

A Comprehensive 3-D Electromagnetic Model of Package Sealing Inductors by means of Surface Current Density

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Abstract

A 3-D modeling approach for the consistent computation of electric and magnetic fields in inductors employed in package sealing applications is presented. The inductor consists of a coil, a metallic body, and one or more dielectric materials. The time-varying magnetic field generated by the coil induces eddy currents in a thin aluminum workpiece embedded in a multilayer packaging material, resulting in Joule heating that enables sealing through polymer welding. Among the dielectric materials employed in the inductor to insulate the coil from the body, magnetodielectric materials are of particular interest, as they are used to guide and concentrate the magnetic flux in the workpiece sealing region while minimizing eddy current losses in the magnetic core. Comprehensive analysis of the device requires the simultaneous evaluation of electric and magnetic fields, workpiece eddy currents, magnetic and dielectric losses in magnetodielectrics, and, therefore, of the overall power dissipation. Along with these requirements, the capability to directly prescribe conductors voltage boundary conditions is needed to reproduce, in the model, the inductor bias scheme. To this purpose, a computational framework based on the A-V formulation is implemented in COMSOL Multiphysics® using the *Electric and Magnetic Fields (mef)* interface of the AC/DC Module, where the electric scalar potential and the magnetic vector potential are solved simultaneously. By including the electric potential as a dependent variable, voltage can be prescribed on both the coil terminals and the metallic body, allowing the representation of the operating bias scheme, including the high-voltage configurations used in dielectric breakdown tests. Furthermore, the “full-Maxwell” formulation enables a consistent evaluation of electric and magnetic fields together with all relevant electromagnetic loss mechanisms, allowing the total electromagnetic power dissipated in the inductor-workpiece system to be determined. A key aspect of the modeling approach concerns the representation of the aluminum workpiece. Its relatively wide and thin geometry would naturally suggest the use of a Transition Boundary Condition (TBC). However, TBC leads to numerically challenging systems, with convergence ranging from extremely slow to unattainable depending on the solver formulation. On the other hand, in the induction sealing systems at hand, the workpiece thickness is much smaller than the skin depth, such that the thin-sheet limit of TBC reduces to a Surface Current Density (SCD) condition. Notably, SCD predicts the correct electromagnetic behavior of the workpiece while exhibiting robust and faster convergence across all tested solver formulations. The methodology is demonstrated on a simplified inductor model retaining the key physical features of induction sealing devices. Finally, the computational requirements associated with the analysis of an industrial inductor are reported.

Keywords: induction sealing, transition boundary condition, surface current density, A-V formulation.

Introduction

Induction sealing is a technology widely used in food packaging filling machines to seal liquid food containers. In this process, eddy currents are induced in a thin electrically-conductive workpiece, typically an aluminum foil, with thickness $th_{wkp} < 10 \mu\text{m}$. The foil is embedded within two plastic films of a laminated packaging material structure mainly composed of paperboard [1]. Joule heating, generated in the foil by the induced eddy currents, raises the temperature of the adjacent plastic above the melting point, enabling sealing through polymer welding. In order to generate the time-varying magnetic field responsible for the development of eddy currents in the aluminum foil, inductors supplied with alternating current are employed [2]. Induction sealing inductors typically comprise: (i) a

coil, made of copper or other metallic alloys, supplied at a frequency $f > 500 \text{ kHz}$, (ii) a metallic body, designed to withstand the high mechanical stress applied in the filling machine jaw system, and (iii) dielectric materials separating the coil from the body, often magnetodielectric (MD), i.e., electrically insulating materials with relative magnetic permeability $\mu_r > 1$. MDs are commonly arranged in a U-shaped configuration to concentrate the magnetic flux in the sealing area while shielding surrounding components, thereby enhancing efficiency and reducing body losses, and, owing to their dielectric nature, limiting parasitic eddy currents within the magnetic core itself. However, MD insulating function could be challenged under high electric field stress promoting the onset of electrical breakdown. The problem may worsen by the adoption of advanced packaging materials with

improved recyclability featuring a reduced aluminum thickness because, in order to maintaining the filling machine production rate, higher operating frequencies may be required. This results in higher inductive voltage drop hence higher electric field, which can increase dielectric losses and local heating, potentially degrading the insulating properties of the MD and eventually leading to electrical breakdown [3], [4]. Since magnetic losses also contribute to heating, and both magnetic and dielectric losses are generally nonlinear functions of temperature, a bidirectional coupling arises among the electric, magnetic, and thermal fields. In this framework, a comprehensive electromagnetic modeling approach for an induction sealing inductor must be able to: (i) predict the eddy currents induced in the workpiece, (ii) evaluate the electric field distribution in the dielectric materials, (iii) calculate magnetic and dielectric losses in MDs, (iv) determine the fields distribution corresponding to a prescribed load power, i.e., the power delivered to the inductor-workpiece system, and (v) reproduce operating and test configurations in which voltage is prescribed at both the coil terminals and the metallic body. The latter capability is particularly relevant because induction sealing tests are typically conducted at a prescribed load power, yet different voltage bias schemes can deliver the same sealing power while producing different electric field distributions, with important implications for insulation performance hence device reliability. To address these requirements, a modeling approach based on the A-V formulation of the *Electric and Magnetic Fields (mef)* interface of the AC/DC Module of COMSOL Multiphysics® is adopted, enabling the fully coupled computation of electric and magnetic fields. Since the device dimensions are much smaller than the wavelength at the operating frequency, radiation effects can be neglected, and an AC/DC formulation is sufficient. Moreover, *mef* solves for the electric scalar potential, enabling the direct specification of voltage boundary conditions. A further key aspect of the model concerns the representation of the thin aluminum workpiece. Notably, preliminary simulations showed that the more general Transition Boundary Condition (TBC) may lead to poorly conditioned systems and slow convergence. Therefore, the thin workpiece is modeled using a Surface Current Density (SCD) formulation, which reduces the coupling introduced by the TBC across the thin conducting layer, resulting in a more robust and computationally efficient solution. Application of SCD is justified in the case at hand because $th_{wkp} \ll \delta$, where δ is the skin depth, such that the asymptotic thin-sheet limit of TBC reduces to a SCD.

The paper is organized as follows. First, the simplified inductor device under investigation is introduced. The capabilities and limitations of the *Electrostatics*, *Electric Currents*, *Magnetic Fields*, and *Magnetic and Electric Fields* interfaces of the AC/DC Module are then discussed with respect to the present application, progressively motivating the adoption of the proposed *mef*-based formulation. The resulting model is then applied to the simplified inductor, and the obtained results are discussed. A comparison between TBC and SCD representations of the workpiece follows. Finally, the computational requirements of the proposed methodology for an industrial induction sealing device are reported.

A Dummy Inductor Device

In order to identify an efficient modeling framework avoiding unnecessary complications, a dummy inductor is considered first. It features a very simplified geometry whose mesh can be accurately and easily controlled, whilst retaining the key physical features required to investigate the modeling approach. In the device, see Figure 1, a relatively thick rectangular copper coil generates the time-varying magnetic field inducing eddy currents in the nearby (distance from coil ~ 1 mm) flat, wide and thin aluminum workpiece. An MD material (i.e., relative dielectric constant $\epsilon_r > 1$, relative magnetic permeability $\mu_r > 1$, electrical conductivity $\sigma = 0$) channels the magnetic flux density on the workpiece regions closer to the coil conductors, and electrically insulates the coil from the metallic body. The voltages prescribed on the positive and negative coil terminals are denoted by V_+ and V_- , respectively, while the body is assumed to be grounded, i.e., $V_b = 0$.

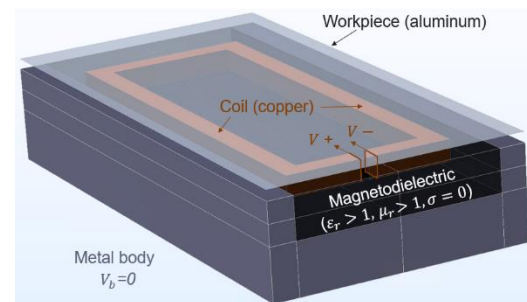


Figure 1. Dummy inductor model under investigation. V_+ and V_- represent the voltages at the coil terminals, while the metallic body is grounded ($V_b = 0$). The magnetodielectric and the workpiece are shown in transparent view to visualize the coil.

In industrial inductors, MDs are typically employed alongside other dielectric materials and are confined only to the regions where magnetic flux concentration is required to enhance heating.

Nevertheless, as the focus of this work is the physics modeling setup rather than the optimization of the MDs distribution, a single domain of MD is assumed to fill the whole inductor. As a starting point, the dummy inductor model neglects all loss mechanisms in MD considering $\sigma = \varepsilon_r'' = \mu_r'' = 0$, where ε_r'' and μ_r'' are the imaginary parts of the complex permittivity (dielectric losses) and of the complex permeability (magnetic losses), respectively. In addition, the dependence of the real part of the magnetic permeability μ_r' on the magnetic field is disregarded. Besides, since heat transfer is, for the time being, outside the scope of this work, the temperature dependence of the material properties is not considered either. The materials assumed in the dummy model are therefore described by constant parameters, thus the resulting electromagnetic problem is linear.

Limits of the *Electrostatics* and *Electric Currents* Interfaces

Among the available AC/DC formulations, *Electrostatics* (*es*) constitutes the simplest approach for estimating electric field distributions. Despite neglecting inductive phenomena, electrostatic analyses are widely employed for capacitance extraction and electric field evaluation in electrically small structures, even when the excitation is time-harmonic [5]. However, in *es*, conductors are treated as equipotential surfaces, therefore the voltage drop developing along the coil cannot be reproduced.

The *Electric Currents* (*ec*) interface overcomes this limitation by allowing the voltage to vary continuously along the coil. This is achieved by applying voltage boundary conditions at coil terminals and by solving for the electric potential according to charge conservation and Ohm's law within the conductive domains. As a result, a voltage drop along the coil is determined, and the corresponding 3-D electric field can be computed. For the considered device, the resulting voltage distribution exhibits the expected smooth variation along the coil, see Figure 2. Consequently, by allowing the voltage to vary continuously along the coil, the *ec* interface enables a meaningful 3-D analysis of the electric field. Nevertheless, *ec* does not obviously provide access to the magnetic field. Therefore, the inductive contribution to the device impedance is neglected, preventing a consistent description of coil voltage and current, as well as eddy currents and loss mechanisms within the load. These considerations motivate the transition to a more comprehensive electromagnetic formulation.

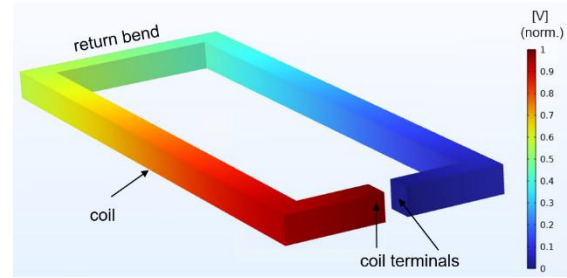


Figure 2. Voltage drop on the coil calculated by the *Electric Current* (*ec*) interface; boundary conditions: $V_+ = 1$ (normalized reference value), $V_- = 0$.

Limits of the *Magnetic Fields* Interface

Applicable to a wide range of problems of small electromagnetic size where radiation can be neglected, as in the case at hand, the frequency-domain *mf* formulation features a full-Maxwell approach, through which both the electric field $\mathbf{E}$ and the magnetic field $\mathbf{H}$ can be derived in agreement with Faraday's and Ampère's laws. More specifically, in *mf*, the dependent variable is the magnetic vector potential $\mathbf{A}$, such that the magnetic flux density $\mathbf{B} = \nabla \times \mathbf{A}$. From Faraday's law, the electric field can generally be expressed as $\mathbf{E} = -j2\pi f \mathbf{A} - \nabla V$, where V is the electric scalar potential. In *mf*, the Weyl gauge $V=0$ is applied, hence $\mathbf{E} = -j2\pi f \mathbf{A}$. For induction heating applications, such as induction sealing, *mf* is usually the tool of choice, allowing to compute eddy currents and other inductive phenomena such as the proximity effect, also known, with reference to 3-D systems, as ring effect. Figure 3 illustrates the surface current density distribution in the coil as computed by *mf*; the ring effect, and the proximity effect with the workpiece, leading to a concentration of current density toward the top surface of the coil, are clearly observed (workpiece not shown in the figure).

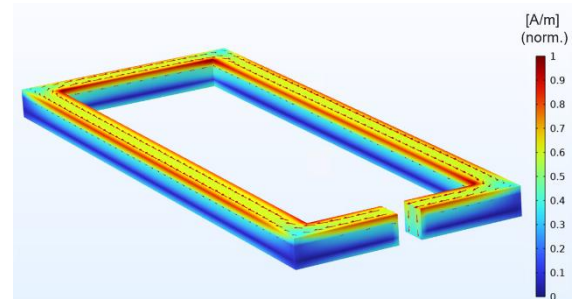


Figure 3. Normalized surface current density on coil calculated by the *Magnetic Fields* (*mf*) interface; the ring effect and the proximity effect with the workpiece (not shown) are evident.

In *mf*, the relatively thick coil and body conductors can be modeled using an "Impedance Boundary Condition" (IBC), also known as the Leontovich

boundary condition [6]. In this formulation, the tangential electric and magnetic fields are related through a surface impedance parameter. This approximation is valid when the skin depth is much smaller than the characteristic dimensions of the structure, and allows avoiding the explicit meshing of the conductor interior. Conversely, the relatively wide and thin workpiece can be represented through a “Transition Boundary Condition” (TBC). The TBC models a thin conductive layer through an equivalent surface that relates the fields on its two sides, thereby reproducing the layer response with th_{wkp} entering as a model parameter.

For TBC to be applicable, it is sufficient that the layer thickness is relatively small compared to the characteristic dimensions and curvature of the modeled object, it does not require the thickness to be strictly smaller than the skin depth. Owing to its generality, the TBC is typically adopted as a default modeling approach for relatively thin conductive layers [7]. Being a full-Maxwell formulation, *mf* naturally captures the electric field distribution associated with the inductive property of the device, resulting in an electric field that is stronger near the coil terminals than in the return bend region, see Figure 4.

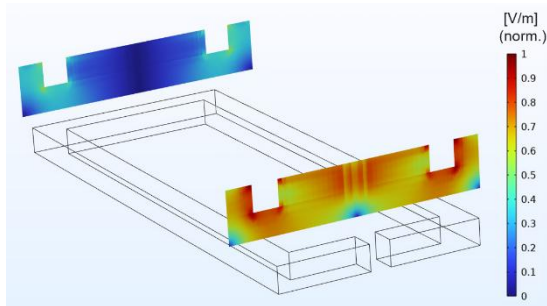


Figure 4. Normalized electric field distribution calculated by *mf*. The higher field at coil terminals compared to the field at the coil return bend is correctly captured. Since V_b cannot be prescribed, the resulting electric field distribution corresponds to a floating-body condition where the field is symmetric with respect to the coil centerline.

These findings agree with experimental observations indicating a higher breakdown probability in regions closer to the coil terminals. However, *mf* does not incorporate any electric scalar potential. Consequently, electric potential boundary conditions cannot be prescribed. In particular, the metallic body cannot be explicitly grounded, such that the resulting electric field can only represent a floating-body condition, in which the electric field between coil and body is governed by capacitive coupling. The resulting electric field distribution is thus symmetric with respect to the coil centerline (in turn, this symmetry would allow

only half of the geometry to be modeled).

Conversely, as discussed in the next section, the *mef* interface enables the direct specification of voltages on conductive terminals, thereby overcoming the limitation of the *mf* interface.

A Modeling Setup based on the *Magnetic and Electric Fields* Interface

The *Magnetic and Electric Fields* (*mef*) interface is another full-Maxwell formulation available in the AC/DC module. It implements an A-V formulation in which both the magnetic vector potential $\mathbf{A}$ and the electric scalar potential V are retained as dependent variables, leading to $\mathbf{E} = -j2\pi f \mathbf{A} - \nabla V$. In the static limit, the inductive contribution due to $\mathbf{A}$ vanishes and V coincides with the electrostatic potential. As already discussed, an advantage of *mef* is that, since V is a dependent variable of the electromagnetic problem, electric potential boundary conditions can be prescribed directly; the introduction of the additional variable, however, requires solving for four unknowns: the three components of $\mathbf{A}$ plus the additional scalar V . Since voltage is prescribed as a boundary condition in the proposed formulation, it is useful to briefly discuss under which circumstances it constitutes a meaningful excitation quantity. A terminal voltage is well defined whenever the line integral of the electric field between the terminals is effectively independent of the integration path. This condition is rigorously satisfied in electrostatic systems and in ideal TEM structures, and remains valid to engineering accuracy for electrically small circuits and ports, as well as for quasi-TEM devices. In the present case, depending on the operating frequency, the inductor supply system can be described using either lumped circuit theory or TEM transmission line concepts. Consequently, the operating bias scheme can be specified in terms of terminal voltages. These voltages provide boundary conditions for V , whereas the electric field inside the device is determined by both V and $\mathbf{A}$.

In the present *mef* implementation, boundary conditions for V at coil terminals are achieved by means of “Electric Potential” or “Ground” nodes instantiated under the IBC feature, such that a prescribed voltage is applied to selected edges belonging to the IBC boundaries. Specifically, $V+$ and $V-$ are applied to the rectangular edges corresponding to the coil terminals (see Figure 1). Conversely, grounding of the body is achieved through the external boundary belonging to the bottom surface of the computational domain. In fact, since a “Magnetic Insulation” condition set to the default “Ground” potential reference is assigned to this boundary, it is sufficient for the body to share one or more edges with it to enforce $V_b = 0$.

Furthermore, to guarantee current conservation, the coil terminals are galvanically connected through “Magnetic Insulation” features set to “Electric Insulation”, thereby ensuring a continuous current path throughout the coil. When one of the coil terminals is grounded, the same approach is adopted to establish a galvanic connection to the external ground.

Moreover, it is worth noticing that, while *mf* is a gauged formulation by default, *mef* is ungauged. Consequently, a “Gauge Fixing for A-field” node is required in *mef* when using direct solvers. This feature removes the gauge freedom associated with the magnetic vector potential by enforcing the Coulomb gauge $\nabla \cdot \mathbf{A} = 0$. Iterative solvers, on the other hand, are able to determine the correct electric field even without gauge fixing by minimizing the residual and converging despite the problem not being unique. The adoption of gauge fixing yields a smoother and more easily interpretable representation of V , whilst it typically incurs a higher computational cost. Finally, it is observed that, although the geometry of the device under test is symmetric, a full 3-D model is required since electric potential boundary conditions cannot be symmetric.

Surface Current Density Boundary Condition

A TBC to be applied to the workpiece is also available in the *mef* interface; however, it leads to numerical issues. With gauge fixing and direct solvers, the TBC formulation fails to converge. Although convergence can be achieved using the FGMRES (Flexible Generalized Minimal Residual Method) iterative solver with a Multigrid preconditioner, this results in very long computational times. In the present case the issues cannot be resolved by increasing the number of elements in the swept mesh applied in the gap between coil and workpiece. Furthermore, the problem appears to be insensitive to the coil-workpiece gap size, at least in the investigated range (from 0.5 to 20 mm). Conversely, when the TBC feature is disabled, i.e., unloaded inductor, the model consistently converges even with direct solvers. This observation suggests that the numerical difficulties originate from the TBC. Therefore, to identify a suitable alternative to TBC, it is useful to observe that under the assumptions of (i) electrically “very thin” workpiece, i.e., $th_{wkp} \ll \delta$, and (ii) conduction-dominated current, i.e., workpiece electrical conductivity $\sigma_{wkp} \gg 2\pi f \epsilon_0$, both of which hold for the considered application,

the TBC reduces to a local surface current density of the form $\mathbf{J}_s = th_{wkp} \sigma_{wkp} \mathbf{E}_t$ [A/m], where $\mathbf{E}_t$ is the tangential electric field on the sheet surface [8]. Indeed, whilst the general TBC formulation leads to hyperbolic field distributions across the conductive layer, in the thin-sheet limit considered here, the fields are practically uniform across the thickness, such that $\mathbf{J}_s$ can be obtained simply by multiplying the current density $\sigma_{wkp} \mathbf{E}_t$ [A/m²] by th_{wkp} . Therefore, in the considered application, the TBC can be replaced by the available “Surface Current Density” (SCD) node¹. With this approach, the resulting system can be solved using either direct or iterative solvers. Remarkably, when iterative methods are employed, the computational time is reduced by approximately a factor of five compared to the model with TBC. Unlike the *mef* case, no convergence issues associated with TBC were observed in the investigated *mf* models. The different numerical behavior of TBC in *mf* and *mef* remains unclear. However, it is well documented that boundary conditions introducing additional coupling beyond standard continuity conditions, such as TBC, may adversely affect the conditioning and convergence properties of the resulting system [9]. In contrast, surface-based representations such as SCD often exhibit improved numerical robustness and computational efficiency [10]. To conclude, the rationale leading to the proposed *mef*-SCD formulation is summarized in Figure 5.

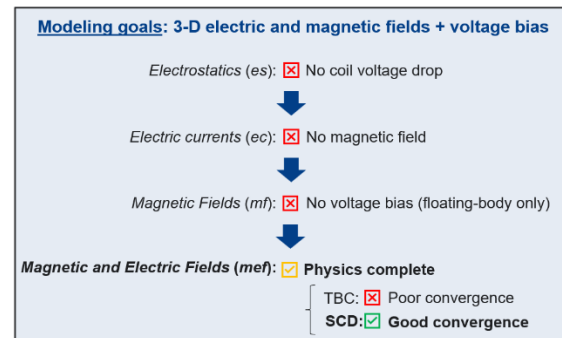


Figure 5. Rationale leading to the selection of the *mef* interface based on SCD.

Results on Dummy Inductor

In this paragraph, a few outcomes of the simulations performed on the dummy inductor using the described *mef* interface are presented. Figure 6 shows the electric scalar potential on the coil (gauge fixing applied), the electric field in the MD, and the surface current density in the coil, body, and workpiece for a frequency-domain study

¹ available as “Surface Current Density” in COMSOL Multiphysics® version 6.4, or as “Surface Current” in version 6.3.

where $V_+ = 1$ (reference normalized value), $V_- = V_b = 0$, $f > 1$ MHz and $th_{wkp} \ll 10 \mu\text{m}$. The proposed *mef*-SCD formulation captures the main electromagnetic features of the device. Indeed, a continuous voltage drop develops along the coil, giving rise to a nonuniform electric field distribution. In particular, the electric field is stronger near the coil terminals rather than in the return bend region, with the highest values occurring between V_+ and the body. The model also correctly reproduces the proximity effect in the current density distribution, whereby electromagnetic interactions cause opposite currents to concentrate on facing surfaces (coil-coil, coil-workpiece, coil-body, and body-workpiece), thereby minimizing their separation distance.

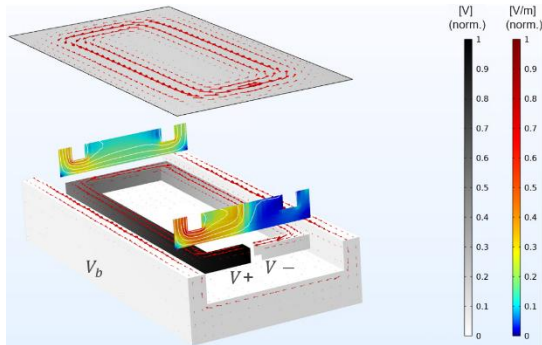


Figure 6. Output of the “Magnetic and Electric Fields” (*mef*) model based on the Surface Current Density (SCD) boundary condition: electric scalar potential on the coil (gauge fixing applied), electric field in the MD, and surface current density in the coil, body, and workpiece. Bias scheme: $V_+ = 1$ (reference normalized value), $V_- = V_b = 0$. Legends: electric scalar potential [V] and electric field magnitude [V/m], both normalized; contour lines of the electric scalar potential are also shown. Red arrows: surface current density, with lengths set proportional to their magnitude (using different scale factors for the various model entities). For visualization purposes, both the electric field and the contour lines in the MD, and the surface current density on the workpiece, are vertically offset.

Electrostatic Cross-Sectional Analysis

Inspection of the 3-D *mef* results shows that the electric field within the magnetodielectric is predominantly transverse. Indeed, the high conductivity of both the coil and the workpiece implies that the longitudinal field components required to sustain conduction currents remain relatively small, suggesting a quasi-TEM behavior of the device. To investigate further, 2-D simulations were performed using both the *es* and *mef* interfaces under an equivalent bias scheme. Notably, the resulting electric field distributions were found to be virtually identical, indicating that the electric field predicted by a full-Maxwell

formulation on a transverse cross section coincides with the solution of the corresponding electrostatic problem. The observed agreement is consistent with a quasi-TEM description of the structure, for which the transverse electric field is fully determined by the electrostatic potential [11]. This comparison therefore suggests that the inductive term $-j2\pi f \mathbf{A}$ is primarily associated with the longitudinal voltage drop along the inductor. Once suitable electric potential boundary conditions are known, 2-D electrostatic analyses may therefore be employed for rapid comparison of alternative device geometries, dielectric materials, or insulation designs. Nevertheless, the *mef* formulation remains necessary whenever the complete electromagnetic behavior of the device, including magnetic fields and frequency-dependent magnetic and dielectric losses, must be evaluated.

Application to an Industrial Inductor Model

Finally, the proposed *mef*-SCD formulation is applied to a model representing an induction sealing inductor designed for industrial filling machines. In this case, in order to ensure a more uniform eddy current distribution and, consequently, a more even temperature field in the sealing region, the presence of folds and overlaps in the workpiece requires a more complex coil geometry, as well as the use of multiple MD parts. This leads to a much finer mesh, resulting in a significant increase in the number of degrees of freedom. The model is solved using COMSOL Multiphysics® version 6.4 on a machine equipped with a dual-socket CPU system based on AMD EPYC 9354 processors (64 cores total). Since the mesh required to accurately represent the highly detailed geometry is already sufficiently fine, first-order shape functions can be used without loss of accuracy. Together with an optimized FGMRES solver configuration based on Direct Preconditioning and Auxiliary-Space Algebraic Multigrid (AMG), the use of linear shape functions allows the problem to be solved, in about 5 minutes and 75 GB of RAM. The same solver configuration remains effective after introducing the μ_r' dependence on B and material loss mechanisms in the MD, resulting in comparable computation times. Specifically, a $\mu_r'' > 0$ was introduced using the “Magnetic Losses” feature within the “Ampère’s Law in Solids” node. Concurrently, both conductive and dielectric losses were incorporated into an effective imaginary permittivity ϵ_r'' through the “Dielectric Losses” option. The approach is consistent with frequency-domain material characterization, which typically provides an effective complex permittivity where conductive and dielectric loss contributions are not generally distinguishable. This is

particularly true for composite MDs, where interfacial polarization and percolative transport mechanisms make the concept of a unique DC conductivity of limited physical significance [12]. Taken together, these results indicate that the relatively low computational cost of the proposed model makes it well suited for future multiphysics extensions, such as coupling with heat transfer.

Conclusions and Perspectives

A modeling approach for the description of the electromagnetic behavior of an induction sealing device loaded with a thin aluminum workpiece has been presented based on the AC/DC Module's *Magnetic and Electric Fields (mef)* interface and a Surface Current Density (SCD) boundary condition. The *mef* A-V formulation enables the direct representation of the inductor voltage bias scheme and, consequently, of high-voltage breakdown test configurations within a full-Maxwell framework, where magnetic and electric fields, together with associated loss mechanisms, are computed self-consistently. In the considered induction sealing application, the thin aluminum workpiece can be accurately represented by the SCD formulation, which corresponds to the thin-sheet limit of the more general TBC. Importantly, SCD overcomes the convergence difficulties associated with TBC in *mef* and provides robust and efficient numerical convergence. Future modeling work will focus on coupling heat transfer to evaluate the role of magnetic and dielectric losses in heating and insulation-related degradation phenomena in the magnetodielectric materials. In parallel, experimental work will focus on identifying and validating the dependence of magnetodielectric properties on temperature and electromagnetic field, as well as the corresponding material models.

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