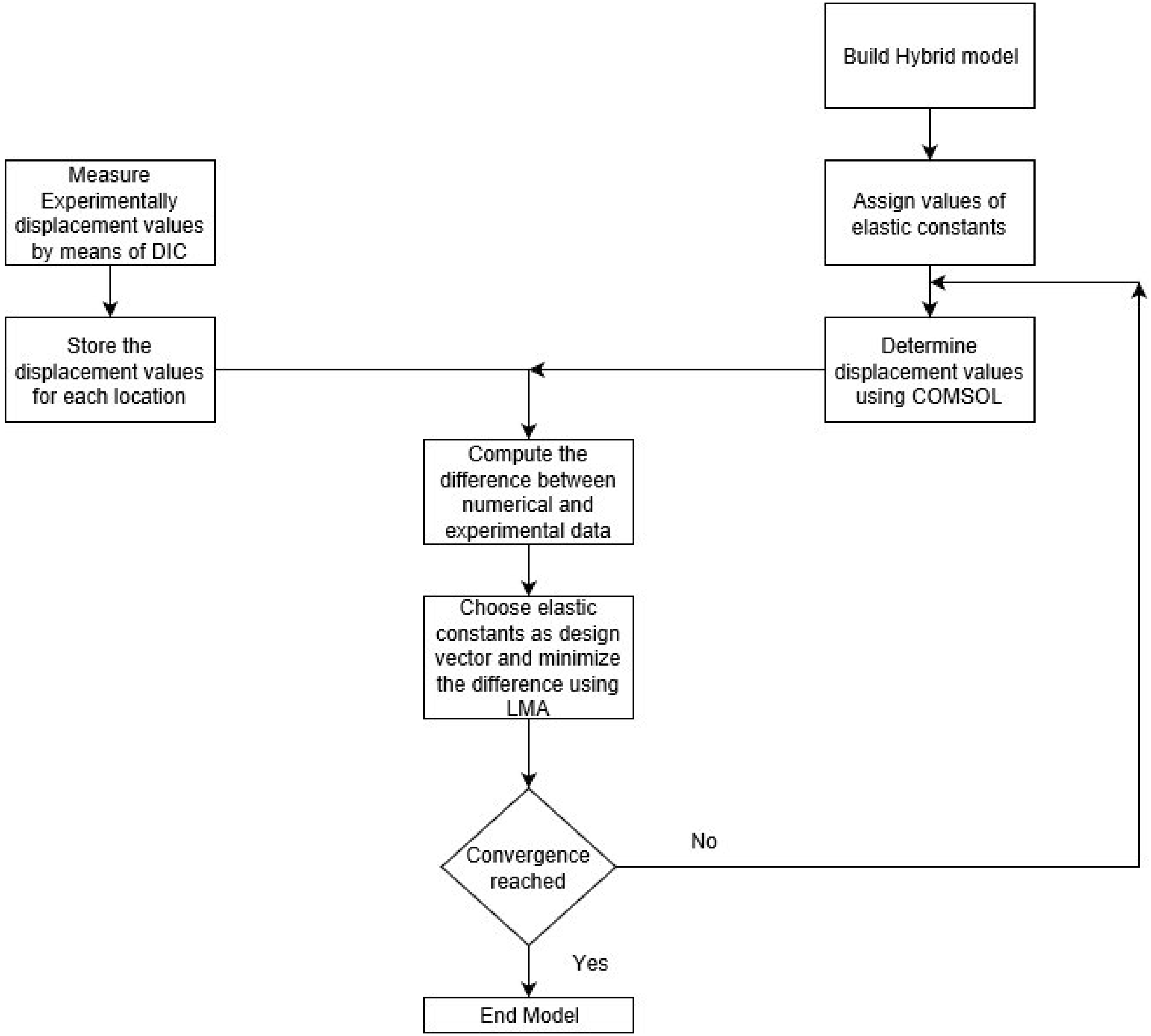
J and J^T are Jacobian and Jacobian transpose, determined by backward difference, $J_{m,n} = \frac{\partial r_m}{\partial P_n}$

m is number of nodal S^* values

n is number of constitutive parameters (6 in this work)

λ is non-negative damping factor, adjusted each iteration step, adjusts between Steepest Descent Method and Gauss-Newton Method

Inverse Method Procedure



Inverse Method Procedure

- Initial Guesses

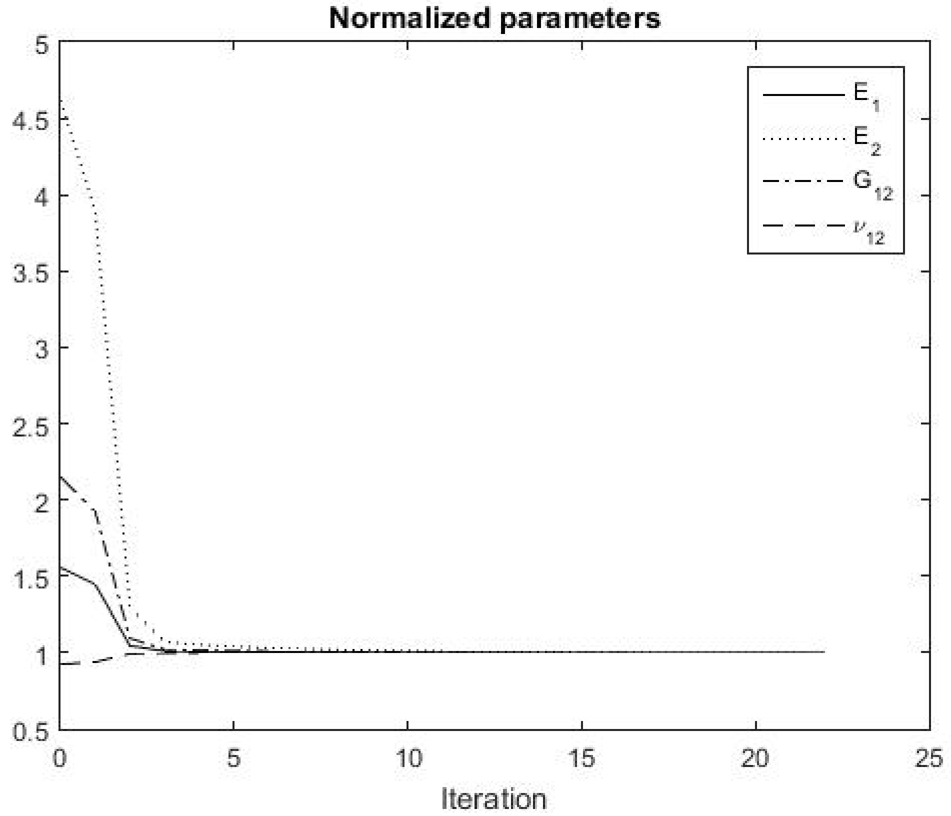
$$\begin{Bmatrix} \boldsymbol{E_x^0} \\ \boldsymbol{E_y^0} \\ \boldsymbol{G_{xy}^0} \\ \boldsymbol{\nu_{xy}^0} \end{Bmatrix} = \boldsymbol{E} \begin{Bmatrix} \boldsymbol{R_1 E_x} \\ \boldsymbol{R_2 E_y} \\ \boldsymbol{R_3 G_{xy}} \\ \boldsymbol{R_4 \nu_{xy}} \end{Bmatrix}$$

$\boldsymbol{E}$ is the maximum absolute random error (user specified), 50, 100, 200 and 400% were used

$\boldsymbol{R_1}, \boldsymbol{R_2}, \boldsymbol{R_3}$, and $\boldsymbol{R_4}$ are independent generated random numbers ($\boldsymbol{0 \leq R_i \leq 1}$, $\boldsymbol{i = 1, 2, 3, 4}$)

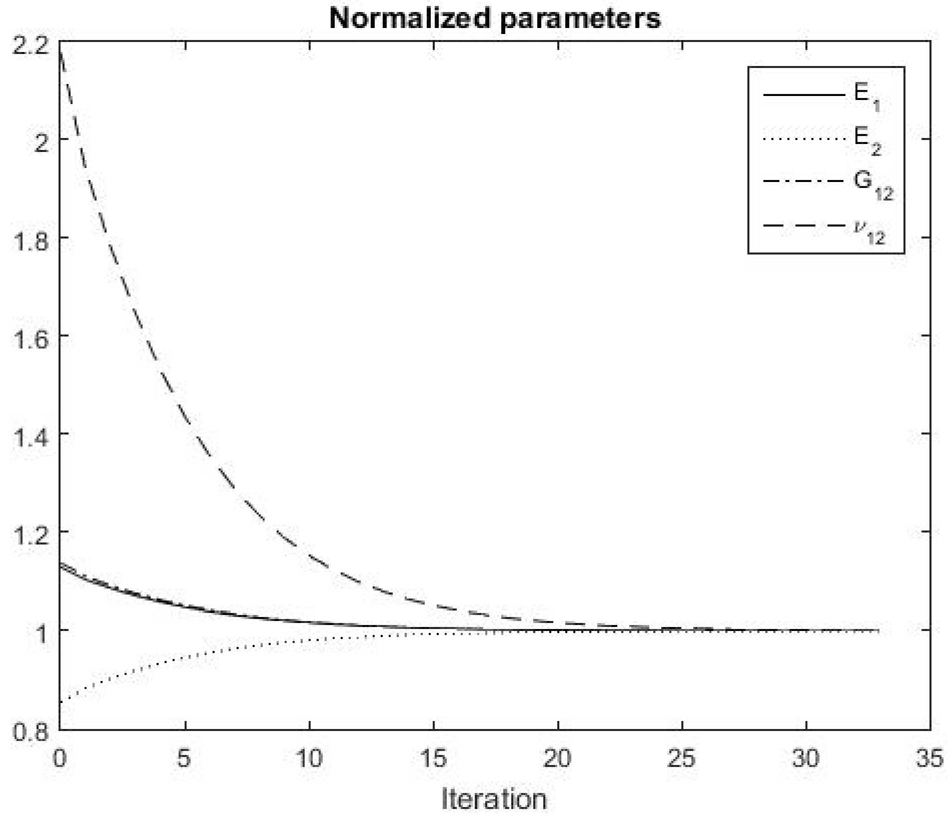
Numerical Experiment

- Using COMSOL v-displacement to verify the reliability and robustness of the inverse method.



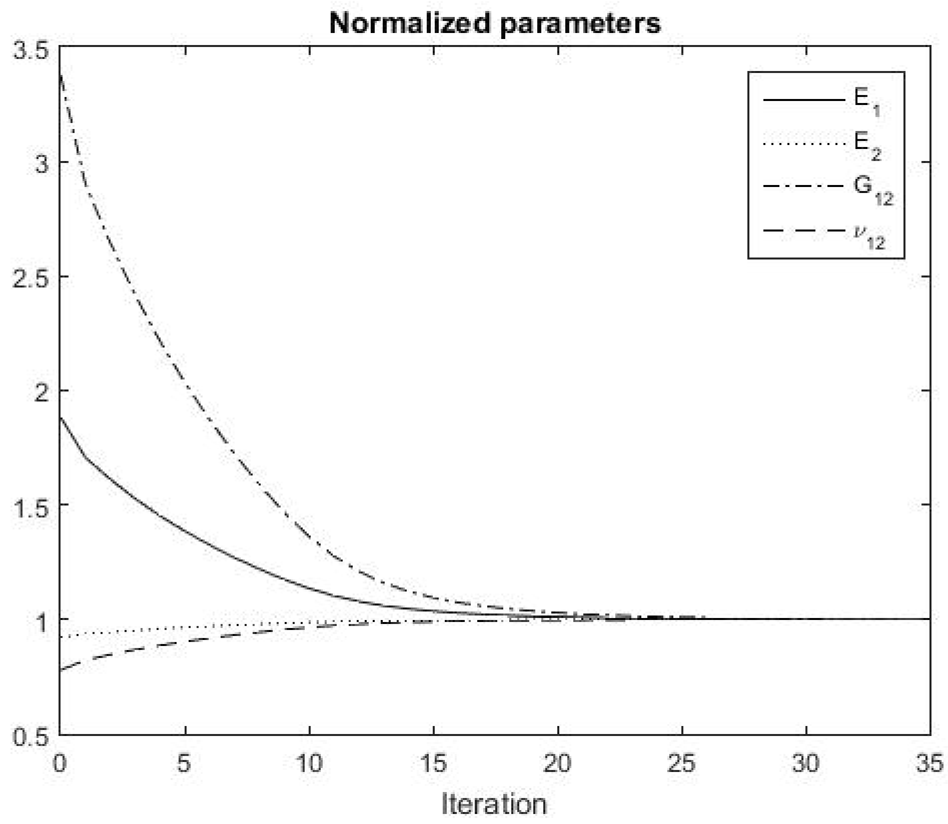
Numerical Experiment

- Using COMSOL v-displacement to verify the reliability and robustness of the inverse method.



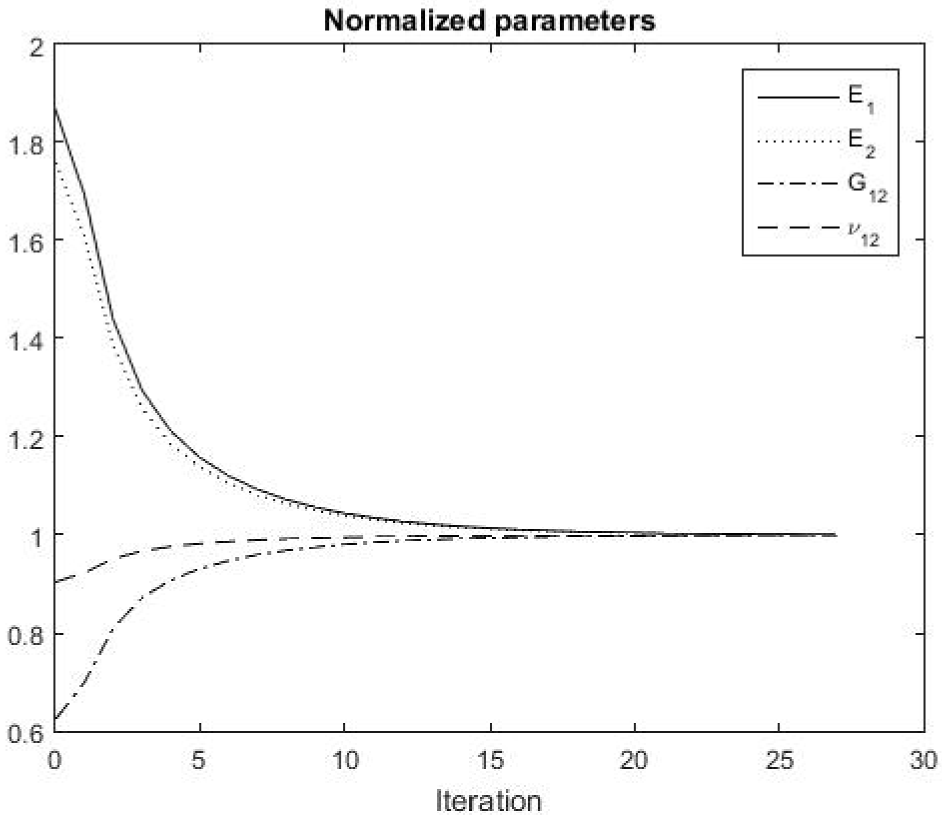
Numerical Experiment

- Using COMSOL v-displacement to verify the reliability and robustness of the inverse method.

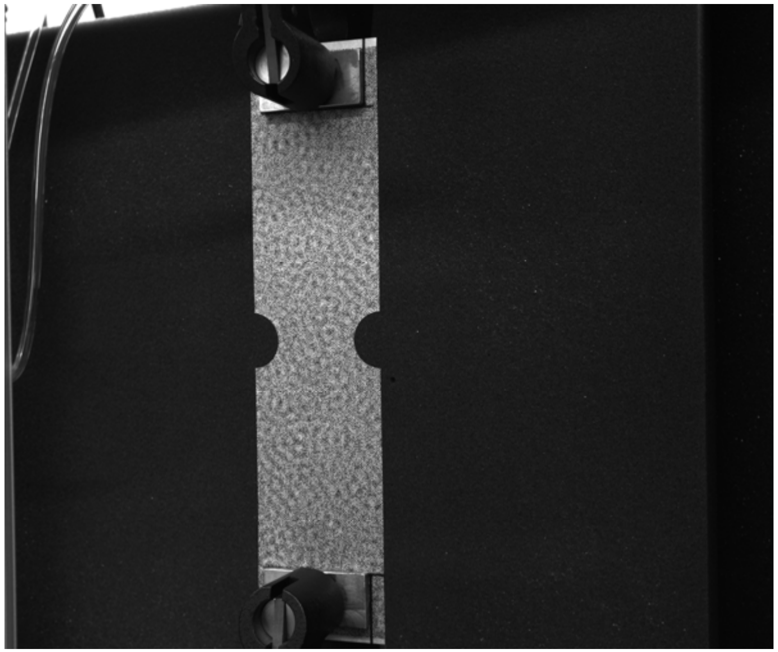


Numerical Experiment

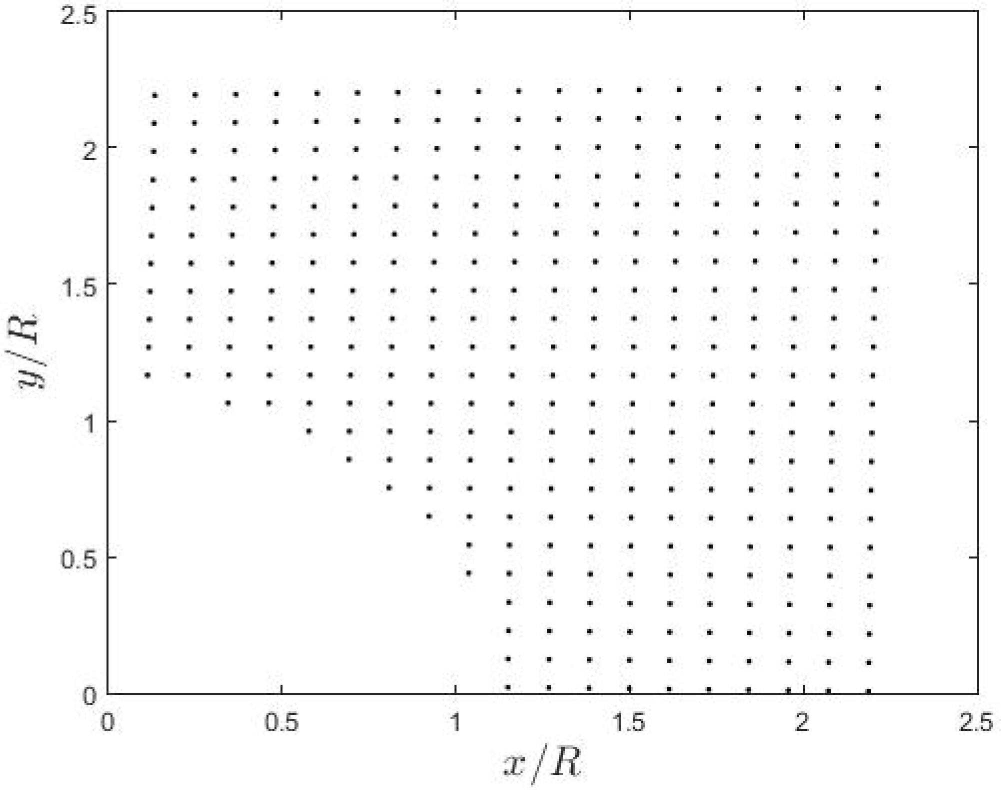
- Using COMSOL v-displacement to verify the reliability and robustness of the inverse method.



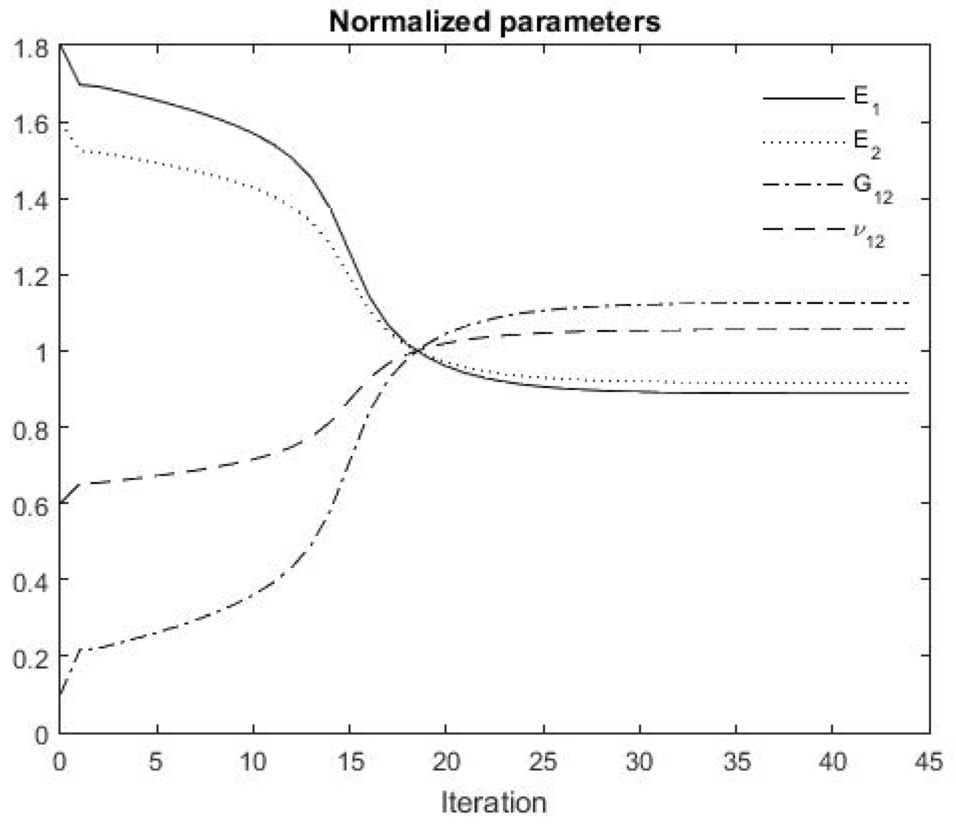
DIC Results



DIC Results

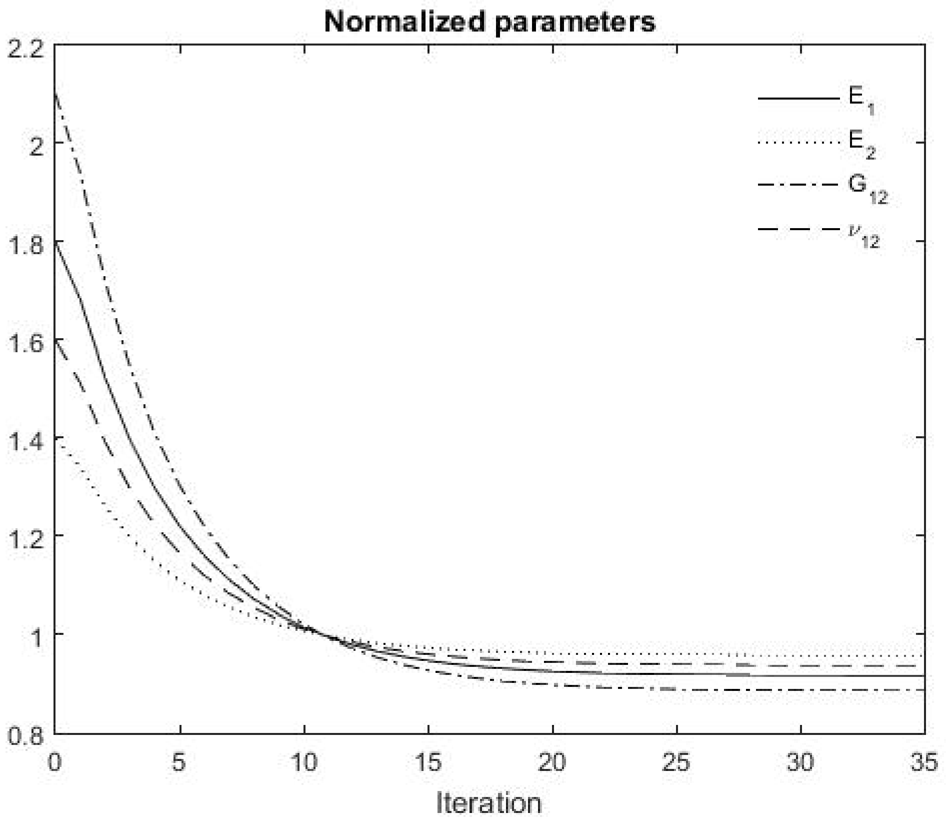


DIC Results



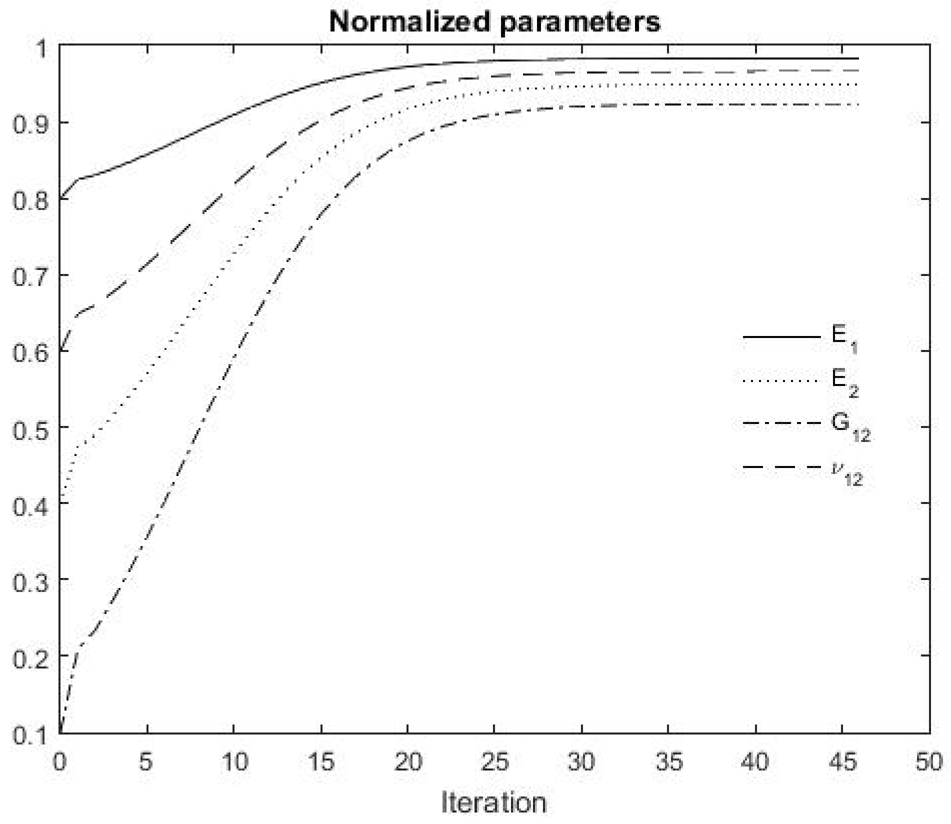
Normalized parameters
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
30
35
40
45
Iteration
E_1
E_2
G_{12}
ν_{12}

DIC Results



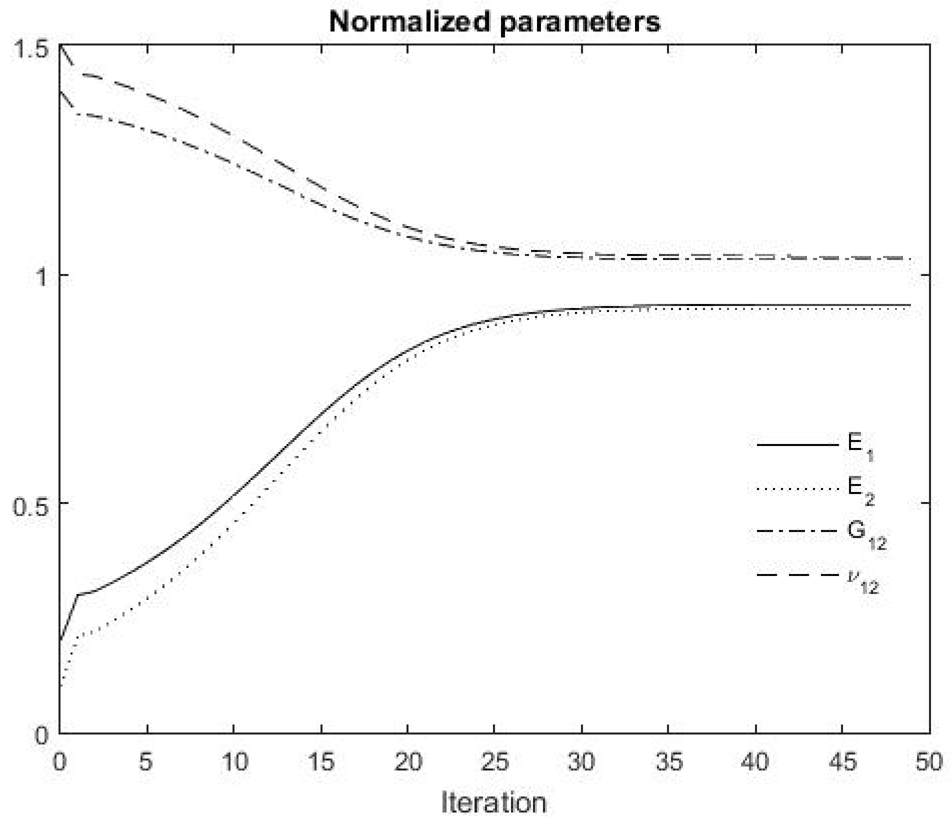
Normalized parameters
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0
5
10
15
20
25
30
35
Iteration
E1
E2
G12
ν12

DIC Results



Normalized parameters
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
5
10
15
20
25
30
35
40
45
50
Iteration
E1
E2
G12
ν12

DIC Results



Normalized parameters
1.5
1
0.5
0
0
5
10
15
20
25
30
35
40
45
50
Iteration
E1
E2
G12
ν12

DIC Results

Predicted values of constitutive properties of paperboard using inverse method and COMSOL LiveLink for different number of generated initial guesses

# It.	Error (%)	E_x (GPa)	E_y (GPa)	G_{xy} (GPa)	ν_{xy}
10	50	2.21 ± 0.3110	4.23 ± 0.0769	1.33 ± 0.1787	0.4007 ± 0.0658
20		2.20 ± 0.3258	4.26 ± 0.1035	1.26 ± 0.1408	0.3931 ± 0.0293
50		2.29 ± 0.4498	4.28 ± 0.1283	1.29 ± 0.1802	0.3928 ± 0.0580
100		2.14 ± 0.3661	4.29 ± 0.1270	1.27 ± 0.2300	0.3745 ± 0.0865
10	100	2.18 ± 0.2426	4.28 ± 0.1151	1.31 ± 0.1553	0.3702 ± 0.0427
20		2.19 ± 0.5010	4.19 ± 0.3907	1.33 ± 0.1449	0.3935 ± 0.0634
50		2.25 ± 0.5233	4.24 ± 0.2705	1.31 ± 0.2294	0.3936 ± 0.0573
10	200	1.94 ± 0.3848	5.59 ± 2.9432	1.62 ± 0.5192	0.4718 ± 0.2119
20		2.14 ± 0.9159	4.90 ± 2.0482	1.54 ± 0.5450	0.4519 ± 0.1707
50		2.21 ± 0.7392	4.97 ± 2.0956	1.38 ± 0.6429	0.4459 ± 0.1879
10	400	2.07 ± 0.2670	4.28 ± 0.0820	1.26 ± 0.2541	0.3685 ± 0.0266
20		2.42 ± 1.3748	4.35 ± 6.5064	2.69 ± 2.1909	0.7786 ± 0.5229
50		2.19 ± 1.4226	4.45 ± 6.9325	1.29 ± 2.6975	0.3685 ± 0.5964

Target values: E_x = 2.12 GPa, E_y = 4.52 GPa, G_{xy} = 1.27 GPA, ν_{xy} = 0.3838.

Conclusion

- Determination of Constitutive Properties using
  - COMSOL LiveLink
  - Levenberg-Marquardt Algorithm
  - One single measured displacement data
- Advantages:
  - Direct use of single test to determine stresses and displacements
- Error less than 10%.
- Applicable for any loading and boundary conditions.

Thank you