COMSOL CONFERENCE ROTTERDAM 2013

Computing the Influence Functions of an Adaptive Optics Large Deformable Mirror: the Numerical Method and the Experimental Data

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Outline

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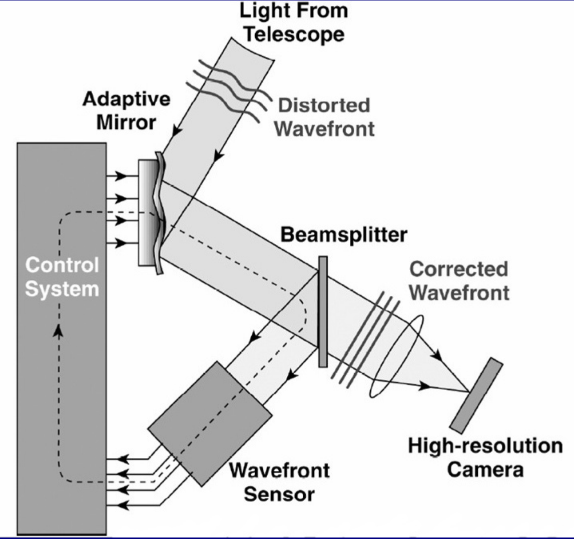

Light From Telescope
Adaptive Mirror
Distorted Wavefront
Beamsplitter
Corrected Wavefront
Control System
High-resolution Camera
Wavefront Sensor

Outline

- A *feed-forward*, open-loop correction dramatically increases the closed-loop response of the servo system
- This correction is based on the DM stiffness matrix . . .
- . . . operatively defined by arbitrarily displacing one actuator, while all the others are constrained at 0, and calculating the reaction forces
- The *influence function* (IF) is the shape of the DM when poking a single actuator

The Control System

Improving the Closed-Loop Response

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AdOpt IF

Del Vecchio et al.

Rationale
The AO Principle
Motivation
Static assumptions and simplifications
The model
Local CS's
Results
LBT
VLT
Experimental validation
Set-up and procedure
Comparison
Summary

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The Case Studies

2 Zerodur DM's

R_o/R_i	physical outer/inner radii
K_f/K_b	front/back surface conic constants
R_f/R_b	front/back surface optical radii
t_m	mean thickness
N	total number of actuators

	LBT	VLT
R_o	455.5 mm	558 mm
R_i	28 mm	48 mm
t_m	1.6 mm	2.0 mm
R_b	1994.9 mm	4575.30 mm
K_b	0	0
R_f	1974.24 mm	4575.3 mm
K_f	−0.7330	−1.66926
N	672	1170

Concave Large Binocular Telescope DM [Riccardi et al., 2010]
Convex Very Large Telescope DM [Biasi et al., 2012]

Outline

Approximation I: Functioning the Geometry

From the Optical Parameters to the Full Axi-symmetric Shell

- Matlab generation of $z(r)$, $r = \sqrt{x^2 + y^2}$

  front surface $z_f = z_f(r, K_f, R_f)$
  back surface $z_b = z_b(r, K_b, R_b)$

- fitting with polynomials of degree $M = 9$:

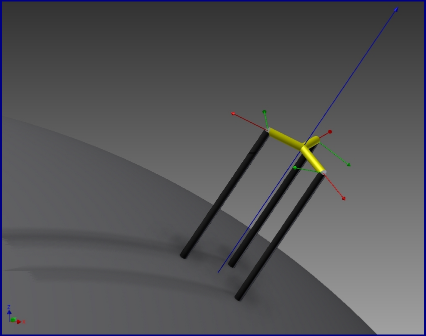
$$\text{mean surface} \quad z = \frac{1}{2}(z_f + z_b) = \sum_{i=1}^{M+1} V(i) r^{M+1-i}$$

$$\text{normal} \quad \varphi = -\arctan\left(\frac{dz}{dr}\right) = \sum_{i=1}^{M+1} P(i) r^{M+1-i}$$

$$\text{thickness} \quad t = |z_f - z_b| = \sum_{i=1}^{M+1} Q(i) r^{M+1-i}$$

Approximation II: Replacing the Magnet

From the 3D Puck to a 6-Elements Load Spreader



K_p (the puck axial stiffness under a body load) is computed via a full 3d model

- 3 trusses, $K = EA/l = K_p/3 \hookrightarrow$ 3 glue contacts, $r = 50\ \mu\mathrm{m}$
- trusses $\Leftarrow$ 3 beams, $K = \infty \Rightarrow$ actual push/pull node
- 3 local r and 3 local θ constraints

Approximation III: Replacing the Trusses

What the FEM Looks Like

One more approximation

fake beams ($I_{yy} = I_{zz} = J \approx 0$) instead of trusses

(EVEN IF A COMSOL WORK-AROUND IS AVAILABLE)

	LBT	VLT
shell elements	19144	32824
beam elements	4032	7020
dof's	253×10^3	434×10^6

Outline

The *global-to-local* and *local-to-global* transformation matrices **G2L** and **L2G** = **G2L**$^{-1}$

$$\mathbf{G2L} = \begin{bmatrix} \cos(\varphi)\cos(\theta) & \cos(\varphi)\sin(\theta) & \sin(\varphi) \\ -\sin(\theta) & \cos(\theta) & 0 \\ -\cos(\theta)\sin(\varphi) & -\sin(\varphi)\sin(\theta) & \cos(\varphi) \end{bmatrix}$$

$$\mathbf{L2G} = \begin{bmatrix} \cos(\varphi)\cos(\theta) & -\sin(\theta) & -\cos(\theta)\sin(\varphi) \\ \cos(\varphi)\sin(\theta) & \cos(\theta) & -\sin(\varphi)\sin(\theta) \\ \sin(\varphi) & 0 & \cos(\varphi) \end{bmatrix}$$

BUT ... *Comsol hangs up*

Avoid 672 or 1170 auxiliary coordinate systems

Define constraints and forces analytically (*functioning*)

Respecting the DM Convexity or Concavity

Too Many Coordinate Systems Cause Comsol to Hang

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Functioning Restraint and Force Equations I

The Coordinate Definitions

P_I	$N \times 3$	interface nodes (truss/beam intersection)	$X_{I_{i,j}}, Y_{I_{i,j}}$
A_I	$N \times 3$	interface angles (truss/beam intersection)	ψ_j
P_F	N	actuation nodes (beam intersections)	X_{F_i}, Y_{F_i}

$$i = 1, 2, \ldots, 3N$$
$$j = 1, 2, 3$$

$\psi = 0, \pm(2/3)\pi$ for $j = 1, 2, 3$ are defined wrt the actuation axis

Functioning Restraint and Force Equations II

The Interpolation Function

Matlab generation of a $4N \times 5$ matrix whose rows from i to $i+4$ are defined as

$$\begin{bmatrix} X_{I_{i,1}} & Y_{I_{i,1}} & X_{F_i} & Y_{F_i} & \psi_1 \\ X_{I_{i,2}} & Y_{I_{i,2}} & X_{F_i} & Y_{F_i} & \psi_2 \\ X_{I_{i,3}} & Y_{I_{i,3}} & X_{F_i} & Y_{F_i} & \psi_3 \\ X_{F_i} & Y_{F_i} & X_{F_i} & Y_{F_i} & 0 \end{bmatrix}$$

This matrix is used as table data source of the Comsol *nearest neighbor* interpolation function $\Gamma(x, y)$.

$$\Gamma(\zeta_{i,j}, \eta_{i,j}), \text{ where } (\zeta_{i,j} = X_{F_i}, \eta_{i,j} = Y_{F_i})$$

- associates to the P_I coordinates the coordinates of the correspondent P_F
- defines for each P_I the three angles $\Theta_{i,j} = \psi_j$.

Functioning Restraint and Force Equations III

The Final Implementation

- Defining
  - $\rho = \sqrt{\zeta^2 + \eta^2}$
  - $\theta' = \arctan(\eta/\zeta)$
  - $\varphi' = \sum_{i=1}^{M+1} P'(i)\rho^{M+1-i}$
- Recalling
  - $\mathbf{u} = [u; v; w]$ *displacement vector in the global cs*
- Substituting in **G2L**
  - θ with θ'
  - φ with φ'

local displacement in the cs relative to each actuation axis

$$\mathbf{u}_l = [u_l; v_l; w_l] = \mathbf{Gu}$$

Functioning Restraint and Force Equations IV

The Pointwise Constraints

- *N pointwise constraints* in (1) apply the strokes to P_F
- *$3N$ pointwise constraints* (2) and (3) radially and tangentially (in the cylindrical local cs of each actuator) apply the constraints to P_I

$$w_{I_k} = \begin{cases} 0 & \text{if } k \neq i \\ w^* & \text{if } k = i \end{cases} \tag{1}$$

$$u_{I_k} \cos(\psi_j) + v_{I_k} \sin(\psi_j) = 0 \qquad j = 1, 2, 3 \tag{2}$$

$$-u_{I_k} \sin(\psi_j) + v_{I_k} \cos(\psi_j) = 0 \qquad j = 1, 2, 3 \tag{3}$$

w^* displacement of the kth ($k = 1, 2, \ldots, N$) actuator along its axis

Adding a small set of analytic functions allows the computation of the N IF's avoiding any additional auxiliary coordinate system

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The Matlab Solving Function

Running the Loop (without Involving the Comsol GUI)

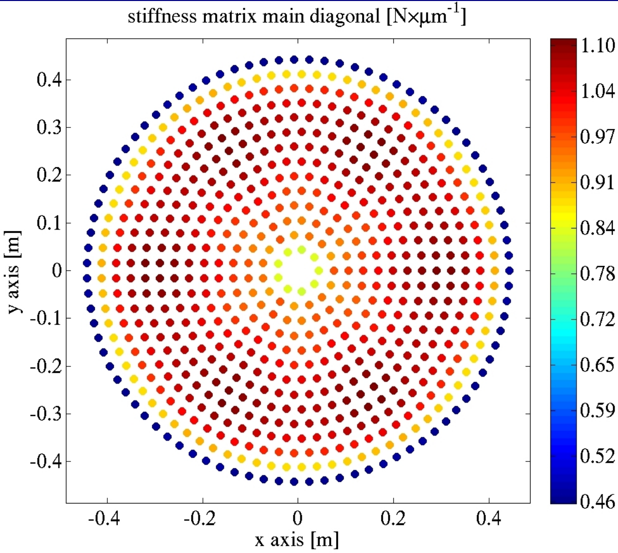
The Matlab function

- loads the model file
- runs the IF, taking the actuator number(s) as input vector
- computes the forces and the displacements along the DM r, φ, and θ

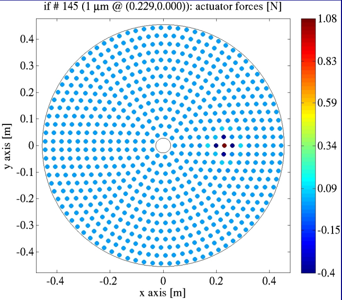
Outline

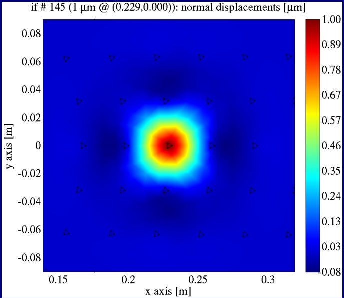
The LBT Results I

The Main Diagonal of the 672 by 672 LBT DM Stiffness Matrix vs. the Actuator Geometry



The LBT Results II
The IF # 145

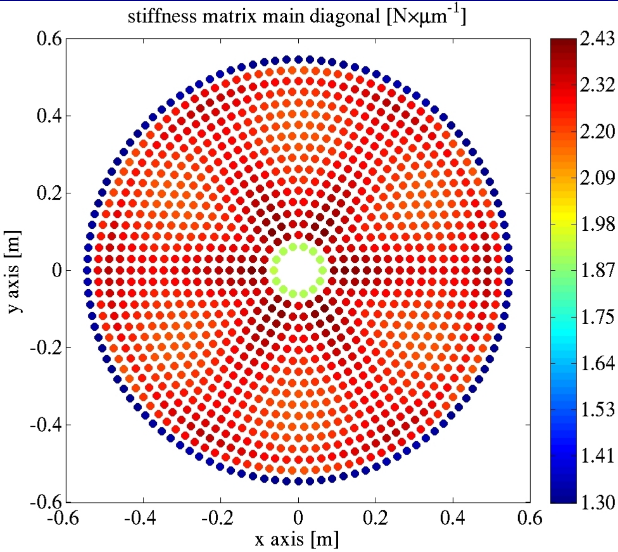




Outline

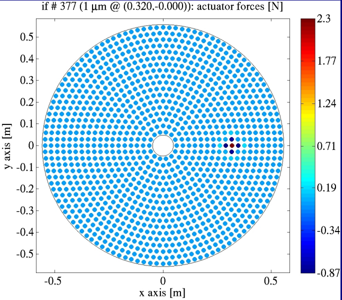
The VLT Results I

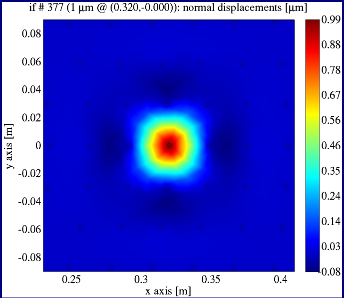
The Main Diagonal of the 1170 by 672 1170 DM Stiffness Matrix vs. the Actuator Geometry



The VLT Results II

The IF # 377

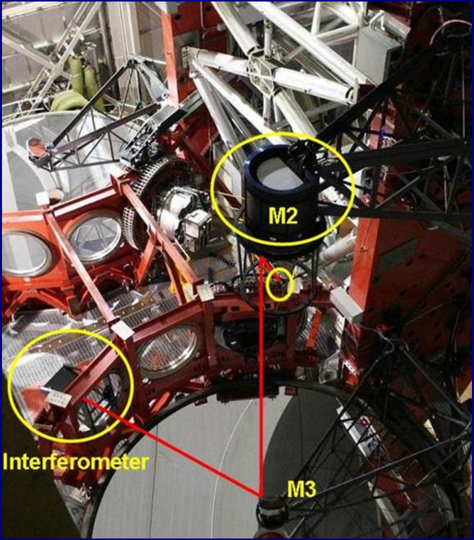




Outline

The optical train

- Interferometer
- Telescope M3
- DM (M2)
- Retro-reflecting mirror
- DM (M2)
- Telescope M3
- Interferometer



The Sampling Procedure
Getting the Deformation Map

Synchronization: Mirror command & imaging

Actuator poked

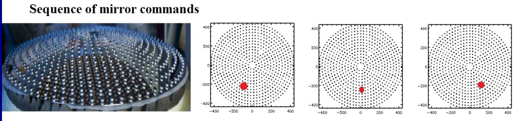
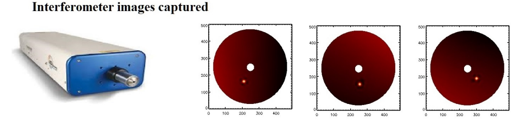


Image captured

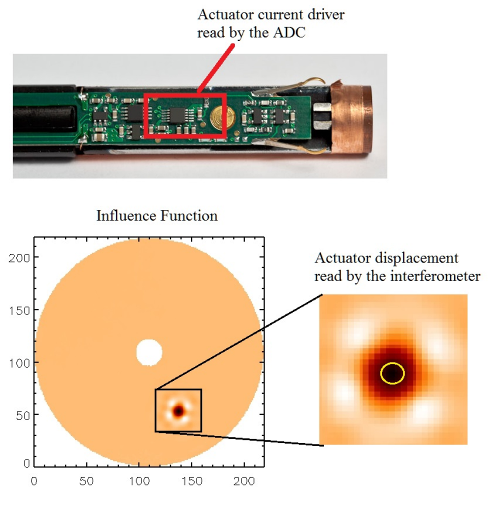


Frame rate = 25 Hz to reduce noise

Push-Pull to increase SNR

Analysis Procedure

Getting the Stiffness

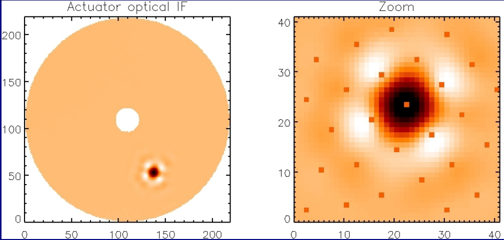


$K = F/d$:

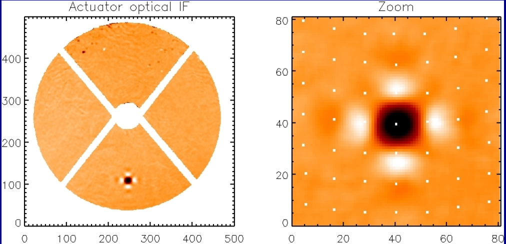
- Force read by the actuator current driver
- Displacement measured with the interferometer

Interferometric Example

Typical Deformation Maps



LBT

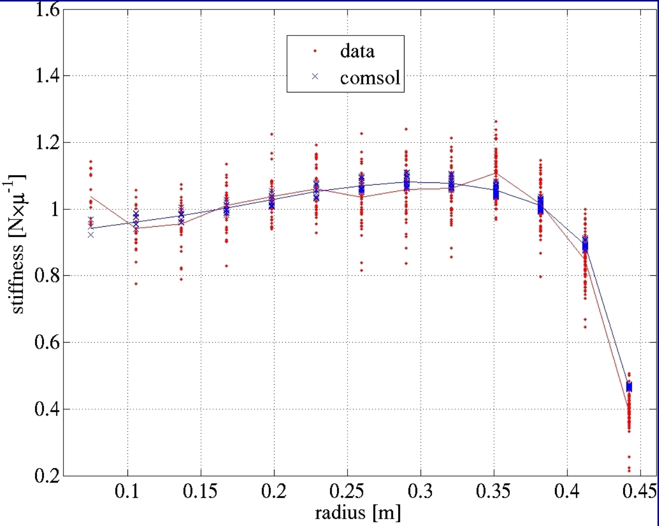


VLT

Outline

Matching the Comsol Results

The Concordance and the Limitations



LBT

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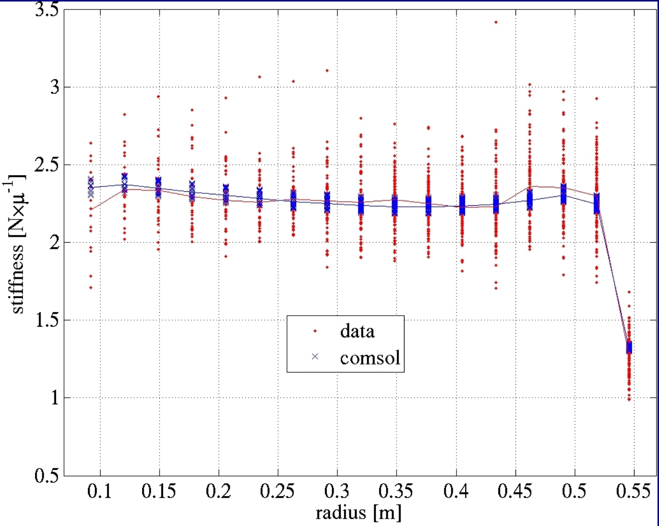

VLT

Matching the Comsol Results

The Concordance and the Limitations

- inter-actuator force calibration
- poor image resolution
- uncertainty of the imaged actuator locations
- poor IF visibility on the edges
- effects of malfunctioning actuators

Lessons Learned & Future Work

Exploiting the unrestrainedness

Although originated by a flaw of the code, functioning of a definition reveals the powerful of the flexibility of Comsol

The availability of the Matlab tools allows

- (general) geometry generation
- (compact) definition of pointwise constraints via $\Gamma(x, y)$
- (*for* loop) solving N cases

Lessons Learned & Future Work

The accuracy of the results is demonstrated by the experimental data

The powerful of functioning

The influence functions of an Adaptive Optics Deformable Mirror can be truthfully evaluated by numerical methods.

A powerful and reliable computational tool is available for the opto-mechanical design.

For Further Reading I

Biasi, R., Andrighettoni, M., Angerer, G., Mair, C., Pescoller, D., Lazzarini, P., Anaclerio, E., Mantegazza, M., Gallieni, D., Vernet, E., Arsenault, R., Madec, P.-Y., Duhoux, P., Riccardi, A., Xompero, M., Briguglio, R., Manetti, M., and Morandini, M. (2012).
VLT deformable secondary mirror: integration and electromechanical tests results.
In Ellerbroek, B. L., Marchetti, E., and Véran, J.-P., editors, *Adaptive Optics Systems III*, volume 8447 of *Proc. SPIE*. SPIE.

For Further Reading II

Riccardi, A., Xompero, M., Briguglio, R., Quirós-Pacheco, F., Busoni, L., Fini, L., Puglisi, A., Esposito, S., Arcidiacono, C., Pinna, E., Ranfagni, P., Salinari, P., Brusa, G., Demers, R., Biasi, R., and Gallieni, D. (2010).
The adaptive secondary mirror for the large binocular telescope: optical acceptance test and preliminary on-sky commissioning results.
In Ellerbroek, B. L., Hart, M., Hubin, N., and Wizinowich, P. L., editors, *Adaptive Optics Systems*, volume 7736 of *Proc. SPIE*. SPIE.