Presented at the COMSOL Conference 2010 Boston

Modeling an Optical "Black Hole" with True Gaussian Beam Illumination

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Nanophotonics and Metamaterial Group

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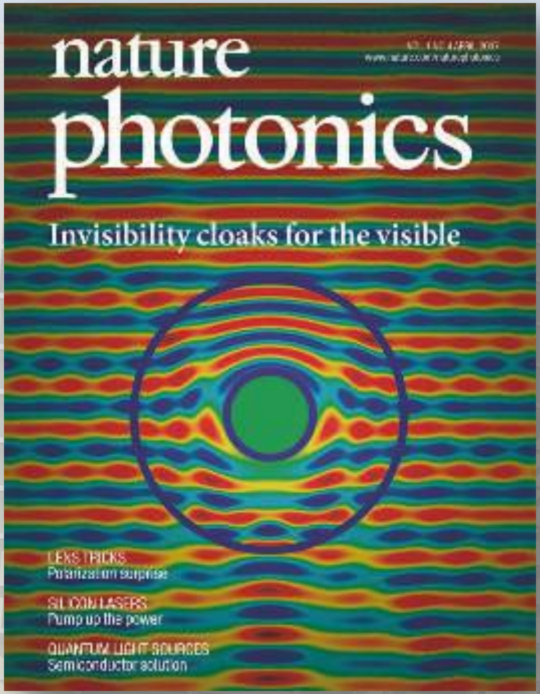
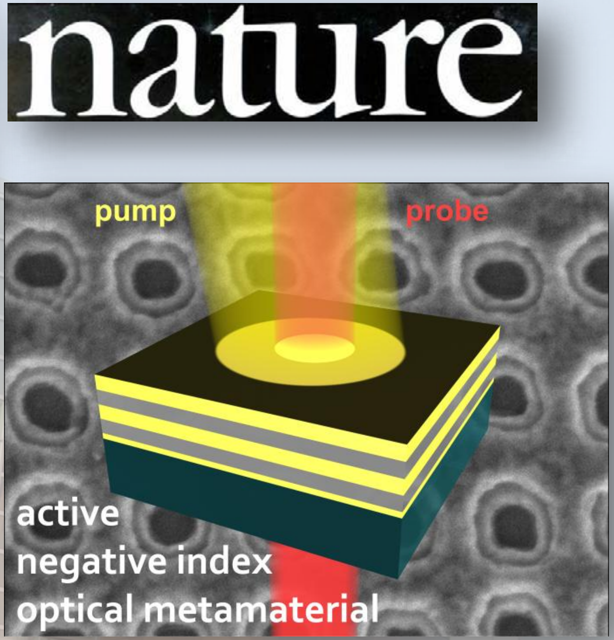
- Q. Song, et al, *Opt. Lett.* **35**(15), 2624-2626 (2010)
- Y. Sivan, et al, *Opt. Express* **17**(26), 24060-24074 (2009)
- S. Xiao, et al, *Opt. Lett.* **34**(22), 3478-80 (2009)
- J. Borneman, et al, *Opt. Express*, **17**(14), 11607-17 (2009)
- Z. Liu, et al, *Metamaterials* **2**, 45-51 (2008)
- ...



W. Cai, U. K. Chettiar, A. V. Kildishev, and V. M. Shalaev, *Nature Photonics* **1**, 224-27 (2007)

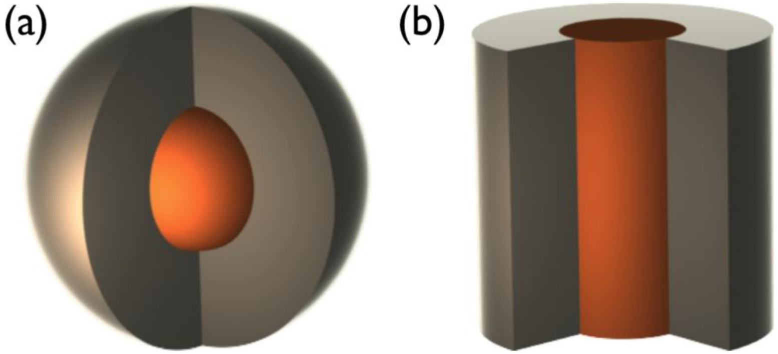


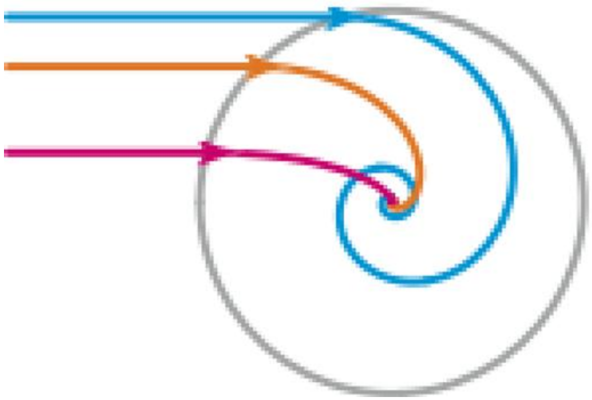
S. Xiao, V. P. Drachev, A. V. Kildishev, X. Ni, U. K. Chettiar, H.-K. Yuan, and V. M. Shalaev, *Nature* **466**, 735-738 (2010)

A Black Hole

Optical "Black Hole"

- broadband omnidirectional light absorber
- absorbs surrounding light like a real black hole
- already made experimentally in microwave region



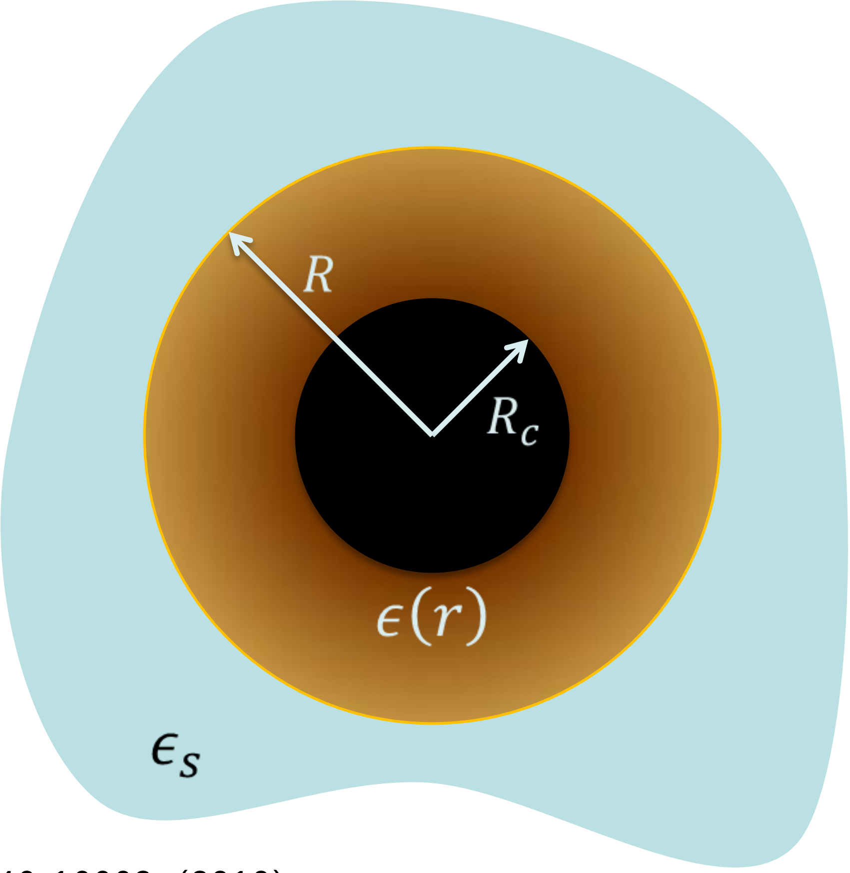


E. E. Narimanov and A. V. Kildishev, *Appl. Phys. Lett.* **95**, 041106 (2009)

Modeling the Materials

$$\epsilon(r) = \begin{cases} \epsilon_s, & r > R \\ \epsilon_s \left(\frac{R}{r}\right)^p, & R_c < r \le R \\ \epsilon_c + i\gamma, & r \le R_c \end{cases}$$

$$\epsilon_c = \epsilon_s \left(\frac{R}{R_c}\right)^p$$



A. V. Kildishev, L. J. Prokopeva, et al, *Opt. Express* **18**, 16646-16662, (2010)

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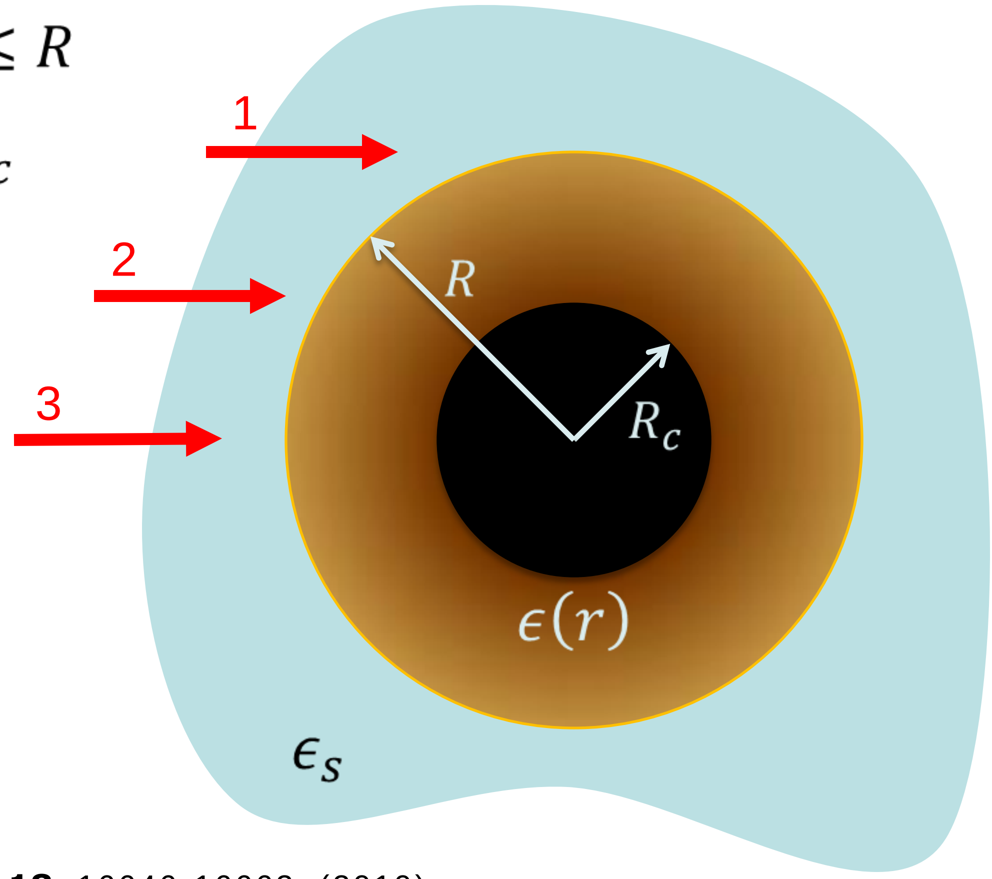

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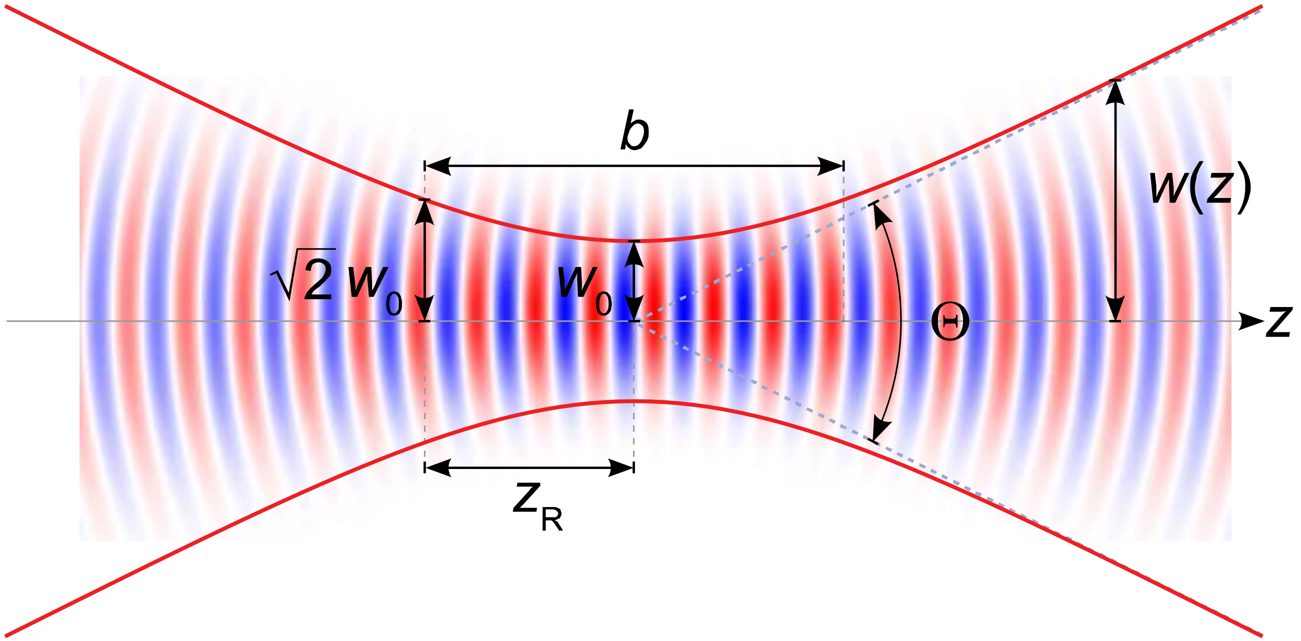
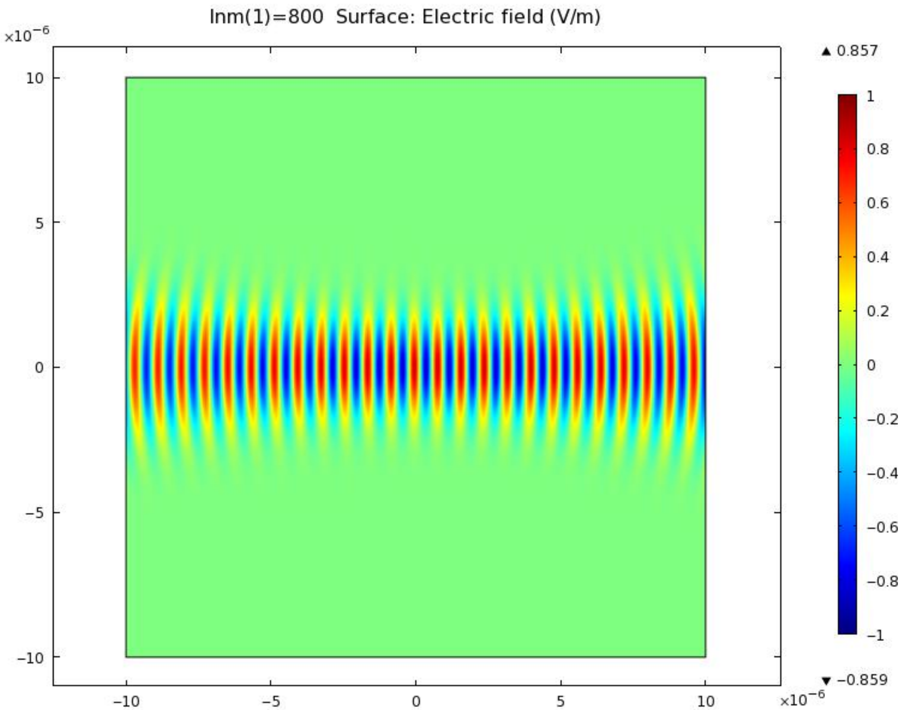
Modeling the Gaussian Beam

- the classical paraxial approximation

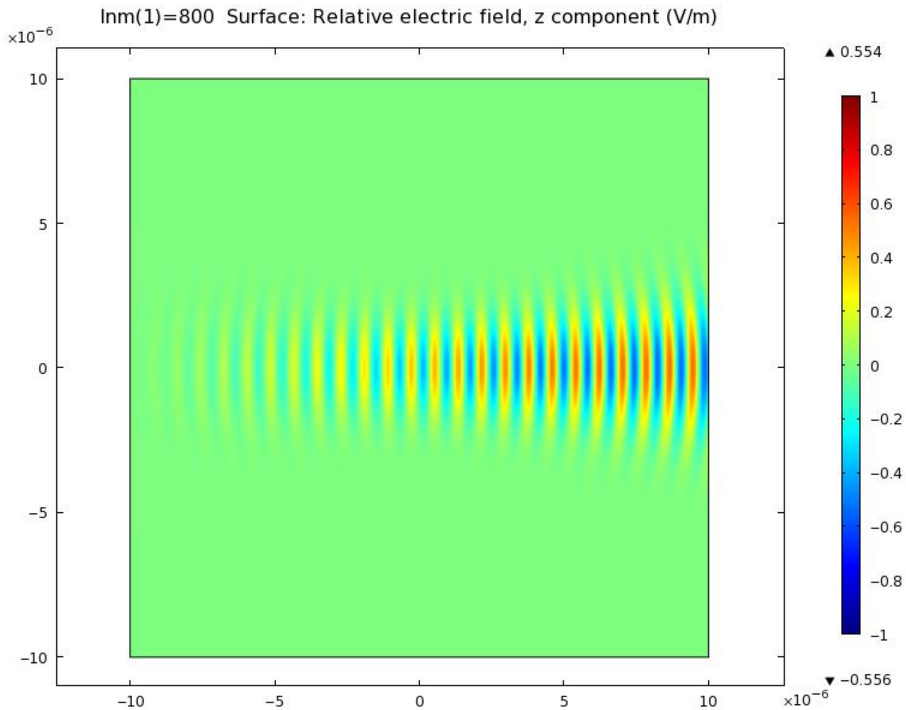
$$E(r,z) = E_0 \frac{w_0}{w(z)} \exp\left(\frac{-r^2}{w^2(z)}\right) \exp\left(-ikz - \frac{ikr^2}{2R(z)} + i\zeta(z)\right)$$



However …

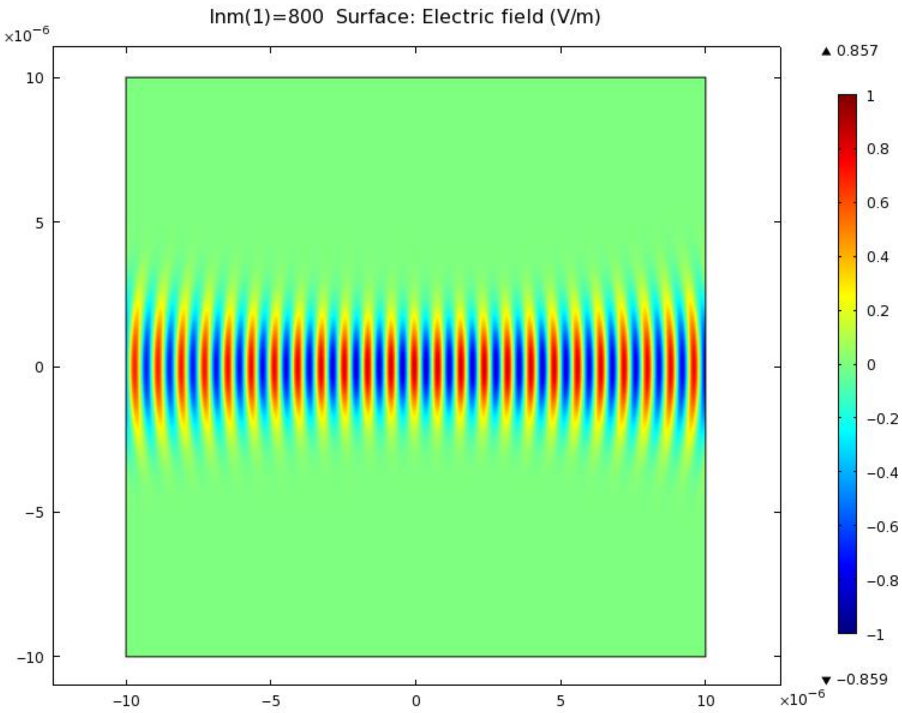


total field

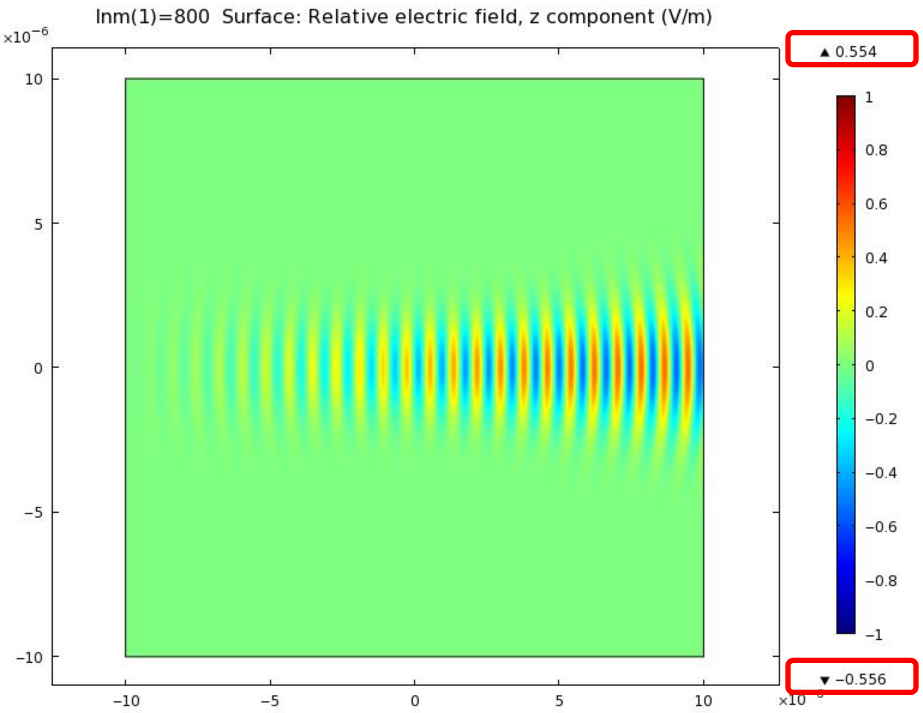


scattered field

However ...



total field



~ 0.55

scattered field

Plane Wave Expansion

- Helmholtz wave equation

$$\begin{cases} \nabla^2 f + k^2 f = 0 \\ f(0,y) = \exp\left[-(y/w)^2\right] \end{cases}$$

$$f(x,y) = \int_{-1}^{1} a_q \exp\left[\imath k\left(qy + \sqrt{1-q^2}\,x\right)\right] dq + \int_{|q|>1} a_q \exp\left[\imath kqy - k\sqrt{q^2-1}\,|x|\right] dq$$

$$a_q = \tfrac{1}{2\sqrt{\pi}} kw \exp\left[-\tfrac{1}{4}(kwq)^2\right]$$

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$\sim 10^{-19}$

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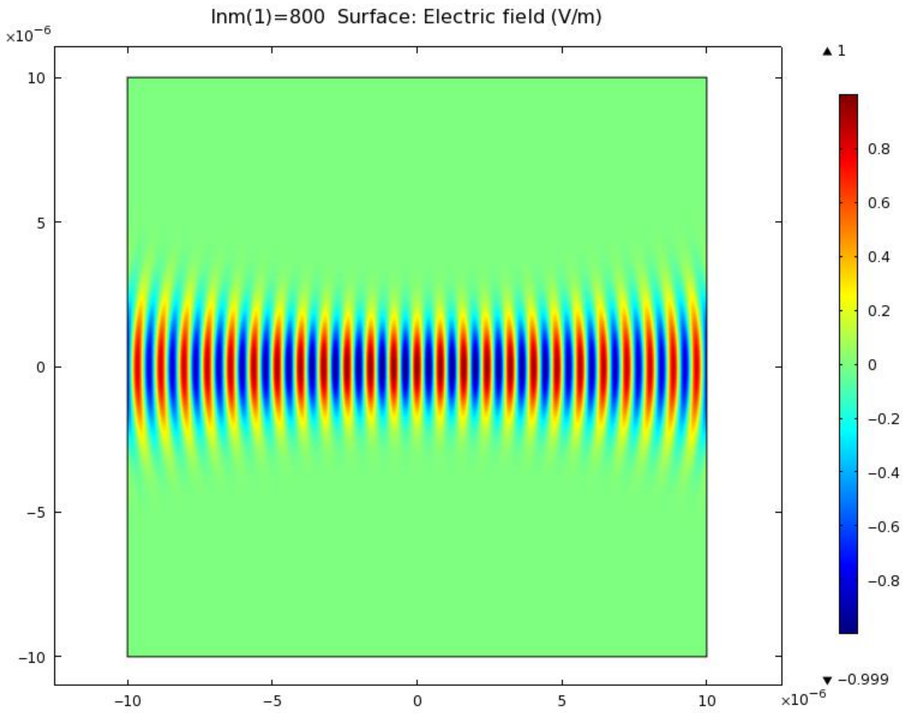
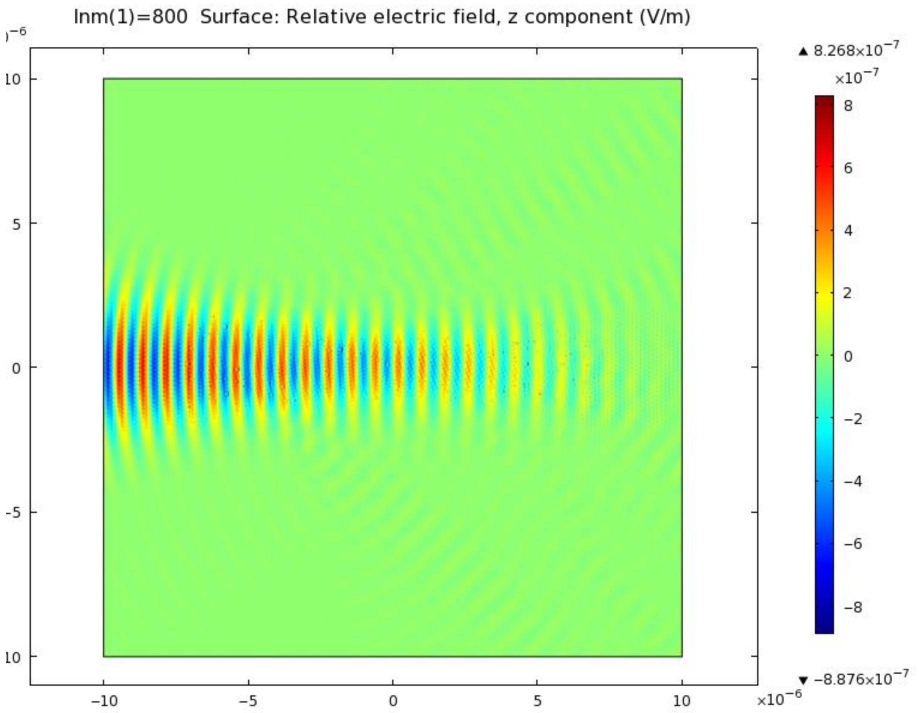
Improved Modeling

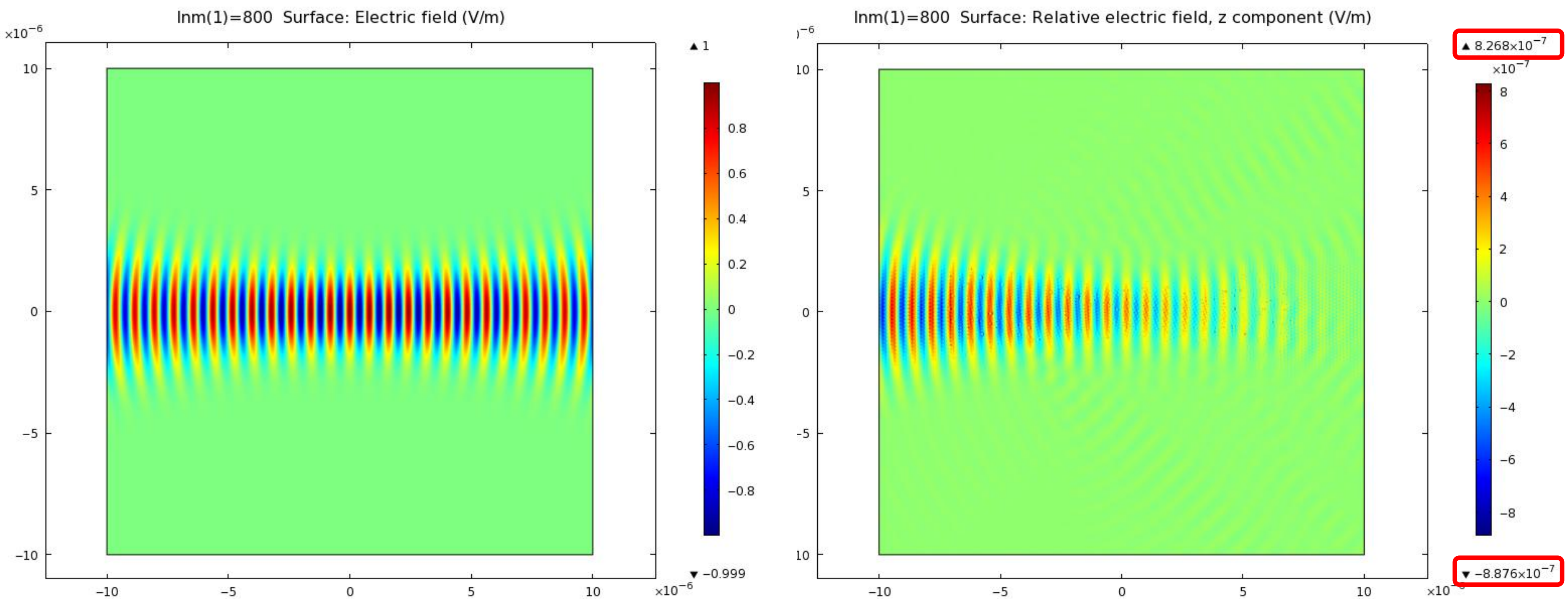


total field



scattered field

Improved Modeling

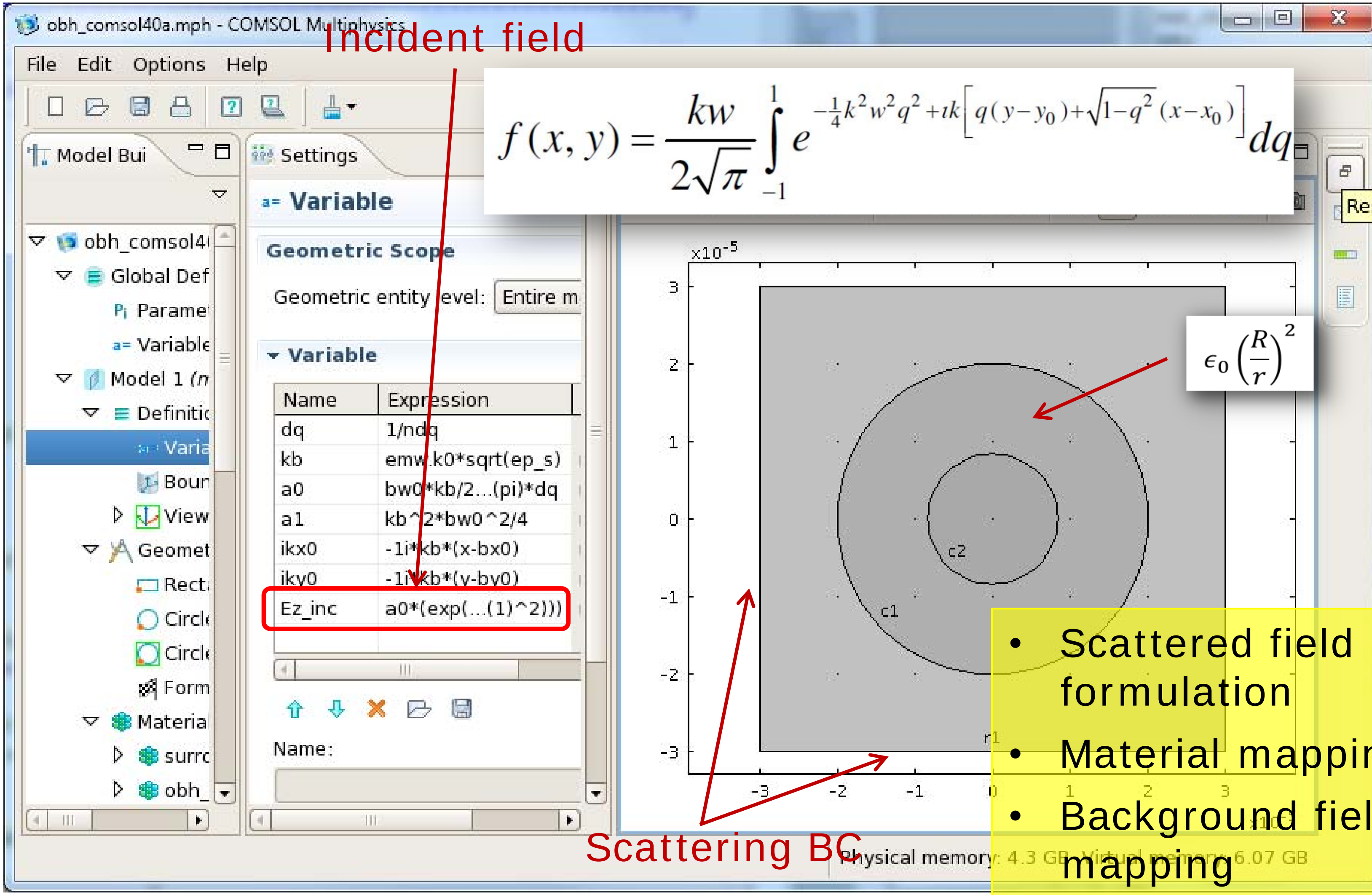


total field

scattered field

$\sim 10^{-7}$

Model the Optical "Black Hole" in COMSOL

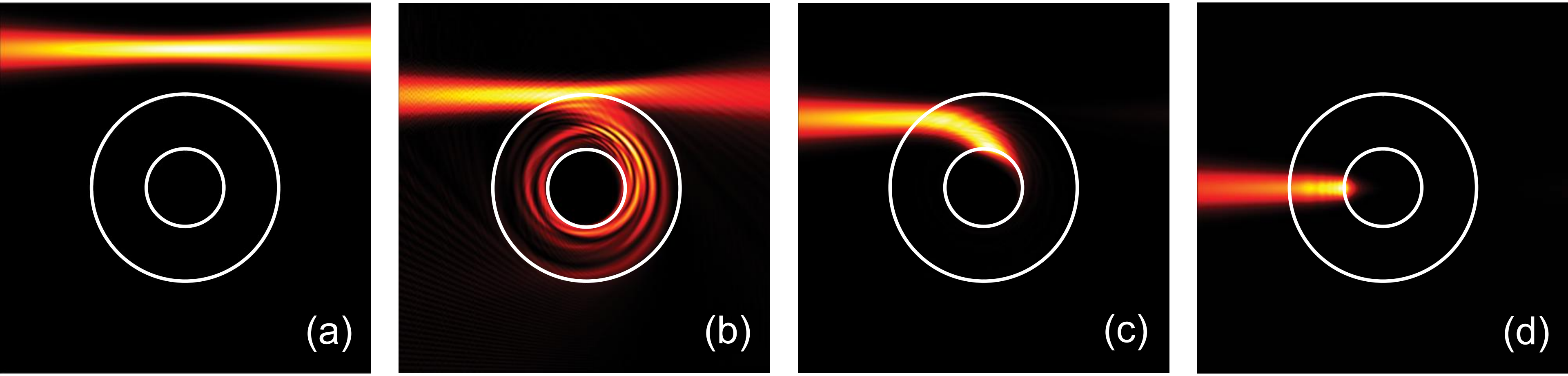


$$f(x,y)=\frac{kw}{2\sqrt{\pi}}\int_{-1}^{1} e^{-\frac{1}{4}k^2w^2q^2+\imath k\left[q(y-y_0)+\sqrt{1-q^2}\,(x-x_0)\right]}dq$$

$$\epsilon_0\left(\frac{R}{r}\right)^2$$

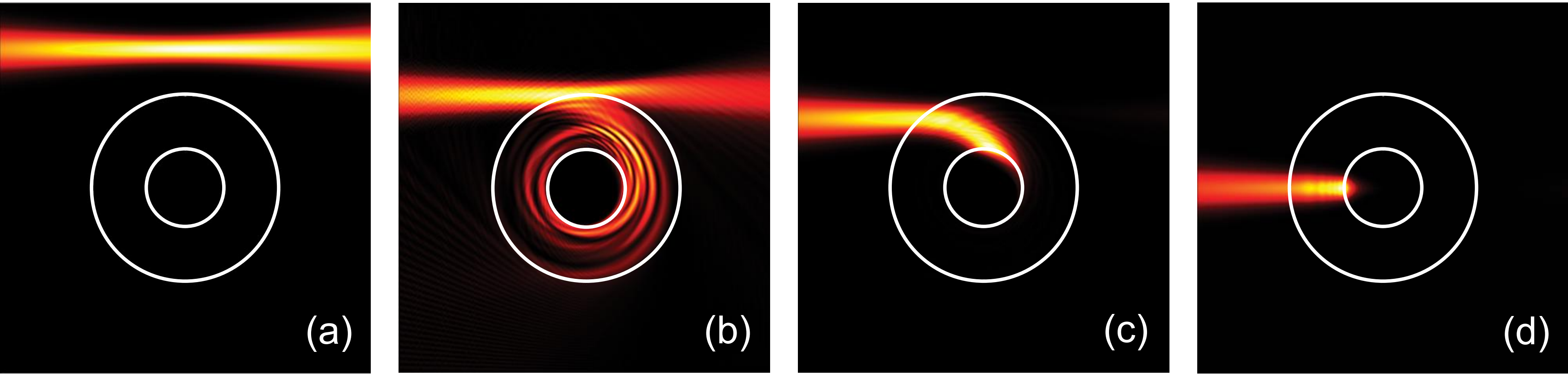
- Scattered field formulation
- Material mapping
- Background field mapping

Optical "Black Hole" with Gaussian Beam



Simulation results of an ideal optical black hole with outer radius R = 20 μm, and inner radius R_c = 8.367 μm. The Gaussian beam with free-space wavelength λ = 1.5 μm and minimum waist width $w = 2\lambda$ is focused at x_0 = 0, and (a) $y_0 = 1.5R$; (b) $y_0 = R$; (c) $y_0 = 0.75R$, and (d) $y_0 = 0$.

Optical "Black Hole" with Gaussian Beam



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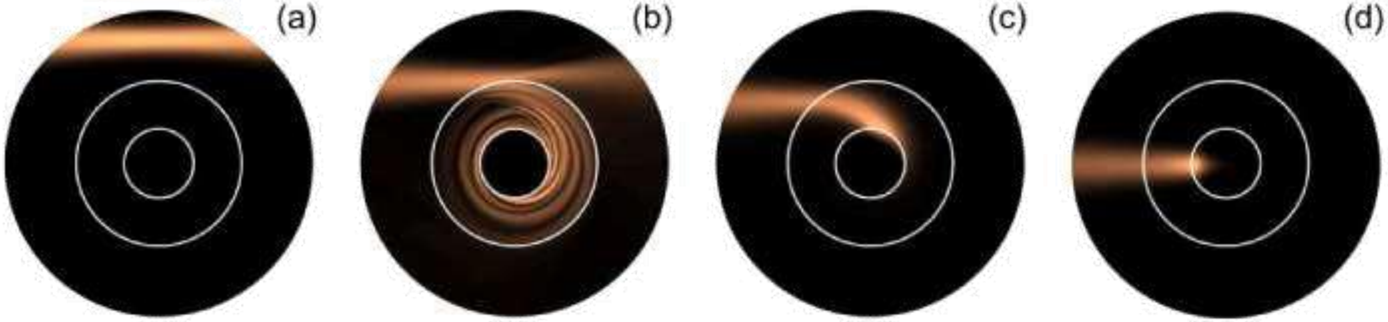
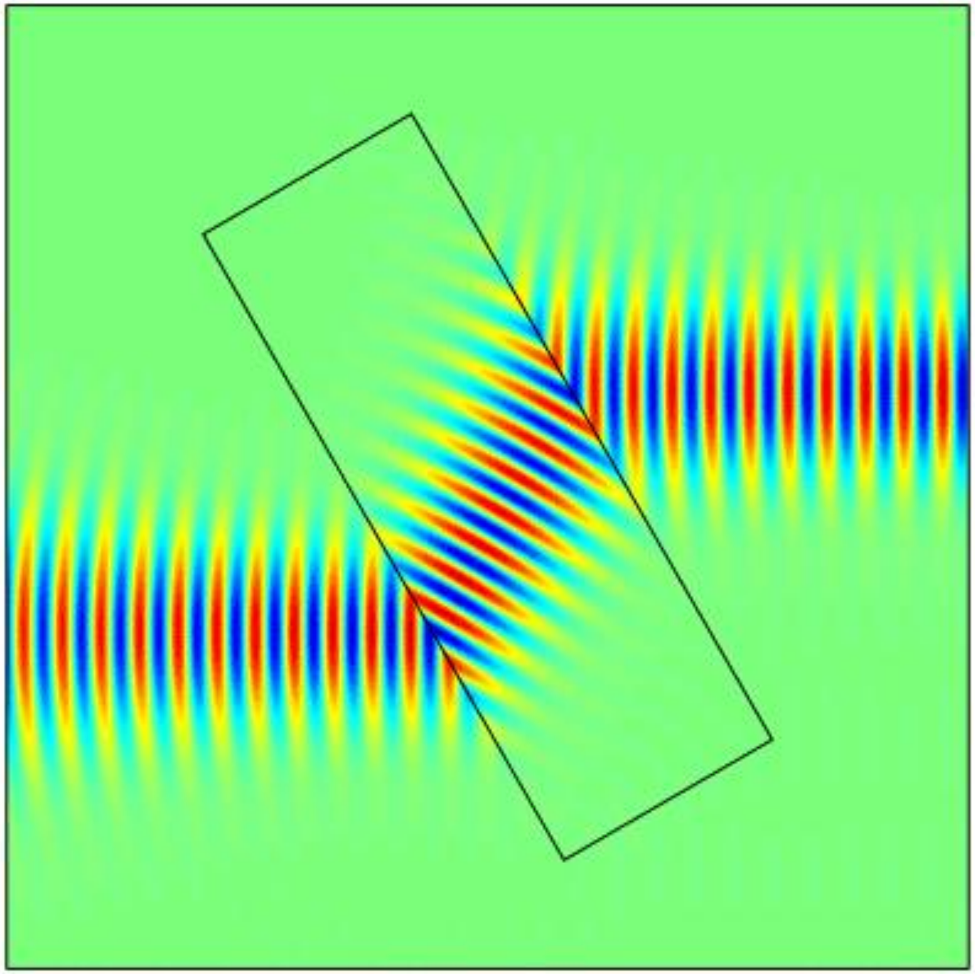


Another Application – Negative Index Metamaterials



Summary

- modeled an ideal optical “black hole” device
- used a new method to precisely model the Gaussian beam illumination
- the simulation results of the optical “black hole” device shows expected performance