Numerical modelling of moisture related mechanical stress in wooden cylindrical objects using COMSOL: a comparative benchmark

H.L. (Henk) Schellen (PhD), A.W.M. (Jos) van Schijndel (PhD)
Eindhoven University of Technology, Department of Building and Architecture, VRT 6.29; P.O. Box 513; 5600 MB Eindhoven; The Netherlands; H.L.Schellen@tue.nl

Abstract: For preservation of artefacts in a museum the indoor climate is often restricted to a very narrow interval for temperature, but most of all for relative humidity (1,7). In old buildings the museum conditions of artefacts, e.g. near cold walls, mostly are not in line with museum recommendations.

To have an impression of indoor museum climates in old buildings, a large number of case studies were carried out in several Dutch museums. For at least one year, temperature and relative humidity were recorded in different rooms and at different external wall surfaces of the museums. The results of this measurement campaign reveal that there were a large number of indoor climate conditions that did not satisfy the originally formulated restricted climate (1).

To have an impression of the impact of the measured indoor climates on the conservation properties of objects, a numerical simulation model was needed. The model should be able to determine the temperature and moisture content gradients in organic objects and the related mechanical stress due to changing environmental indoor climate conditions. For this purpose a three dimensional numerical simulation model was constructed in COMSOL. The partial differential equations for heat and moisture transport in objects were implemented in COMSOL and coupled to the mechanical stress module of COMSOL. The results were temperature and moisture profiles in three dimensional objects due to dynamically changing (measured) environmental conditions. The resultant mechanical stresses were calculated and compared to maximum allowable threshold stresses to stay in the safe elastic strain regime.

Keywords: Museum indoor climate, modeling, conservation, museum objects, stress, moisture induced damage.

1. Introduction

The indoor climate conditions in a number of about 25 different museums in The Netherlands have been monitored (4). One of the key questions was to determine the possible risk on damage to objects in relation to the varying indoor climate. From literature it is known that varying climate conditions, especially fluctuating relative humidity (RH) conditions, result in varying moisture contents in hygroscopic materials like wood. These varying moisture contents induce deformations of the wooden materials. When these deformations stay in the elastic range of the wooden materials no damage is expected. When a maximum stress level is reached, the response of the materials, however, is no longer reversible. The inelastic region is reached and the deformations will result in an irreversible response. S. Jakiela et al. (2) developed a numerical model of moisture movement and related stress in a lime wood cylinder. The model was developed to determine maximum stress levels in a wooden cylinder, representing objects of art exposed to changing climate conditions.

The object of this work was to develop a similar model in COMSOL to determine the stress levels in a similar object under measured boundary conditions in the different museum environments. In this way it was possible to compare different museum indoor climates regarding the damage they may induce in objects of value.

2. COMSOL model

The COMSOL model is based on a model of a lime wooden cylinder with diameter 0.13m. The coordinate system is cylindrical and the material model is assumed to be orthotropic.

2.1 Hygro thermal equations

The COMSOL model is based on the following equations:

Thermal transport:

$$\rho c_p \frac{\partial T}{\partial t} = \nabla(k\nabla T) \qquad (1)$$

ρ = Density [kg/m^3]

c_p = Constant pressure specific heat [J/kg.K]
k = Thermal conductivity [W/m.K]

Water vapor transport by diffusion:

$$\frac{\partial P}{\partial t} = \nabla(D(P)\nabla P) \qquad (2)$$

P = Vapor pressure [Pa]
D(P)= Moisture diffusion coefficient [m^2/s]

2.2 Boundary conditions

Thermal:

$$q = h \cdot (T_o - T_s) \qquad (3)$$

q = Heat flux at surface [W/m^2]
h = Heat transfer coefficient [$W/m^2.K$]
h = 7.7 $W/m^2.K$

Hygric:

$$g = \beta \cdot (P_o - P_s) \qquad (4)$$

g = Vapor flux at surface [kg/m^2s]
β = Vapor transfer coefficient [s/m]
β = 1.5E-6 s/m
P_o = $RH_o * P_{sat}(T_o)$ [Pa]

Indices:
o : environmental
s : surface
sat : saturated

Environmental conditions
t=0:
T_o=20 °C
RH_o= 70%RH
t=∞:
T_o=20 °C
RH_o= 30%RH

2.3 Mechanical stress and strain

In a first approximation the deformation of wood is considered to be elastic and orthotropic. Partial differential equations (PDE) for the linear stress and strain description by Hooke's Law will be coupled to PDE's for heat and moisture description.
For a 3-D volume element in a continuous body in equilibrium the differential equations of mechanical equilibrium in Cartesian coordinates are described by (3). In a first approximation wood can be considered as an orthotropic material. Due to the strength in the axial direction the displacements in this direction can be neglected.

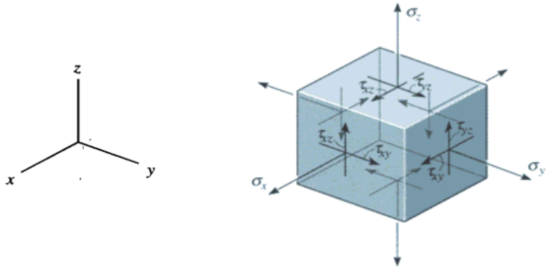

Figure 1: General state of stresses (Hibbeler 1997)

The components of stresses in a continuous plane in equilibrium under the action of surface (without external body forces) satisfy two differential equations of equilibrium. These equations expressed in Cartesian coordinates have the form:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$

$$\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0$$

where
$\sigma_{x,y}$ = the normal components of stress [N/m^2]
τ_{xy}, τ_{yx} = the shear components of stress associated with two axes [N/m^2]

The relation between stress and strain is described by the generalized Hooke's law and for an anisotropic material it can be written as:

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_y} & 0 \\ -\frac{\nu_{yx}}{E_x} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix} + \begin{pmatrix} \alpha_x \\ \alpha_y \\ 0 \end{pmatrix} \Delta\theta + \begin{pmatrix} \kappa_x \\ \kappa_y \\ 0 \end{pmatrix} \Delta w$$

where
ε_x, ε_y = the normal strain components [-]
γ_{xy} = the shear strain component associated with two axes [-]
υ_{xy}, υ_{yx} = Poisson's ratio [-]
E_x, E_y = moduli of elasticity or Young's moduli [N/m^2]

G_{xy} = the shear modulus [N/m^2]
α_x, α_y = the linear thermal expansivity [m/m.K]
$\Delta\theta$ = a temperature increment [K]
w = moisture content [kg/m^3]

κ_x, κ_y = the linear relative deformation (shrinkage or swelling) due to moisture content changing [m/m(kg/m^3)]

In case of small displacements of a continuous body, we can write

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y} \quad \text{and} \quad \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

Furthermore the matrix must be symmetrical, therefore

$$\frac{\nu_{yx}}{E_x} = \frac{\nu_{xy}}{E_y}$$

For the elastic description of an orthotropic material, 4 independent elastic material properties must be known: E_x, E_y, G_{xy} and one of υ_{xy} or υ_{yx}.

In fact, the material was treated as an anisotropic continuous body, with orthotropic mechanical and deformation material properties.

Inverting the two-dimensional matrix and combining the relations above, we obtain the displacement equations, which can be written:

$$-\nabla\cdot(c\otimes\nabla u) = 0$$

Where c is a rank four tensor, which can be written as four 2-by-2 matrices c_{11}, c_{12}, c_{21} and c_{22}:

$$c_{11} = \begin{pmatrix} 2G+\mu & 0 \\ 0 & G \end{pmatrix}$$

$$c_{12} = \begin{pmatrix} 0 & \mu \\ G & 0 \end{pmatrix}$$

$$c_{21} = \begin{pmatrix} 0 & G \\ \mu & 0 \end{pmatrix}$$

$$c_{22} = \begin{pmatrix} G & 0 \\ 0 & 2G+\mu \end{pmatrix}$$

Where G, the shear modulus, is defined by

$$G = \frac{E}{2(1+\nu)}$$

And μ in turn is defined by

$$\mu = 2G\frac{\nu}{1-\nu}$$

2.4 Material properties

The material properties of lime wood were taken from Jakiel (2):

The mean dry density of the specimens was 530 kg/m^3. Poisson's ratio υ_{xy}= υ_{yx}=0.4.The equilibrium moisture content and the moisture diffusion coefficients were taken from graphs presented in (2) and were implemented in COMSOL in interpolation tables, as is presented in table 1.

Table 1: Moisture diffusion coefficient D and equilibrium moisture content EMC of lime wood as a function of equilibrium relative humidity RH

RH [%RH]	D(RH) [m^2/s]	EMC(RH) [m%]
0	1.0E-13	0
10	1.0E-13	2.5
20	5.0E-12	4.0
40	3.5E-12	6.7
60	5.0E-12	10.5
80	4.0E-12	14.5
90	3.7E-12	19.0
100	3.5E-12	24.0

The modulus of elasticity properties of lime wood were copied from a table presented by Jakiel (2) into an interpolation table in COMSOL:

Table 2: Modulus of elasticity of lime wood in tangential direction (ET) and radial direction (ER)

RH [%RH]	ET [MPa]	ER [MPa]
20	600	1120
35	490	900
50	450	820
65	420	770
80	400	760

The dimensional changes of lime wood in the radial and tangential direction across the entire range of EMC were α_R=0.13 and α_T=0.28. These values were related to the change in RH.

3. Verification

The results of the COMSOL model were compared with the results from Jakiel (2). These results are presented in figures 1 and 2:

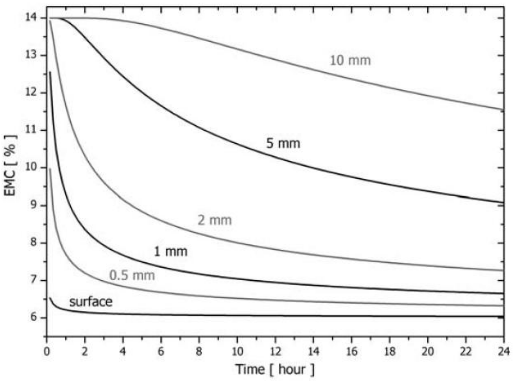

Figure 2: Change in the distribution of moisture content at selected distances from the external surface of a wooden cylinder for a step RH variation from 70 to 30% (2)

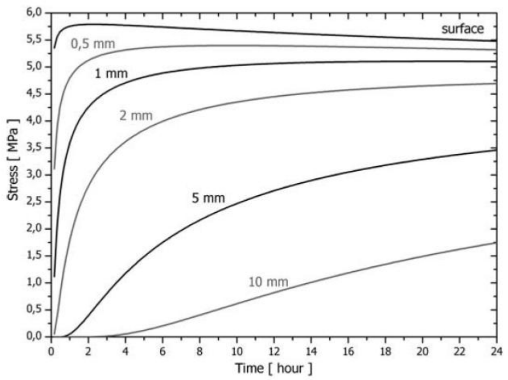

Figure 3: Tangential stress developing in wood as a result of the gradient of moisture content shown in figure 1 (2)

Figure 2 represents the changing moisture content of the cylinder of wood on different depths from the surface as a function of time. The initial equilibrium moisture content EMC at a starting relative humidity of 70 %RH is 14%. The step variation in RH is from 70 to 30%RH. Figure 3 shows the tangential stress in the wooden cylinder due to the changing moisture content. The largest stress is developing at the surface, due to the shrinking of wood at the surface, induced by the drying at the surface.
Figure 4 and 5 can be compared to figure 2 and 3. The increment of the curves depth is 1 mm, starting from the surface.

The corresponding figures from the COMSOL model are presented below:

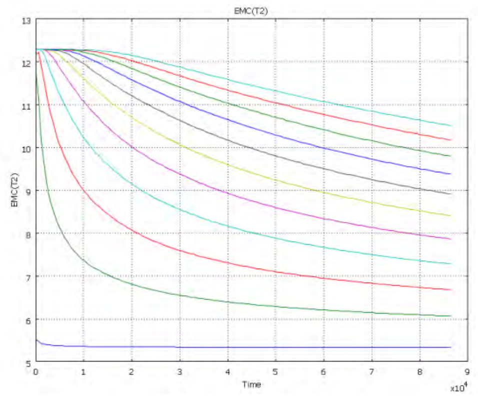

Figure 4: COMSOL calculation of the change in the distribution of moisture content at selected distances from the external surface of a wooden cylinder for a step RH variation from 70 to 30%

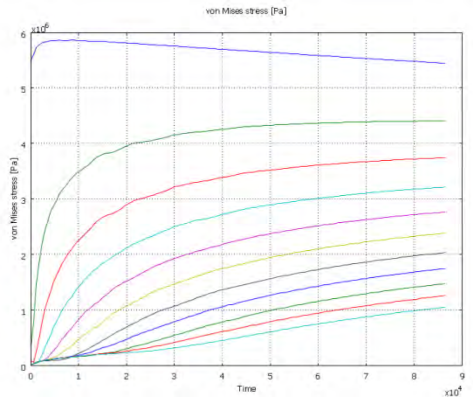

Figure 5: COMSOL calculation of the tangential stress developing in wood as a result of the gradient of moisture content shown in figure 3

The equilibrium moisture content at the starting relative humidity of 70%RH slightly differs in the COMSOL calculations. This is the result of the interpolation tables, which were taken from the curves from (2).
Figures 6 and 7 show moisture gradient and stress gradient by Jakiel (2) after 24 h (a) and 10 days (b). The lightest tone corresponds to the initial moisture content of 14% and lack of stress. The darkest tone corresponds to the final moisture content of 6 % and the stress level of 5.75 MPa. Figures 8 and 9 show COMSOL results.

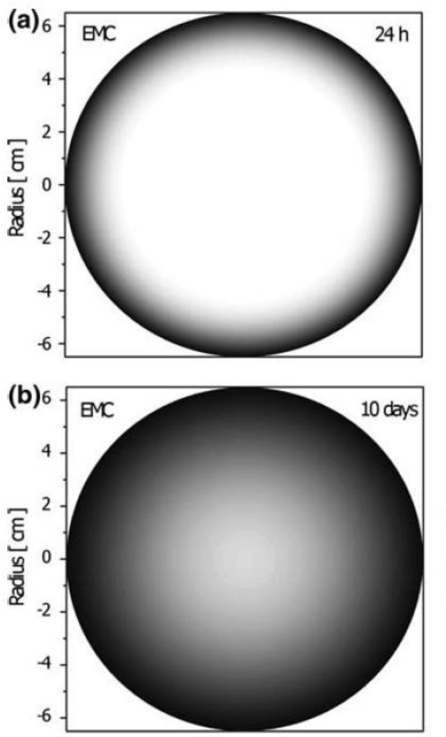

Figure 6: Moisture content gradient (2)

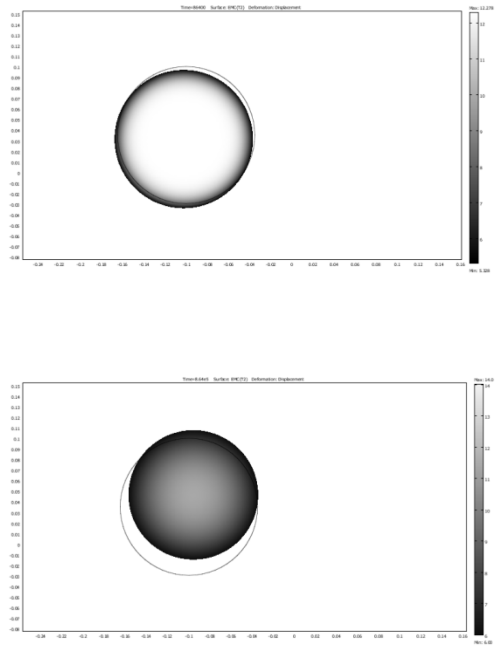

Figure 8: COMSOL results for moisture gradient

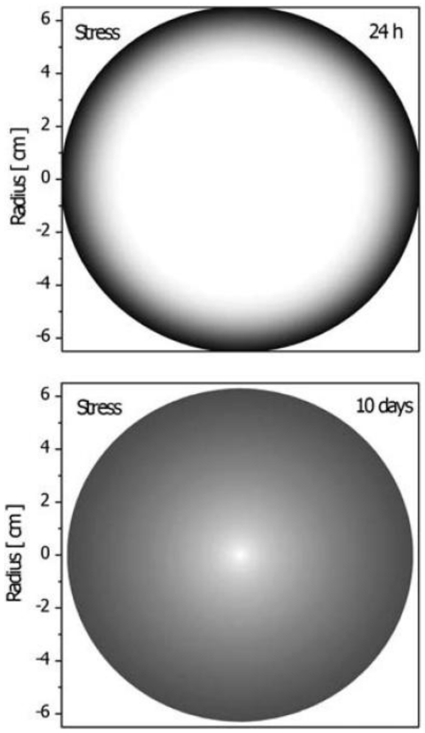

Figure 7: Evaluation of stress gradient

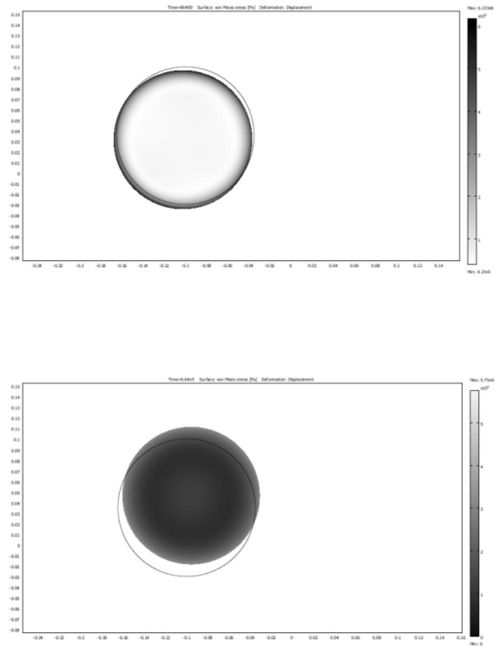

Figure 9: COMSOL results for stress gradient

4. Conclusion

The implementation of a numerical model in COMSOL for calculating moisture induced mechanical stress in wooden objects seems to be promising. There was a clear equivalence between the results, presented by Jakiela and the results from the COMSOL model. In future the COMSOL model will be used to calculate mechanical stress in objects, induced by a a varying indoor climate, as has been measured in several museums.

5. References

1. ASHRAE. Museums, Libraries and archives, in: 2007 ASHRAE Handbook,: Heating, ventilating, and air-conditioning applications, SI edition, American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc., 2007, Chapter 21, p.p. 21.1-21.23. (2007)

2. Jakiela, S., Bratasz, L., Kozlowski, R.: Numerical modeling of moisture movement and related stress field in lime wood subjected to changing climate conditions. Wood Sci Technol (2008) 42: 21-37 (2007)

3. Lekhnitskii, S.G. Theory of Elasticity of an Anisotropic Elastic Body. Holden-Day, Inc. San Francisco (1963)

4. Martens, M.H.J., Schellen, H.L. A sound indoor climate for a museum in a monumental building. Thermal performance of the building exterior envelopes of whole buildings XI International Conference. Florida. (2010)

5. Mecklenburg, M.F., Tumosa, C.S., ‘Temperature and relative humidity effects on the mechanical and chemical stability of collections’, in: ASHRAE Journal, p.p. 77-82. (1999)

6. Schellen, H.L. Heating monumental churches. Indoor climate and preservation of cultural heritage. PhD thesis. Eindhoven University of Technology. ISBN: 90-386-1556-6 (2002)

7. Thomson, G. The Museum Environment, 1st edition, Butterworths, in association with the International Institute for Conservation of Historic and Artistic Works, London, United Kingdom, 1978, p.p. 114. (1978)

5. Acknowledgement

This work was supported by European Commission funding through the EU Climate for Culture project 226973 within FP7-ENV-2008-1