Integrated Model for Ocean Waves Propagating over Marine Structures on a Porous Seabed

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Abstract: In this paper, an integrated model for ocean waves propagating over a submerged coastal structure, based on COMSOL Multiphysics, is presented. In the model, Navier-Stoke equations, Biot's poro-elastic theory, and structural mechanics theory are solved for wave propagation, seabed response and structure deformation, respectively. The new feature of this model is to integrate wave, soil and structure modes into one model. This can be achieved within COMSOL Multiphysics, but will be a very challenge task with the conventional models. In this paper, we first present the model of ocean wave generation over a porous seabed. Then we further consider two coastal engineering problems: (1) ocean waves propagating over a submerged breakwater on a porous seabed; and (2) waves over a deformable structure on a porous seabed, which can be applied to wave energy converter.

Keywords: Ocean waves; Poro-elastic model; Integrated model; Marine structures; COMSOL.

1. Introduction

Marine structures on a porous seabed have been widely constructed for the coastal protections, oil production transport and offshore wind farm foundation. The existence of these structures (such as breakwaters, vertical walls, pipelines and mono-piles, etc.) will largely interact with the water surface waves, and consequently affect the wave-induced seabed responses around the marine structures.

In the past few decades, considerable effort has been devoted to the wave-soil-structure interaction (WSSI) phenomenon. The major reason for this growing interest is that many marine structures have been damaged by the wave-induced seabed response, rather than from the construction deficiencies [1-3]. To have a better understanding of the functionality and stability of marine structures, the wave motion and seabed responses around these structures must be determined.

Numerous investigations for the wave-seabed-structure interactions have been carried out since the 1980s. A detailed review of previous research in the area can be found in [4]. Most of them have been focused on the individual approaches [5-8], rather than an integrations of wave, seabed and structure models. However, the phenomenon of the wave-seabed-structure interactions will not been fully captured without a consideration of all components together.

This study, based on COMSOL Multiphysics, is to develop an integrated model for ocean waves propagating over a marine structure on a porous seabed. The theoretical formulations together with the use of COMSOL Multiphysics are given in §2, while the numerical results are presented and discussed in §3. The remarking conclusions and future works are drawn in §4.

2. Theoretical Formulations

An integrated model for WSSI is developed in this study. An assumption is made that the poro-elastic deformations in seabed are very small and do not affect the wave transformations or its induced pressure on the surface of the poro-elastic seabed [9]. This assumption simplifies the boundary conditions at the seabed interface where the water pressure and shear stress calculated from the wave field are passed into the seabed. This integrated model includes three main components: (i) wave mode on the basis of the Navier-Stokes (N-S) equations; (ii) seabed mode on the basis of the Biot's consolidation equations with poro-elastic theory; and (iii) structure mode on the basis of structural mechanics theory.

2.1 Governing equations of wave mode

Navier-Stokes (N-S) equations are utilized to describe motion of the water liquid phase. Starting with the momentum balance in terms of stresses, the generalized equations in terms of transport properties and velocity gradients are

$$\rho\frac{\partial\vec{u}}{\partial t}-\nabla\bullet[\eta(\nabla\vec{u}+(\nabla\vec{u})^T)] + \rho(\vec{u}\bullet\nabla)\vec{u}+\nabla p_f=\vec{F} \quad (1)$$

$$\nabla\bullet\vec{u}=0 \quad (2)$$

where η is the dynamic viscosity of fluid, ρ is the fluid density, $\vec{u}$ is the velocity field, p_f is the pressure, t is the time, and $\vec{F}$ is a volume force such as gravity.

2.2 Governing equations of seabed mode

The consolidation equation for the flow of a compressible pore fluid in a compressible porous medium can be given as [10]

$$\nabla\bullet(K\nabla p)-\gamma_w n'\beta\frac{\partial p}{\partial t}=\gamma_w\frac{\partial\varepsilon_s}{\partial t} \quad (3)$$

where p is the pore pressure, K is the permeability matrix of the soil, γ_w is the unit weight of pore water, n' is the soil porosity, and $\varepsilon_s=\nabla\bullet\vec{u}_s$ (where $\vec{u}_s$ is the soil displacement) is the volume strain of soil matrix. The compressibility of pore fluid (β) is defined as

$$\beta=\frac{1}{K_w}+\frac{1-S}{P_{w0}} \quad (4)$$

in which K_w is the true modulus of elasticity of water (taken as 2×10^9N/m^2), P_{w0} is the absolute water pressure and S is the degree of saturation.

The relationships between soil displacement and pore pressure are given as

$$G\nabla^2\vec{u}_s+\frac{G}{1-2\mu_s}\nabla\varepsilon_s=\nabla p \quad (5)$$

where G is the shear modulus related to the Young's modulus (E) and the Poisson's ratio (μ_s) in the form of $E/(2(1+\mu_s))$.

2.3 Governing equations of structure mode

Based on the small-displacement assumption, the relationships between strain components and displacement at a point of marine structure are given as follows

$$\varepsilon_x=\frac{\partial u_m}{\partial x} \quad \varepsilon_{xy}=\frac{1}{2}(\frac{\partial u_m}{\partial y}+\frac{\partial v_m}{\partial x}) \quad (6)$$

$$\varepsilon_y=\frac{\partial v_m}{\partial y} \quad \varepsilon_{yz}=\frac{1}{2}(\frac{\partial v_m}{\partial z}+\frac{\partial w_m}{\partial y}) \quad (7)$$

$$\varepsilon_z=\frac{\partial w_m}{\partial z} \quad \varepsilon_{xz}=\frac{1}{2}(\frac{\partial u_m}{\partial z}+\frac{\partial w_m}{\partial x}) \quad (8)$$

The strain tensor ε and stress tensor σ are

$$\varepsilon=\begin{bmatrix}\varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz}\\ \varepsilon_{yx} & \varepsilon_y & \varepsilon_{yz}\\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_z\end{bmatrix}, \quad \sigma=\begin{bmatrix}\sigma_x & \tau_{xy} & \tau_{xz}\\ \tau_{yx} & \sigma_y & \tau_{yz}\\ \tau_{zx} & \tau_{zy} & \sigma_z\end{bmatrix} \quad (9)$$

The stress-strain relationship for linear conditions reads

$$\sigma=D_m\varepsilon \quad (10)$$

where D_m is the elasticity matrix.

The structural mechanics theory in this study is based on a weak formulation of the equilibrium equations expressed in the global stress components.

$$-\nabla\bullet\sigma=\vec{F}_m \quad (11)$$

in which $\vec{F}_m$ denotes the volume forces (body forces).

2.4 Boundary conditions

When solving the governing equations, appropriate boundary conditions at external boundaries and internal interfaces for these three modes are required (see Figure 1).

In the wave mode, a piston wave maker is used in the left-hand-side boundary (Γ_1) of computational domain to generate wave and a sponge layer is located in the right-hand-side boundary (Γ_2) to avoid/reduce the wave reflection. Zero pressure is applied on the water free surface (Γ_3), while no-slip condition is adopted at the solid surface, such as sea floor (Γ_4) and surface of marine structure (Γ_5). In the case of a deformable structure, the impact of the structure deformation on wave motion is considered in term of a deformation of boundary shape (Γ_5).

In the seabed mode, it is commonly accepted that vertical effective normal stresses vanish at the seabed surface while the wave pressure and shear stresses obtained from wave mode are imposed as boundary conditions of seabed surface (Γ_4). In this study, the seabed is considered as a porous medium of a finite thickness and rests on an impermeable rigid bottom, indicating that zero displacements, zero gradient of pore pressure and no vertical flow occur at the horizontal bottom (Γ_6). When two side boundaries (Γ_7 and Γ_8) of seabed are far away from the concerned region (such as the region around a marine structure), they can be assumed to have zero displacement.

In the structure mode, the displacement and velocity at the surface (Γ_5) of a deformable structure are dominated by the wave pressure acting on the wave-structure interface. For the embedded part (Γ_9) of a structure, it is assumed to have same displacement and velocity as those of ambient soil (updated from seabed mode).

2.5 Use of COMSOL Multiphysics

These three numerical modes are integrated by using COMSOL Multiphysics (3.5a version). The main features of COMSOL Multiphysics adopted to set up the integrated model are listed as follows:

(1) 2D space dimension;
(2) Plane strain mode of structural mechanics;
(3) Coefficient form of PDE mode for seabed mode;
(4) Incompressible Navier-Stokes mode of fluid dynamics;
(5) Arbitrary Lagrangian-Eulerian (ALE) method for mesh movement.

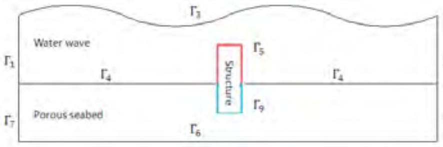
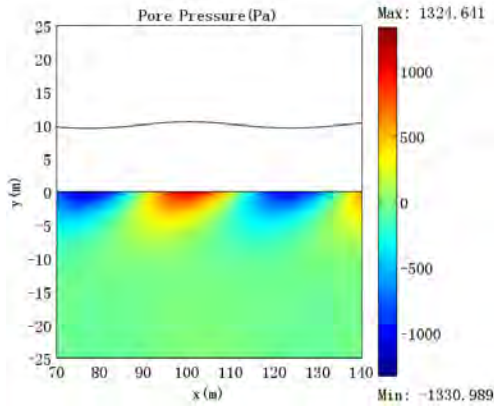


Figure 1. Locations for specification of boundary condition.

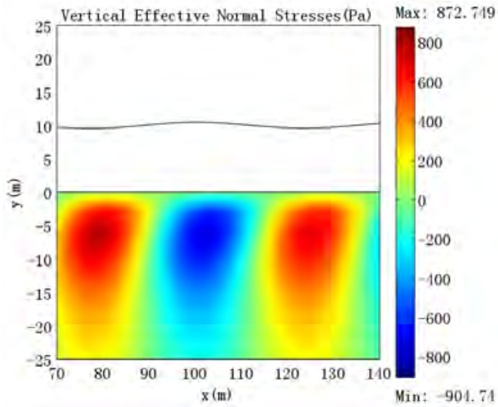
3. Results and Discussions

3.1 Wave-seabed Interaction

As a starting point, this integrated model is applied to predict the wave-seabed interaction without the inclusion of a marine. In the example, a computational domain with a length ($L = 200$ m) is used. The original of the Cartesian coordinate system is located at left-hand-size edge point of sea floor. The incident linear wave is generated with wave height ($H_w = 0.5$ m), wave period ($T = 6.0$ sec) and still water depth ($d = 10.0$ m). For the porous seabed, the seabed thickness, soil porosity, permeability and degree of saturation are taken as $d_s = 25.0$ m, $n' = 0.4$, $K = 0.01$ m/sec and $S = 0.98$, respectively.



(a) pore pressure



(b) Vertical effective normal stress

Figure 2. Distributions of (a) pore fluid pressure and (b) vertical effective normal stress with wave profile and velocity field at time $t = 43.0$ sec.

Figure 2 shows an example of the distributions of wave-induced pore fluid pressure (p) and vertical effective normal stress (σ_z) within the seabed at time $t = 43.0$ sec. The magnitude of pore pressure decreases with depth increases. The magnitude of vertical effective normal stress increases firstly, and then decreases gradually. The comparisons between numerical results and analytical solutions of water elevation [11], maximal pore water pressure and vertical effective normal stress [12, 13] at cross-section $x = 100$ m are shown in Figures 3-5, respectively. In general, there is a good agreement between numerical simulation and analytical theory. It is noted that the wave model used in the previous analytical solutions was based on the potential flow theory, which has no shear stresses along the seabed surface, while the present wave model was based on N-S equations.

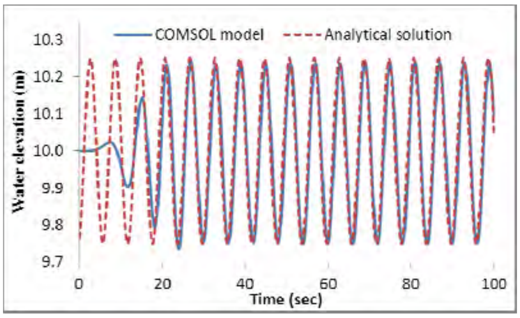


Figure 3. Comparison of simulated water elevation with analytical solution at the cross-section $x = 100$ m.

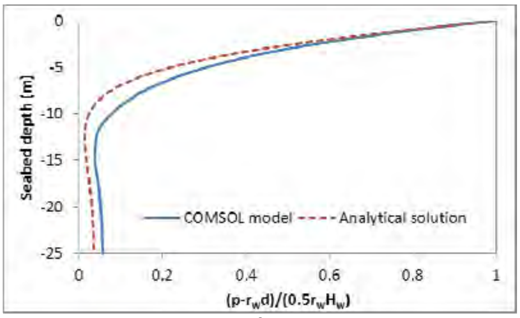


Figure 4. Comparison of simulated maximal pore pressure with analytical solution at the cross-section $x = 100$ m.

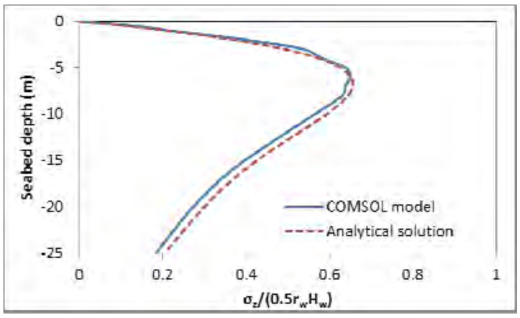


Figure 5. Comparison of simulated maximal vertical effective normal stress with analytical solution at the cross-section $x = 100$ m.

3.2 Wave-seabed Interaction around a Rigid Submerged Breakwater

In this section, a rigid, impermeable and submerged breakwater with a rectangular shape is considered, and its impacts on wave motion and seabed response are analyzed by the integrated model. The structure is 20 m wide and 3 m high, and its central point of bottom line is located at the point (105, 0). Figure 6 shows the distribution of the wave-induced pore fluid pressure around the structure at time $t = 43.0$ sec. As one can expect, the existence of structure can largely affect the wave motion around the submerged breakwater (see Figure 7) and consequently leads to a different distribution of pore pressure from that without a marine structure (see Figure 2(a) and Figure 6). As shown in Figure 7, an obvious wave deformation takes place due to wave-structure interaction when the wave is over the submerged breakwater.

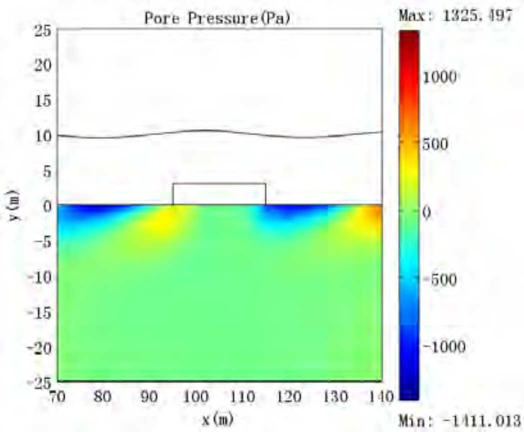


Figure 6. Distributions of the wave-induced pore pressure in a porous seabed at time $t = 43.0$ sec.

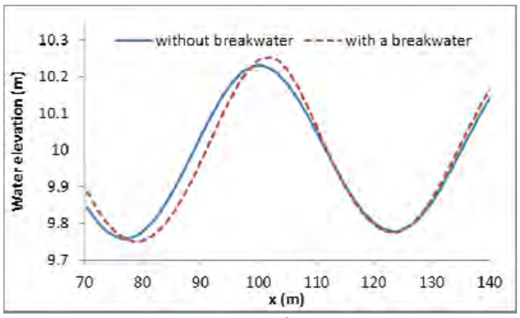


Figure 7. Comparsions of the simulated water elevation with and without the presence of a submerged breakwater at time $t = 43.0$ sec.

3.3 Wave-seabed Interaction around a Deformable Structure

This integrated model has also been used to study the wave propagation and seabed response around a deformable impermeable structure (such as wave energy converter). Two types of structure with different embedded depths are investigated, as depicted in Figure 8. One is simply fixed on the sea floor with zero embedded depth, and the other is embedded into the seabed with an embedded depth of 2 m. The width and height above seabed are kept the same in these structures, and they are taken as 1 m and 5 m, respectively.

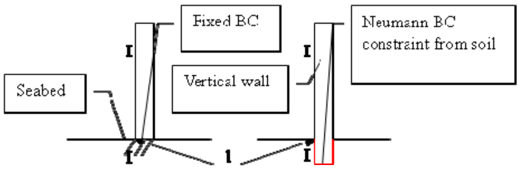


Figure 8. The sketch vertical structure and seabed model

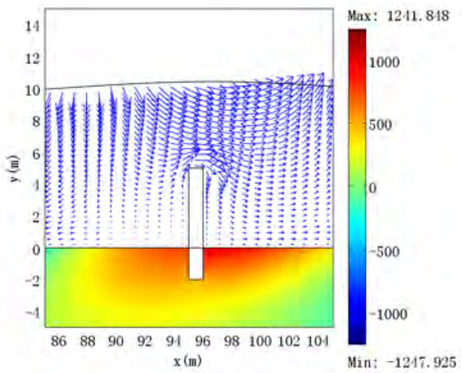
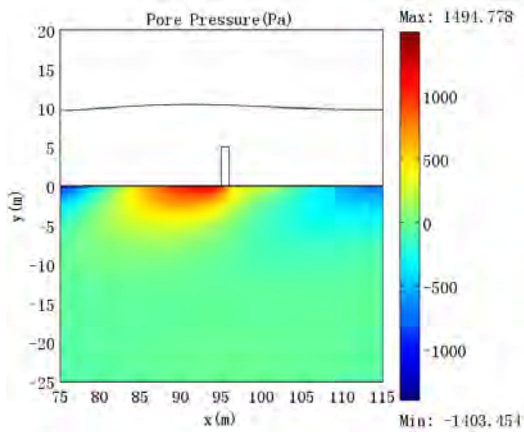


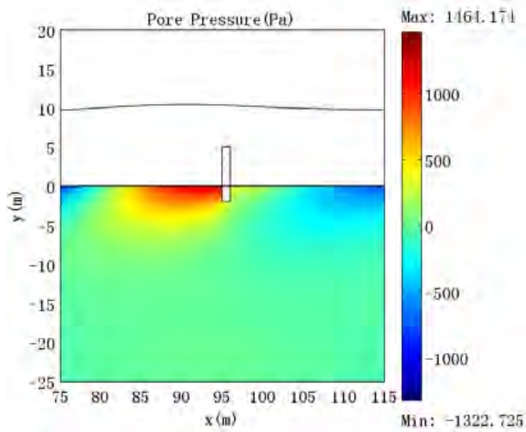
Figure 9 The sketch of velocity profile in the system of wave-vertical wall-soil interactions-Case 2 (wave period=8s, wave height=0.5m).

Figure 9 illustrates the effects of structure obstacle on fluid velocity field, from which significant changes of velocity pattern around the structure can been observed. Furthermore, in this example, the wave crest arrives at the structure, which creates a positive pressure on the seabed surface, and result in compaction of the seabed.

Figure 10 shows the effects of embedded depth on the distribution of wave-induced pore pressure around foundation. As shown in the figure, there is about 2% of increment of the pore pressure amplitude in case 1, compare with case 2. The comparison indicates the embedded part of structure may disturb/block the development of pore pressure. This implies that the embedded pile will reduce the pore pressure, and enhance the stability of the structure.



(a) Case 1



(b) Case 2

Figure 10. Effect of embedded depth of deformable structure on pore pressure.

4. Conclusions and future work

An integrated model, based on COMSOL Multiphysics, has been developed to study the WSSI phenomenon in this study. To validate this model, the simulated wave profile, pore fluid pressure and vertical effective normal stress in the case without any marine structure are compared with those from analytical theory. The comparison results show a good agreement between numerical simulation and analytical theory. This validated model is then applied to study the impacts of rigid or deformable submerged structure on the wave motion and seabed response. The results indicated: (i) COMSOL Multiphysics has a good ability in simulating WSSI; (ii) the existence of a marine structure may significantly increase the wave crest height and its induced pore pressure within seabed; and (iii) the embedded depth of a deformable structure slightly affect the wave propagation but has a large impact on pore pressure around the structure foundation.

In this paper, only the preliminary results for the wave motion and its induced seabed response around a marine structure (which can be rigid or deformable) are presented. More detailed parametric studies for the influences of wave, soil and structure characteristics will be conducted in the future study.

5. References

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