

# Optimal Design of Slit Resonators for Acoustic Normal Mode Control in Rectangular Room

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**Abstract:** The present article presents a method to redistribute the acoustic modes of a rectangular enclosure in the low frequency range using slit resonators. The objective of the present work is to compare different strategies of optimal design in order to determine the dimensions of the resonators. The method of the finite elements will be used to model the acoustic physical behavior of the room. In addition a neuronal network will estimate the loudness level perceived by the auditor. The different strategies of design are: First, a strategy of design will be implemented based on the minimization of the fluctuations of the sound level pressure. Second, the optimization will be based on the diminution of the variations of the loudness level. Finally, two methods of optimization, genetic algorithm and differential evolution will be compared. The three different strategies from optimization will be compared generally and of it will determine the design variables that are critics in this process.

**Keywords:** Slit resonators, normal mode control, optimal design.

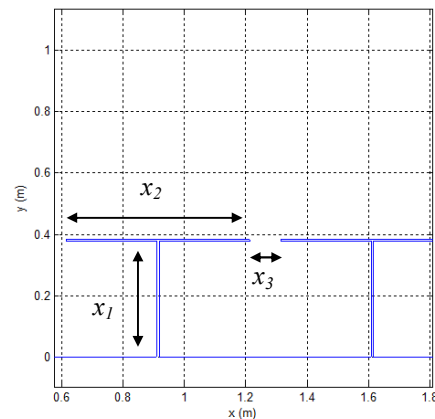
## 1. Introduction

The sound field of an enclosure is characterized by the interaction between the source and the acoustic properties of the room. The frequency response and the balance of the timbre depend on the geometry and the materials of the enclosure. The objective of this article is to decrease the effects of the resonances at low frequencies and to suitably distribute the normal modes of vibration using optimal slit resonators which dimensions are optimized. This type of resonators is of great interest in architectural acoustics due to easy construction.

Slit resonators are composed by a periodic structure of T-like plates. It can be described using three physical dimensions. The height of

the supporting plate, the width of the supported plate and the distance between the nearest extremes of the supported plates are denoted as  $x_1$ ,  $x_2$  and  $x_3$  respectively. This is shown in Fig1.

The resonant frequency and absorption characteristics of this type of devices have been studied by Pedersen [1]. Mechel [2] has included viscous and thermal losses to this formulation. Geometric modifications and the effects of grazing flow have been accounted for in [3-5].



**Figure 1.** Dimensional characteristics of slit resonators.

The room dimensions' optimization has been studied by Cox et al [6] and Zu et al [7-8]. In these works, the fluctuations of the sound pressure level have been minimized. Instead, Floody and Venegas [9-10] have proposed the optimization of the room dimensions based on minimizing the loudness level fluctuations. In this work, these two approaches are used to optimize the dimensions of slit resonators. Two different optimization algorithms are considered and compared. These correspond to the genetic algorithm and the differential evolution algorithm.

A cubical enclosure of 5.1 m side with and without slit resonators is considered as a case of study. The source and the reception point are located in opposite corners. Vertically-oriented slits are considered. Their length coincides with the height of the room. The sound field is modeled for frequencies ranging from 20 Hz to 200 Hz using a mixture between the finite element method and a classic analytical solution. This choice has been made to decrease the computational cost.

## 2. Theory and Governing Equations

### 2.1 Formulation of the Problem and Application of the Method of Separation of Variables

The enclosure is excited by a flat spectrum point source. This problem is governed by the Helmholtz's equation when considering harmonic solution. This is shown in Eq. 1 along with the respective boundary condition. In order to simplify the problem the stationary solution in the frequency domain will be studied only.

$$\nabla^2 P + k^2 P = 0, \quad \nabla P \cdot \hat{n} = 0 \quad (1)$$

By using the method of separation of variables the following equations and boundary conditions are obtained

$$P(x, y, z) = P_{xy}(x, y)P_z(z) \quad (2)$$

The dependency in  $z$  is given by:

$$\frac{\partial^2 P_z}{\partial z^2} + k_z^2 P_z = 0, \quad \left( \frac{\partial P_z}{\partial z} \right)_{z=0} = \left( \frac{\partial P_z}{\partial z} \right)_{z=L_z} \quad (3)$$

And for the  $(x, y)$  dependency:

$$\frac{\partial^2 P_{xy}}{\partial x^2} + \frac{\partial^2 P_{xy}}{\partial y^2} + k_{xy}^2 P_{xy} = 0, \quad \nabla P_{xy} \cdot \hat{n} = 0 \quad (4)$$

It should be satisfied that:

$$k^2 = k_{xy}^2 + k_z^2 \quad (5)$$

The equation and the boundary condition Eq. (3) have a well-known solution [11]. Eq. (4) and

its respective boundary condition can be solved by using the finite element method [12]. Triangular linear lagrangian elements have been used to model the pressure in the 2D part of the equation. The finite element formulation for the equation Eq. (4) is the following eigenvalue problem solved with Comsol<sup>®</sup> Multiphysics:

$$K\phi = k_{xy}^2 M\phi \quad (6)$$

Where,  $K$  and  $M$  are the acoustics stiffness and mass matrices, and  $\phi$  is the eigenvector. Thus, the natural frequencies can be calculated using equation Eq. (7). Finally, the sound pressure at any point  $\mathbf{r}$  inside the enclosure produced by a point source located at  $\mathbf{r}_0$  for a frequency  $\omega$  is the result of the combination of the solutions of the equations Eq. (3) and Eq. (4) as is shown in Eq. (8).

$$\omega_{n_x, n_y, n_z} = \sqrt{k_{xy}^2 + k_z^2} \quad (7)$$

$$p(\mathbf{r}, \mathbf{r}_0, \omega) = \sum_{n_x=1}^{\infty} \sum_{n_y=1}^{\infty} \sum_{n_z=0}^{\infty} \frac{A_{n_x, n_y, n_z}(\mathbf{r}, \mathbf{r}_0, \omega)}{\omega^2 - \omega_{n_x, n_y, n_z}^2} \quad (8)$$

$$A_{n_x, n_y, n_z}(\mathbf{r}, \mathbf{r}_0, \omega) = jS_0 \rho_0 c^2 \omega \left( \phi_{r, n_x, n_y} \cos(k_z z) \right) \times \left( \phi_{r_0, n_x, n_y} \cos(k_z z_0) \right)$$

Where  $\rho_0$  is the density of the air and  $U_0$  is the vibration velocity on the source surface.

### 2.2 Determination of the Loudness Levels Using Neural Networks

The loudness may be defined as the sensation that corresponds most closely to the sound intensity of a stimulus [13]. An equal-loudness contour is a curve that ties up sound pressure levels having equal loudness as a function of frequency. In other words, it expresses a frequency characteristic of loudness sensation. In this work a loudness model, implemented using an artificial neural network, has been developed from the equal-loudness-level contours data presented in reference [14]. The procedure described in reference [15] has been followed up. The presented model aims to accurately calculate loudness level at low frequencies. The artificial neural network [16] has been trained with the quasi Newton backpropagation algorithm considering 3000 epochs and an objective goal of

$10^{-5}$ . The final configuration corresponds to a three layer feedforward neural network with 5 neurons in the hidden layer and 1 output neuron. The transfer function of the hidden layer is a sigmoidal hyperbolic tangent function whereas is linear for the output layer. The inputs to the neural network are frequency and sound pressure level. The output is the respective loudness level.

### 2.3 Objective Functions

The optimization techniques are used to determine the best possible design in engineering problems. In this case they are used to determine the optimal dimensions of a set of slit resonators. Since a flat room response is the goal to predict the geometric modifications, a flattest sound frequency response could be well considered as the best frequency response for reference, even though a perfect flat response is practically impossible to get due to the maximums and minimums caused by sparsity of the room modes.

Under this consideration, the chosen objective function is the square root of the sound frequency response deviation from a least square straight line drawn throughout the spectrum as proposed in [6].

$$f_1(\mathbf{x}) = \sqrt{\frac{1}{N} \sum_{i=1}^N [L_p(f_i) - (a_1 f_i + a_0)]^2} \quad (9)$$

Where  $\mathbf{x} = [x_1, x_2, x_3]^T$ , is the design vector based on the dimensions of the slit,  $N$  is the number of points,  $f_i$  is frequency,  $L_p(f_i)$  the sound pressure level,  $a_1$  and  $a_0$  are the coefficients of the linear regression.

The second objective function has the goal of obtaining the *best* psychoacoustic response of the room. This new function to minimize corresponds to the standard deviation of the loudness level in the frequency range previously mentioned [9, 10]. This objective function has been successfully used in the design of the room's geometry [9,10].

$$f_2(\mathbf{x}) = \sqrt{\frac{1}{N} \sum_{i=1}^N [L_L(f_i) - \overline{L_L}]^2} \quad (10)$$

Where  $L_L(f_i)$  is the loudness level and  $\overline{L_L}$  is average the loudness level. For both objectives

functions, dimensional restrictions are imposed to the design variables ( $0.01m \leq x_i \leq 0.60m$ ,  $i = 1, 2, 3$ ). The posed optimization problem is characterized by a strong nonlinear interrelation between the variables and the fitness functions. The functions have many peaks and dips. This makes the solution oversensitive to the dimensions of the slit resonators. For this reason the frequency response curves are smoothed out using the Savitzky – Golay method. Finally for both objective functions the positions of source and receiver will be located in opposed corners of the room, because this represents the worse case.

## 3. Numerical Simulations and Results

### 3.1 Comparison of Results between Genetic Algorithm and Differential Evolution

Using the functions objectives previously detailed. A set of simulations has been run to evaluate the best possible strategy of optimal design. Five simulations have been made using genetic algorithm [17,18] with 100, 200, 300, 400 and 500 generations, for both objective functions. After that the differential evolution algorithm [19, 20] has been used with the same number of generations for both objective functions. The results of the simulations are given in Tables 1 to 5. Where *Gen* is the number of generations.  $f_1(\mathbf{x})Opt$  is the optimum value of the objective function based on the sound pressure level.  $f_2(\mathbf{x})Opt$  is the optimum value of the objective function based on the loudness level.  $f_1(\mathbf{x})Asc$  is the value of the first objective function when the optimization process is based on the loudness level function.  $f_2(\mathbf{x})Asc$  is the value of the second objective function when the optimization process is based on the sound pressure level function.

**Table 1:** Optimization results of the function based on the sound pressure level using genetic algorithm

Genetic Algorithm $L_p$ Optimization					
<i>Gen</i>	$f_1(\mathbf{x})Opt$	$f_2(\mathbf{x})Asc$	$x_1$ (m)	$x_2$ (m)	$x_3$ (m)
100	60,549	565,475	0,287	0,292	0,080
200	51,089	539,627	0,367	0,424	0,013
300	53,593	543,258	0,153	0,596	0,039
400	58,381	573,250	0,359	0,274	0,100
500	54,091	581,153	0,367	0,537	0,099

**Table 2:** Optimization results of the function based on the loudness level using genetic algorithm

Genetic Algorithm $L_L$ Optimization					
Gen	$f_2(\mathbf{x})_{Opt}$	$f_1(\mathbf{x})_{Asc}$	$x_1$ (m)	$x_2$ (m)	$x_3$ (m)
100	358,022	70,490	0,201	0,533	0,080
200	336,248	65,866	0,184	0,583	0,085
300	346,451	65,720	0,153	0,546	0,013
400	348,496	78,182	0,178	0,495	0,023
500	339,674	66,758	0,163	0,555	0,014

**Table 3:** Optimization results of the function based on the sound pressure level using differential evolution

Differential Evolution $L_p$ Optimization					
Gen	$f_1(\mathbf{x})_{Opt}$	$f_2(\mathbf{x})_{Asc}$	$x_1$ (m)	$x_2$ (m)	$x_3$ (m)
100	48,347	561,590	0,377	0,598	0,100
200	39,027	588,773	0,400	0,590	0,091
300	38,230	471,211	0,399	0,590	0,090
400	36,706	483,395	0,400	0,590	0,090
500	37,832	500,325	0,398	0,589	0,090

**Table 4:** Optimization results of the function based on the loudness level using differential evolution

Differential Evolution $L_L$ Optimization					
Gen	$f_2(\mathbf{x})_{Opt}$	$f_1(\mathbf{x})_{Asc}$	$x_1$ (m)	$x_2$ (m)	$x_3$ (m)
100	345,310	70,033	0,193	0,554	0,067
200	322,670	80,272	0,199	0,510	0,026
300	312,440	72,044	0,170	0,541	0,018
400	320,890	76,922	0,186	0,570	0,075
500	321,790	72,072	0,171	0,535	0,018

The differential evolution algorithm is proven to be more efficient than the genetic algorithm. This can be not only seen when comparing the values of  $f_1(\mathbf{x})_{Opt}$ , and  $f_2(\mathbf{x})_{Opt}$  using both methods but also when observing the values  $x_1$ ,  $x_2$  and  $x_3$ . These parameters reach a suitable stability when using the differential evolution method.

Considering the data in the previous tables, the differential evolution is chosen as a definitive optimization strategy, considering both functions objective and 1000 generations. The results are shown in the table 5.

It can be seen that both objective functions are incompatible, i.e. the optimization by sound pressure level does not improve the results in loudness level and vice versa. On the other hand

the purely numerical information does not provide a complete understanding of the effectiveness of both objective functions.

**Table 5:** Results of the optimization results of the function based on sound level pressure and loudness level using evolution differential, 1000 generations

Differential Evolution $L_p$ Optimization					
Gen	$f_1(\mathbf{x})_{Opt}$	$f_2(\mathbf{x})_{Asc}$	$x_1$ (m)	$x_2$ (m)	$x_3$ (m)
1000	37,254	452,781	0,399	0,591	0,091

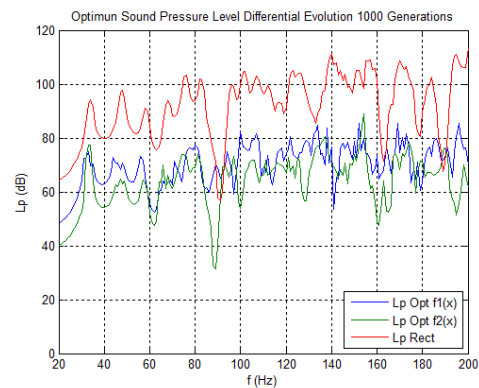
  

Differential Evolution $L_L$ Optimization					
Gen	$f_2(\mathbf{x})_{Opt}$	$f_1(\mathbf{x})_{Asc}$	$x_1$ (m)	$x_2$ (m)	$x_3$ (m)
1000	318,960	80,348	0,189	0,567	0,078

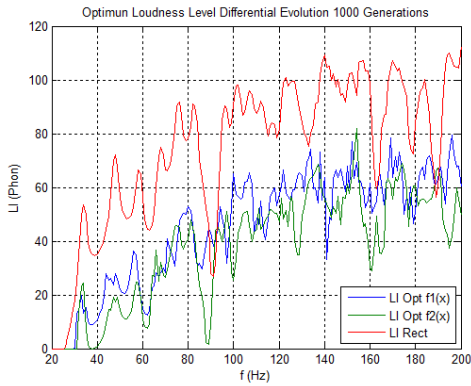
### 3.2 Comparison between the Objective Function Based on Sound Level Pressure and the Objective Function Based on Loudness Level

Figure 2 and 3 show the sound pressure level and the loudness level respectively, for the cubical room and the optimized slit resonators using both objective functions. These plots allow gaining a better understanding of the results. The loudness level function is not efficient at decreasing either the sound pressure level or the loudness level fluctuations. This is particularly noticeable in the anti resonance at 90 Hz.

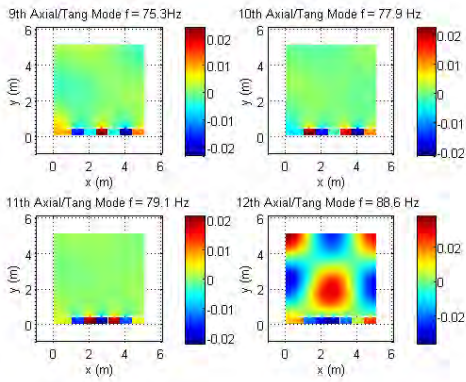
When analyzing the space distribution of the sound pressure in figures 4 - 6, one can observe with more detail the effect of the slit resonators.



**Figure 2.** Sound pressure level – Red line not optimized rectangular room - Blue line  $L_p$  optimized with objective function based on  $L_p$ ,  $f_1(\mathbf{x})$  - Green line. optimized  $L_p$ , objective function based on  $L_L$ ,  $f_2(\mathbf{x})$  - Differential Evolution - 1000 Generations.



**Figure 3.** Loudness Level. – Red line not optimized rectangular room. - Blue line  $L_L$  optimized with objective function based on  $L_p, f_1(\mathbf{x})$ . - Green line, optimized  $L_L$ , objective function based on  $L_L, f_2(\mathbf{x})$  - Differential evolution - 1000 Generations.



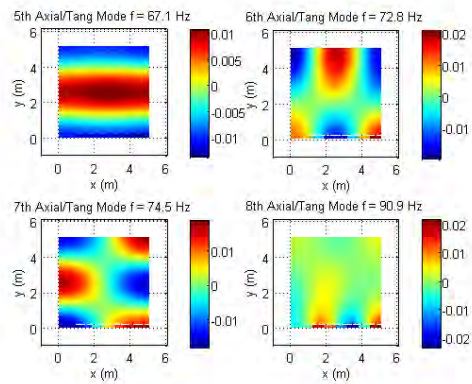
**Figure 4.** Sound pressure space distribution, for axial/tangential modes  $f(n_{xy}, 0)$  - Optimized with objective function based on  $L_p, f_1(\mathbf{x})$  - Frequency band between 70 Hz and 90 Hz - Differential Evolution 1000 Generations

First of all, as it is observed in figure 4, the optimal case using function  $f_1(\mathbf{x})$  depends on the sound level pressure. The distribution of pressure is fairly uniform in at least three modes, especially in the frequency band between 70 Hz and 90 Hz.

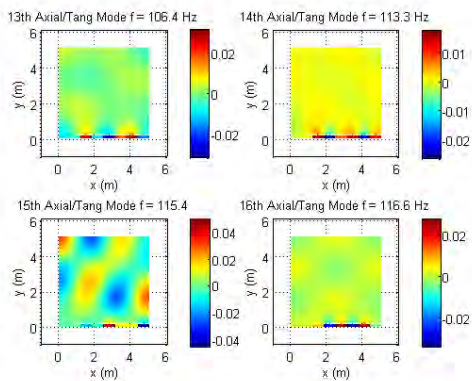
It is seen in this situation that the largest variations of sound pressure occur in the resonators. Although the resonance at 88.6 Hertz does not contribute to the uniformity of the sound field, it allows avoiding any anti resonance in the frequencies around 90 Hertz.

Figure 5 shows the same degree of uniformity of the sound field cannot be reached when minimizing  $f_2(\mathbf{x})$ , which depends on loudness level, in the same frequency band.

In figure 6 one can see how the optimized resonators using the objective function based on loudness level  $f_2(\mathbf{x})$  act in the desired way over the sound field in the frequency range 100 to 120 Hz. This is also seen in the smaller loudness level variability when compared to that show in green curve in figure 3.



**Figure 5.** Sound pressure space distribution, for axial/tangential modes  $f(n_{xy}, 0)$  - Optimized with objective function based on  $L_L, f_2(\mathbf{x})$  - Frequency band between 70 Hz and 90 Hz - Differential Evolution 1000 Generations



**Figure 6.** Sound pressure space distribution, for axial/tangential modes  $f(n_{xy}, 0)$  - Optimized with objective function based on  $L_L, f_2(\mathbf{x})$  - Frequency band between 100 Hz and 120 Hz - Differential Evolution 1000 Generations

## 4. Conclusions

Different methods for optimal design of slits resonators in enclosures have been compared. The best design strategy corresponds to the minimization of the sound level pressure fluctuations using the differential evolution method. This method is substantially more efficient than the commonly used genetic algorithm.

The optimization by loudness level is incompatible with the one based on sound level pressure for the design of this particular type of resonators. This has not been the case when the room optimization is performed over the whole geometry of the room as in [9,10].

The objective function  $f_1(\mathbf{x})$  is much more efficient and stable at simultaneously decreasing the fluctuations of sound level pressure and loudness level. This function tries to eliminate the resonant frequencies smaller than 100 Hz. In this range, the modal density is low and therefore the resonance effects are more notorious, which results in a better physical and psychoacoustic response at higher frequencies. The objective function  $f_2(\mathbf{x})$  tends to better control the resonances at higher frequencies. In this range, however, the effect of these resonances is less noticeable.

Regarding the spatial distribution of the sound pressure level, the optimization based on  $f_1(\mathbf{x})$  is also much better in terms of homogeneity of the sound field.

Considering the results presented in this paper, one may argue that the objective function proposed by Cox et al. [6] is much more efficient. However, it is important to emphasize that the overall enclosure dimensions and the design restrictions are important factors. The study of the influence of these factors is being carried out.

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## **6. Acknowledgements**

R. Venegas gratefully acknowledges an ORSAS award and University of Salford research scholarship