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A Elastic and Hyperelastic Material Model of Joint Cartilage - Calculation of the Pressure Dependent Material Stress in Joint Cartilage

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Abstract: In this paper we introduce a elastic and hyperelastic model to describe the pressure dependent material stress in joint cartilage. We used the pressure dependent E-modulus E = f(s) to calculate the material stress. E = f(s) is a degree 4 polynomial [1]. The indentor was pressed 0.4 mm into the tissue. The results show that the maximal stress at the contact zone between indentor and cartilage account for the elastic model is 1.8 MPa and for the hyperelastic model is 0.9 MPa. Also the stress distribution (sectional view) at the indentor, cartilage and bone at the loaded state for both models were calculated. For both models the stress decreases from the contact zone to the tide mark. The models can predict the deformation of tissue caused by short-term mechanical load as well as the resulting stress distribution within the tissue.

Keywords: Cartilage, E-Modulus, Soft Tissue, biomechanical properties

1. Introduction

The evaluation of joint cartilage occurs by palpation of the cartilage surface area with a surgery tasthaken. In the process the stiffness give information about the health state of the joint cartilage. In this case the pressure dependent material stress σ in joint cartilage is simulated by means of an elastic and hyperelastic material model. For the calculation of the material stress a pressure dependent modulus of elasticity E = f(s) were used. The calculation of the E = f(s) were carried out by the comparison and the iterative approximation between the experimental and simulated force-way-function by a degree 4 polynomial.

2. Experimental Methods

In the experiment, the force-way-diagram was dictated by the pressure of the cartilage tissue by means of an indentor. The geometry and dimensions of the indentor are similar to a 1mm diameter surgery tasthaken. The experiment was carried out on knee joints of pigs (deceased, age: 0.5 years, female). 10 force-way-diagrams were carried out at the femur condyle medial to obtain the averaged force-way-diagram for the comparison with the simulation. The measurements of the cartilage thickness resulted in 1.3 mm.
On the linear stage (acceleration 4 mm/s², speed 3 mm/s) the indentor was pressed 0.4 mm into the tissue. The resulting pressure force was recorded on a force sensor. Figure 1 shows the experimental measuring system to study the biomechanical behaviour of cartilage.

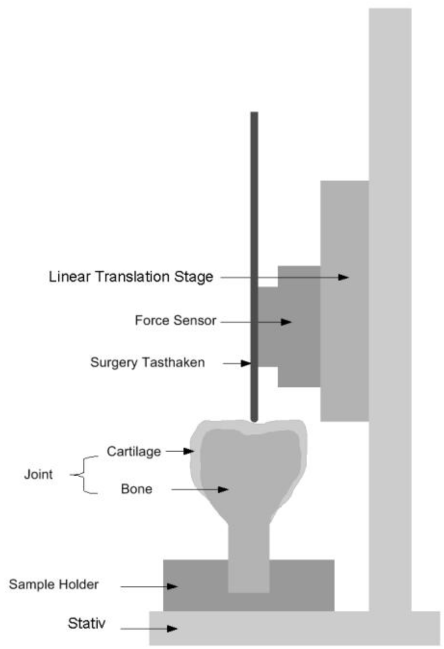

Figure 1. Experimental measuring system to study the biomechanical behaviour of cartilage.

3. Simulation Methods

The preparation of the models is carried out according to the plane stress analysis type. The modelling is compiled into an elastic and hyperelastic material model. The model was configured in concordance with the experiment

so that the indentor was placed on the cartilage containing a bone layer. Then the indentor was adjusted along the Y-axis. This adjustment in Y-direction was ceased in the subdomain constrain settings.
The occurrent stress σ and the deformation of cartilage and bone layer were calculated as a function of the way s of the indentor along the Y-axis. For the construction of the model the indentor as well as the cartilage and bone layer were modelled schematically in the 2D draw mode (figure 2). The lower boundary of the bone layer was chosen fixed in the boundary constraint settings.

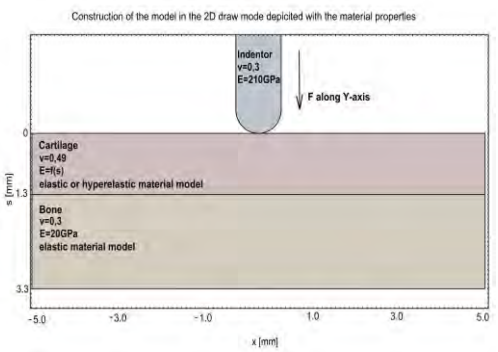

Figure 2. Model geometry and material properties.

3.1 Equations of the elastic model

A linear elastic material model [2] was chosen for the indentor and the bone layer and is described by the following equations:

$$-\nabla\sigma = Fv$$
$$\sigma = \left(S \cdot (I + \nabla u)\right)$$
$$S - S_0 = C : \left(\varepsilon - \alpha\left(T - T_{ref}\right) - \varepsilon_0\right)$$
$$\varepsilon = \frac{1}{2}\left[(\nabla u)^T + \nabla u + (\nabla u)^T \nabla u\right]$$

with

- S – Second Piola-Kirchhoff stress
- ∇u - Displacement gradient
- F – Deformation gradient
- v – Left stretch tensor
- σ – Cauchy stress
- ε – Green strain
- ε_0 – Initial strain
- I – Identity tensor
- T – Present Temperature
- T_{ref} – Stress free reference Temperature
- α – Thermal expansions vector

Also a linear elastic material model was chosen for the cartilage layer and is described by the following equations:

$$-\nabla\sigma = Fv$$
$$\sigma = s$$
$$s - S_0 = C : \left(\varepsilon - \alpha\left(T - T_{ref}\right) - \varepsilon_0\right)$$
$$\varepsilon = \frac{1}{2}\left[(\nabla u)^T + \nabla u\right]$$

3.2 Equations of the hyperelastic model

A linear elastic material model was chosen for the indentor and the bone layer and the equations are described above. For the cartilage layer a hyperelastic material model (Neo-Hookean) [2] was chosen and is described by the following equations:

$$-\nabla\sigma = Fv$$
$$\sigma = \left(S \cdot (I + \nabla u)\right)$$
$$S = \frac{\partial W_s}{\partial \varepsilon}$$
$$W_s = \frac{1}{2}\mu\left(\bar{I}_1 - 3\right) - \mu \ln\left(J_{el}\right) + \frac{1}{2}\lambda\left[\ln\left(J_{el}\right)\right]^2$$
$$\varepsilon = \frac{1}{2}\left[(\nabla u)^T + \nabla u + (\nabla u)^T \nabla u\right]$$

with

- W_s – Strain energy function
- μ, λ – Lame elastic constants
- J_{el} – Elastic deformation gradient
- I_1 – Scalar invariant of C (the right Cauchy-Green deformation tensor)

3.3 Material properties

For both models, the material properties are assigned as follows: Poisson ratio ν, E-modulus and the thickness d of the cartilage layer (ν = 0.49, E = f(s), d_C = 1.3 mm) with the neighboring bone layer (ν = 0.3, E = 20 GPa [3], d_B = 2 mm) as well as for the indentor (ν = 0.3, E = 210 GPa).

3.4 Mesh properties and element quality

Due to the great differences between the E-modulus of the indentor (master) and the cartilage (slave) a contact pair was created. Based on this contact condition, the cartilage boundaries were meshed two times finer than the indentor boundaries [2]. Figure 3 shows the mesh. There are 2117 mesh points, 3922 triangular, 354 boundary and 11 vertex elements. In figure 4 the element quality of the mesh is shown. The minimum element quality is 0.85.

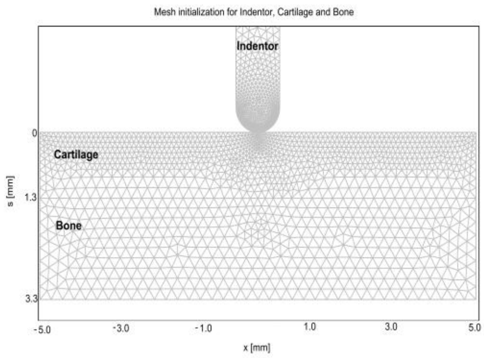

Figure 3. Mesh initialization for Indentor (master), cartilage (slave) and bone.

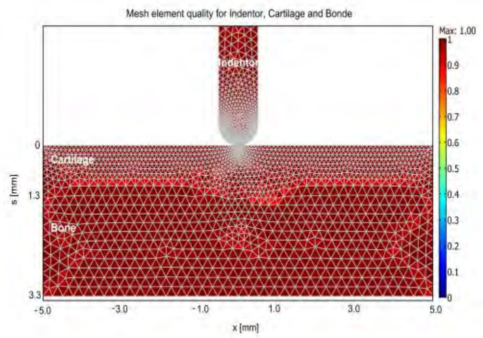

Figure 4. Mesh element quality for Indentor (master), cartilage (slave) and bone.

3.5 Model solving

The model was solved by a parametric solver. It was used the linear system solver Direct UMFPACK. The parametric properties initial step size of 0.005, minimum step size of 0.05 and maximum step size of 0.2 were chosen.

The calculations of the pressure dependent modulus of elasticity were carried out by the comparison and the iterative approach between the experimental and simulated force-way-function by a degree 4 polynomial.

The iterative approach was carried out with MATLAB® 2010b and the simulation with COMSOL® Multiphysics 3.5a.

4. Results

In figure 5 and 6 the stress distribution is depicted for the elastic and hyperelastic material model at a way s = 0.4 mm, a calculated E = f(s) [1] and a Poisson ratio ν = 0.49. The maximal stress at the contact zone between indentor and cartilage account for the elastic model 1.8 MPa and for the hyperelastic model 0.9 MPa.

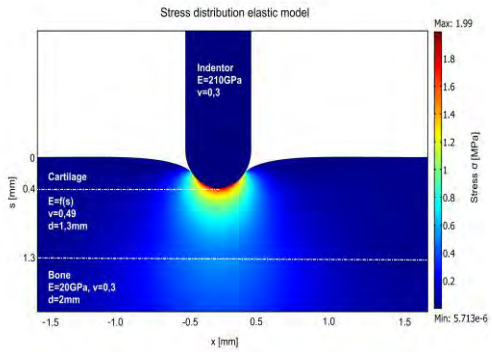

Figure 5. Stress distribution elastic model.

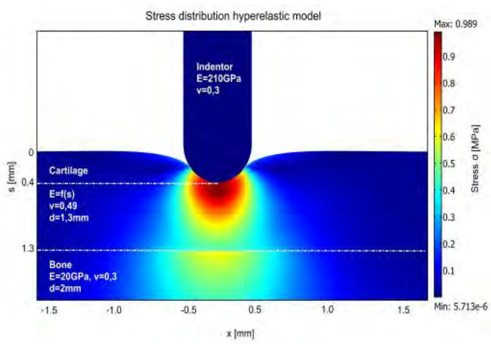

Figure 6. Stress distribution hyperelastic model.

Figure 7 and figure 8 show the stress distribution (sectional view) at the indentor (A), cartilage (B) and bone (C) at the loaded state for the elastic and hyperelastic model. In both models the stress at the indentor is nearly zero. For the elastic model the stress decrease from the contact zone to the tide mark to 0.46 MPa and increase at the tide mark to 0.55 MPa. For the hyperelastic model the stress also decrease from the contact zone to the tide mark. At the tide mark the stress increase from 0.39 MPa to 0.61 MPa.

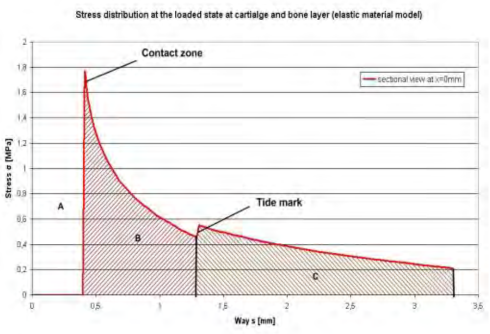

Figure 7. Stress distribution elastic model (sectional view).

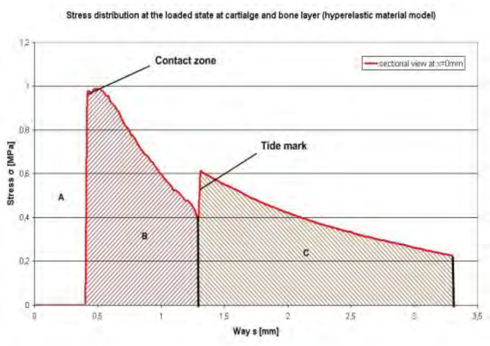

Figure 8. Stress distribution hyperelastic model (sectional view).

5. Conclusions

The introduced simple models allow to simulate the stress distribution in joint cartilage for selected load situations caused by a surgery tasthaken pressed into cartilage tissue. The stress distribution can be qualitatively and quantitatively analyzed for a tissue compression from 0 – 0.4 mm. Through the change of the model geometry the stress of the tissue dependency on the pressure force or the layer composition of the tissue can be examined.

6. References

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