Excerpt from the Proceedings of the 2011 COMSOL Conference in Stuttgart

A Elastic and Hyperelastic Material Model of Joint Cartilage - Calculation of the Pressure Dependent Modulus of Elasticity by Comparison with Experiments and Simulations

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Abstract: In this paper we introduce a elastic and hyperelastic model to describe the biomechanics of joint cartilage. As biomechanical property we calculated the pressure dependent E-modulus E = f(s) to describe the dependence of the biomechanical properties on pressure. The calculation based on the comparison and the iterative approach of the force-way-functions between the experiments and simulations. In this first study we found that the E = f(s) is a degree 4 polynomial. The E-modulus varies between 0 - 2.9 MPa for the elastic and between 0 - 2.2 MPa for the hyperelastic material model by a compression from 0 – 0.4 mm caused by a surgery tasthaken. The pressure dependent E-modulus allows us to simulate the nonlinear behaviour of compressed cartilage tissue.

Keywords: Cartilage, E-Modulus, Soft Tissue, biomechanical properties

1. Introduction

The biomechanical properties have much relevant information for the functional characterisation of cartilage tissue [1]. In the process the modulus of elasticity E is typically evaluated for a defined pressure applied to the cartilage tissue. This single observation neglects the dependence of the biomechanical properties on pressure. In this case the pressure dependent E-modulus E=f(s) with the help of experimental force-way-indentation data by means of an elastic and hyperelastic material model is calculated. The aptitude of the models is evaluated by the comparison with experiments and simulations.

2. Experimental Methods

In the experiment, the force-way-diagram was dictated by the pressure of the cartilage tissue by means of an indentor. The geometry and dimensions of the indentor are similar to a 1mm diameter surgery tasthaken. The experiment was carried out on knee joints of pigs (deceased, age: 0.5 years, female). 10 force-way-diagrams were carried out at the femur condyle medial to obtain the averaged force-way-diagram for the comparison with the simulation. The measurements of the cartilage thickness resulted in 1.3 mm.
On the linear stage (acceleration 4 mm/s², speed 3 mm/s) the indentor was pressed 0.4 mm into the tissue. The resulting pressure force was recorded on a force sensor. Figure 1 shows the experimental measuring system to study the biomechanical behaviour of cartilage.

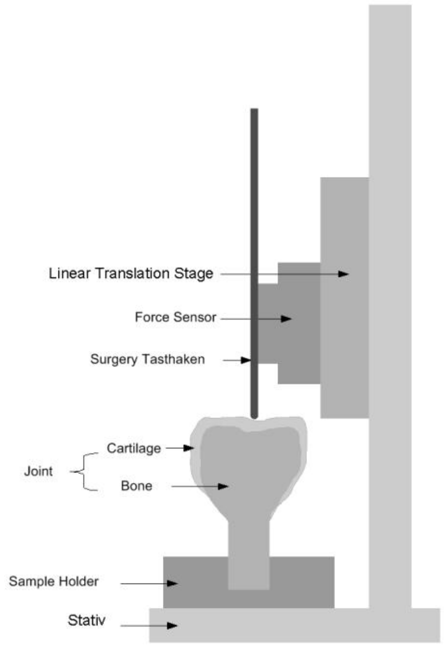

Figure 1. Experimental measuring system to study the biomechanical behaviour of cartilage.

3. Simulation Methods

The preparation of the models is carried out according to the plane stress analysis type.

The modelling is compiled into an elastic and hyperelastic material model. The model was configured in concordance with the experiment so that the indentor was placed on the cartilage containing a bone layer. Then the indentor was adjusted along the Y-axis. This adjustment in Y-direction was ceased in the subdomain constrain settings.
The pressure force F along the Y-axis was calculated by the integration of the stress σ over the contact zone between the indentor and cartilage. For the construction of the model the indentor as well as the cartilage and bone layer were modelled schematically in the 2D draw mode (figure 2). The lower boundary of the bone layer was chosen fixed in the boundary constraint settings.

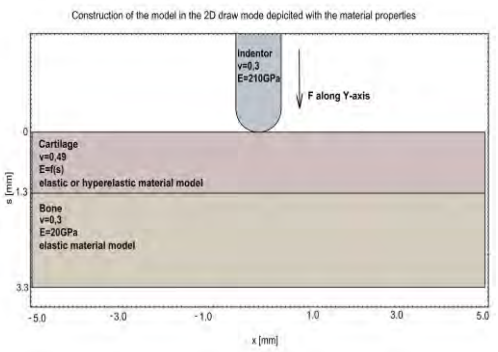

Figure 2. Model geometry and material properties.

3.1 Equations of the elastic model

A linear elastic material model [2] was chosen for the indentor and the bone layer and is described by the following equations:

$$-\nabla\sigma = Fv$$
$$\sigma = \left(S \cdot (I + \nabla u)\right)$$
$$S - S_0 = C : \left(\varepsilon - \alpha\left(T - T_{ref}\right) - \varepsilon_0\right)$$
$$\varepsilon = \frac{1}{2}\left[(\nabla u)^T + \nabla u + (\nabla u)^T \nabla u\right]$$

with

- S – Second Piola-Kirchhoff stress
- ∇u - Displacement gradient
- F – Deformation gradient
- v – Left stretch tensor
- σ – Cauchy stress
- ε – Green strain
- ε_0 – Initial strain
- I – Identity tensor
- T – Present Temperature
- T_{ref} – Stress free reference Temperature
- α – Thermal expansions vector

Also a linear elastic material model was chosen for the cartilage layer and is described by the following equations:

$$-\nabla\sigma = Fv$$
$$\sigma = s$$
$$s - S_0 = C : \left(\varepsilon - \alpha\left(T - T_{ref}\right) - \varepsilon_0\right)$$
$$\varepsilon = \frac{1}{2}\left[(\nabla u)^T + \nabla u\right]$$

3.2 Equations of the hyperelastic model

A linear elastic material model was chosen for the indentor and the bone layer and the equations are described above. For the cartilage layer a hyperelastic material model (Neo-Hookean) [2] was chosen and is described by the following equations:

$$-\nabla\sigma = Fv$$
$$\sigma = \left(S \cdot (I + \nabla u)\right)$$
$$S = \frac{\partial W_s}{\partial \varepsilon}$$
$$W_s = \frac{1}{2}\mu\left(\bar{I}_1 - 3\right) - \mu \ln\left(J_{el}\right) + \frac{1}{2}\lambda\left[\ln\left(J_{el}\right)\right]^2$$
$$\varepsilon = \frac{1}{2}\left[(\nabla u)^T + \nabla u + (\nabla u)^T \nabla u\right]$$

with

- W_s – Strain energy function
- μ, λ – Lame elastic constants
- J_{el} – Elastic deformation gradient
- I_1 – Scalar invariant of C (the right Cauchy-Green deformation tensor)

3.3 Material properties

For both models, the material properties are assigned as follows: Poisson ratio ν, E-modulus and the thickness d of the cartilage layer ($\nu = 0.49$, E = f(s), $d_C = 1.3$ mm) with the neighboring bone layer ($\nu = 0.3$, E = 20 GPa [3], $d_B = 2$ mm) as well as for the indentor ($\nu = 0.3$, E = 210 GPa).

3.4 Mesh properties and element quality

Due to the great differences between the E-modulus of the indentor (master) and the cartilage (slave) a contact pair was created. Based on this contact condition, the cartilage boundaries were meshed two times finer than the indentor boundaries [2]. Figure 3 shows the mesh. There are 2117 mesh points, 3922 triangular, 354 boundary and 11 vertex elements. In figure 4 the element quality of the mesh is shown. The minimum element quality is 0.85.

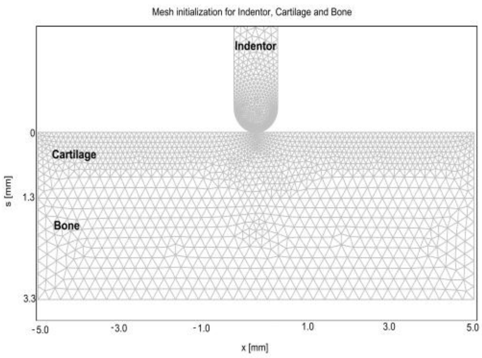

Figure 3. Mesh initialization for Indentor (master), cartilage (slave) and bone.

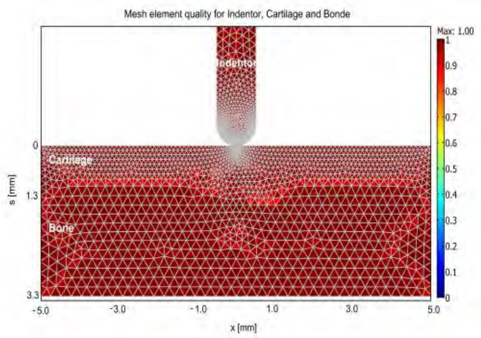

Figure 4. Mesh element quality for Indentor (master), cartilage (slave) and bone.

3.5 Model solving

The model was solved by a parametric solver. It was used the linear system solver Direct UMFPACK. The parametric properties initial step size of 0.005, minimum step size of 0.05 and maximum step size of 0.2 were chosen.

The calculations of the pressure dependent modulus of elasticity were carried out by the comparison and the iterative approach between the experimental and simulated force-way-function by a degree 4 polynomial. For the approach we need an initial polynomial, which is calculated from the slopes dF/ds of the experimental force-way-function and substituted in the equation

$$E(s) = \frac{\left(\left(\frac{dF}{ds}\right) - 0.008\right)}{8.22}$$

for the elastic and

$$E(s) = \frac{\left(\left(\frac{dF}{ds}\right) - 0.022\right)}{10.66}$$

for the hyperelastic material model. These equations were calculated from simulations with E-modulus 1, 5, 10, 15 and 20 MPa and the averaged slopes from the associated simulated force-way-functions.

The iterative approach was carried out with MATLAB® 2010b and the simulation with COMSOL® Multiphysics 3.5a.

4. Results

In figure 5 the experimental context F=f(s) is depicted in comparison with the simulation for both material models. The function F=f(s) is not linear. The simulated values lie within the range of dispersal of the experiment. The simple models observed here show a very good consensus between the simulation and experiment.

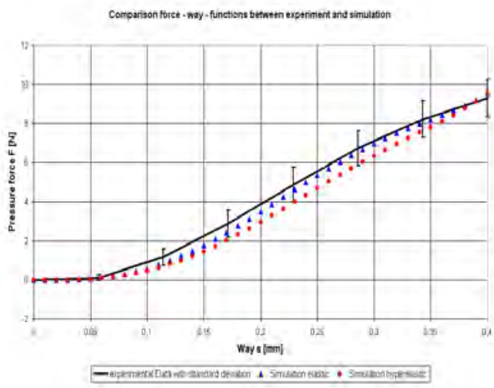

Figure 5. Simulation results in comparison with the experiment for elastic and hyperelastic material model.

The function E = f(s) is depicted for both material models in figure 6.

The E-modulus varies between 0 - 2.9 MPa for the elastic and between 0 - 2.2 MPa for the hyperelastic material model. At s = 0.11 mm, the slope $(dE/ds)_{elastic} = 16$ MPa/mm and $(dE/ds)_{hyperelastic} = 12$ MPa/mm is maximum. From s = 0.34 mm $dE/ds \approx 0$ MPa/mm.

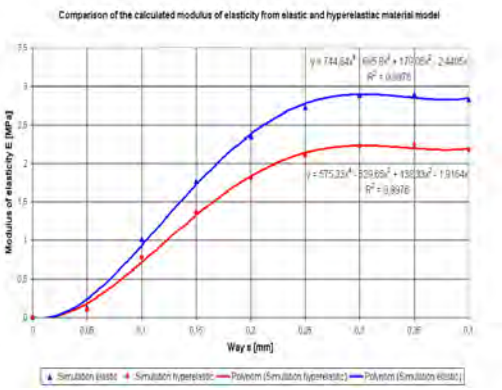

Figure 6. Calculation of the pressure dependent modulus of elasticity for elastic and hyperelastic material model.

5. Conclusions

By means of the elastic and hyperelastic material models and the use of experimental data a pressure dependent E-modulus was determined. For the chosen pressure profile both the material models deliver comparable results. The knowledge of the pressure dependent E-modulus allows for the simulation of tissue deformation as well as the stress distribution in the joint cartilage for dynamic pressure, by which the elastic properties prevail over the viscous properties. Further work will expand the material models for example with the biphasic or triphasic theory [4].

6. References

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7. Acknowledgements

The authors would like to thank the COMSOL support team. The study is part of a research project supported by the German Ministry of Economy and Technology (VF090058).