Frequency Analysis of Si-Wafers with Variable Size and Boundary Conditions

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Abstract:

Silicon wafers represent key elements in modern microelectronics or photovoltaics. Technological fabrications of wafers with large diameters (e.g. 300…450 mm) allow an efficient realization for integrated circuits at low cost. However, this material shows a high sensitivity to vibrations that strongly depend on the positioning as well as orientation of a wafer in a mounting, realized e.g. by single fixed points (typically four [1]) located at the rim of the device.

Numerical calculations of the modes in dependence of variable boundary conditions are thus of high importance for the optimization of handling of wafers during mounting and storage.

Keywords: wafer, frequency

1. Introduction

The influence of the mounting conditions of Silicon wafers on mechanical oscillations is relevant for the material deformation and mechanical load.

Numerical simulations of oscillating behavior and mechanical deformation allows, for given configurations, the analysis of possible eigenfrequencies as well as an estimation of the sensitivity to parameter changes leading to a prediction of device behavior and a determination of critical parameter regimes.

We performed simulations of Silicon wafers of circular symmetry with diameters in the regime 150 mm to 450 mm. For these geometries we systematically varied the number and positioning of fixing point at the rim as realized in typical experimental situations [1]. It could be shown, that the vibration frequencies shift with changes in the mounting. Furthermore the mode shapes can be controlled by suitable positioning and orientation of the wafer in the mounting in agreement to typical experimental observations. In particular, a deliberately chosen mounting configuration may stabilize the system and allow the suppressing of oscillations.

This paper is organized as follows: In section 2, the continuum mechanical description is summarized, section 3 explains the usage of COMSOL, sections 4 and 5 show simulation results for variations in boundaries and material symmetry, section 6 concludes the article.

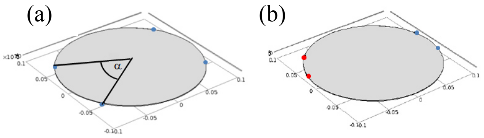

Figure 1. Wafer geometry: (a) symmetric and (b) asymmetric boundaries.

2. Governing equations

Generally, deformation of a continuum can be described by the displacement vector $\mathbf{u}(\mathbf{X},t) = \mathbf{x}(\mathbf{X},t) - \mathbf{X}$ defined as difference between spatial variables $\mathbf{x}$ and material coordinates $\mathbf{X}$, i.e. pointing from the reference position to the current position. In material coordinates (Langrangian formulation) [2,3] we can furthermore define the deformation gradient as $\mathbf{F} = \partial \mathbf{x} / \partial \mathbf{X}$ or, in components, $F_{ij} = \partial x_i / \partial X_j$ The total strain tensor can be expressed in terms of the displacement gradient as [2,3] $\boldsymbol{\varepsilon} = \frac{1}{2}\left(\nabla \mathbf{u} + \nabla \mathbf{u}^T\right)$ with

$$\varepsilon = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}$$

and

$$\nabla \mathbf{u} = \begin{pmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_z}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{pmatrix}$$

Starting from Hooke's law which assumes a linear response of a deformable continuum to tension one can write the linear relation between tension $\mathbf{t}$ and total strain tensor $\boldsymbol{\varepsilon}$ as $\mathbf{t} = \mathbf{C}\boldsymbol{\varepsilon}$ or, in components, as $t_{ij} = C_{ijkl}\varepsilon_{kl}$ with the elasticity tensor $\mathbf{C}$ [2,3] which can, due to the symmetry be represented by a 6×6 matrix as

$$\begin{pmatrix} C_{1111} & C_{1122} & C_{1133} & C_{1112} & C_{1123} & C_{1113} \\ C_{1122} & C_{2222} & C_{2233} & C_{2212} & C_{2223} & C_{2213} \\ C_{1133} & C_{2233} & C_{3333} & C_{3312} & C_{3323} & C_{3313} \\ C_{1112} & C_{2212} & C_{3312} & C_{1212} & C_{1223} & C_{1213} \\ C_{1123} & C_{2223} & C_{3323} & C_{1223} & C_{2323} & C_{2313} \\ C_{1113} & C_{2213} & C_{3313} & C_{1213} & C_{2313} & C_{1313} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{43} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{pmatrix}$$

where we have used the Voigt notation [2,3] for the elastic stiffnesses of the material (i.e. $C_{11} = C_{1111}$, $C_{12} = C_{1122} = C_{2211}$ etc.).

For Silicon, which is an orthotropic material, we have

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{bmatrix}$$

with $C_{11} = 165,7 \cdot 10^9$ Pa, $C_{12} = 63,9 \cdot 10^9$ Pa, and $C_{44} = 79,6 \cdot 10^9$ Pa

3. Use of COMSOL Multiphysics

We use COMSOL Multiphysics to set up the wafer geometry and include our orthotropic material system. Starting for the structural mechanics module we chose the solid state mechanics and select silicon as basis material. We use a three-dimensional model with the diameter (d_1=150 mm, d_2=300 mm and d_3=450 mm) and the height (h_1=0,675 mm, h_2= 0,775 mm h_3= 0,925mm) of the geometrical cylinder as global parameters. For simulations with 4 fixed boundary points (4-point-mounting) we additionally introduce an angle α between the fixed boundaries on both sides (see Fig. 1).

For a crystal cut perpendicular or parallel to the crystal axis we can directly start with the elastic parameters provided (corresponding to i.e. a Silicon wafer in [100] direction, where the brackets refer to the Miller indices [4] of a particular plane). For an arbitrary cut direction (simulating e.g. a [111] wafer) we additionally rotate the local material coordinate system with respect to the wafer geometry by introducing a local coordinate system. This allows us to investigate the influence of anisotropy on the eigenmodes of the system.

4. Frequency Analysis

We investigate the eigenfrequencies in dependence on number of fixed boundaries and angle α between the fixed boundaries for the situation of a 4-point-mounting. Fig.2 summarizes for the five lowest modes the dependence of the maximum displacement and the frequencies on the number of fixed boundary points (equidistantly positioned at the rim of the device, wafer diameter: 300 mm) as realized in a typical mounting [1]. The values converge to the analytical solution of a completely clamped wafer, i.e. Dirichlet boundary condition. For four-point-mountings adding a variable fifth point on request may reduce unwanted oscillations if a system frequency is close to one of the wafer eigenfrequencies.

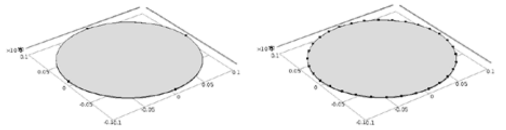

Figure 2. Wafer mounting with variable number of fixed boundary points.

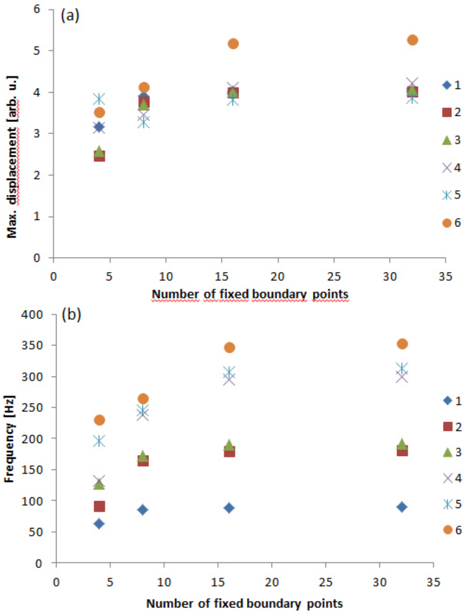

Figure 3: Displacement (a) and frequencies (b) shown for the six lowest eigenmodes.

In a next step, we refer to the four-point mounting and symmetrically vary the angle between the two points on the left and on the right side of the wafer (see Fig. 1 (a)). The results shown in Fig. 4 for a 150 mm wafer (a) and a 450 mm wafer (b) visualize the dependence of the frequency on angle α

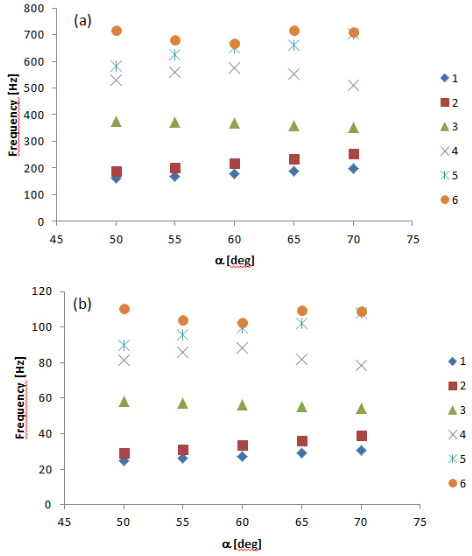

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Figure 4: (a) frequency in dependence on angle α for a wafer diameter of 150 mm (a) and 450 mm (b).

Generally, the lower modes are only slightly distorted (for a visualization see Fig. 5) whereas modes at higher frequencies may significantly change their form when the boundary conditions are changed.

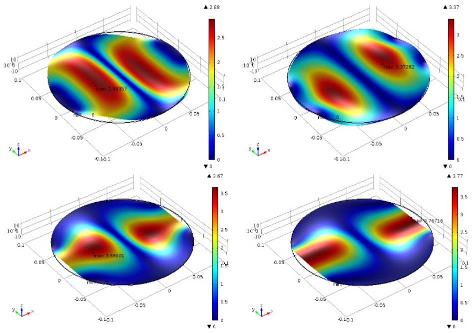


Figure 5: snapshots of a mode for α = 30° (left, top), 40°(right, top), 50°(left, bottom) and 60° (right, bottom) degrees.

As an alternative one can chose a configuration with a constant angle of 30° and move only one pair of boundary points (i.e. left side, see Fig. 1(b)). This configuration, however, leads for large wafer diameters to strong deformations caused by the asymmetry (Fig. 6).

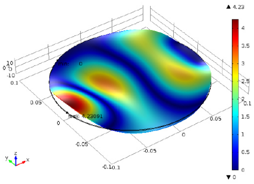
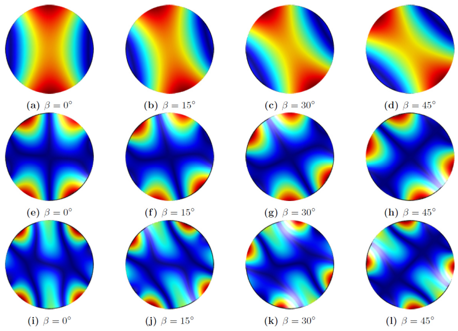


Figure 6: distribution of the displacement for an asymmetric mounting**.**

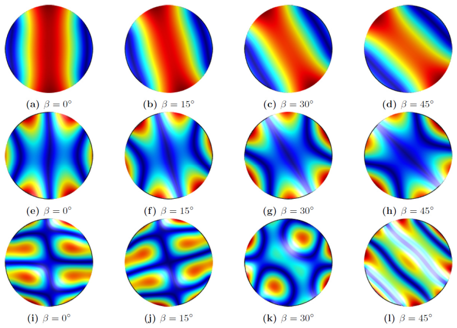
5. Influence of Anisotropy

For a demonstration of the influence of the anisotropy of the material we simulate a Silicon wafer oriented in [111] direction by rotating our local material system with respect to the global system leading to corresponding changes in the elasticity tensor. As a consequence the wafer reacts rather critically to rotations within a 4-point mounting. As an example, Figs. 7 and 8 visualize the dependence of the modes on the rotation angle of the wafer within the mounting plane ($\beta = 0 \ldots 45^\circ$) for a four-point mounting with $\alpha = 50^\circ$ (Fig. 7) and $\alpha = 70^\circ$ (Fig. 8), respectively. The figures clearly show that a larger value for α leads to a higher sensitivity to rotation in the mounting plane. A configuration with smaller α may thus stabilize the system.



(a) $\beta = 0^\circ$ (b) $\beta = 15^\circ$ (c) $\beta = 30^\circ$ (d) $\beta = 45^\circ$
(e) $\beta = 0^\circ$ (f) $\beta = 15^\circ$ (g) $\beta = 30^\circ$ (h) $\beta = 45^\circ$
(i) $\beta = 0^\circ$ (j) $\beta = 15^\circ$ (k) $\beta = 30^\circ$ (l) $\beta = 45^\circ$

Figure 7: 1st, 5th and 10th eigenmode for different orientations (rotation angle β) of the wafer plane within the mounting and $\alpha = 50^\circ$.



(a) $\beta = 0^\circ$ (b) $\beta = 15^\circ$ (c) $\beta = 30^\circ$ (d) $\beta = 45^\circ$
(e) $\beta = 0^\circ$ (f) $\beta = 15^\circ$ (g) $\beta = 30^\circ$ (h) $\beta = 45^\circ$
(i) $\beta = 0^\circ$ (j) $\beta = 15^\circ$ (k) $\beta = 30^\circ$ (l) $\beta = 45^\circ$

Figure 8: 1st, 5th and 10th eigenmode for different orientations (rotation angle β) of the wafer plane within the mounting and $\alpha = 70^\circ$.

6. Conclusions

We investigated the dependence of a circular silicon wafer on size and boundary conditions as realized in typical wafer mountings. Simulations using COMSOL MULTIPHYSICS® reveal that the eigenfrequencies of wafers with a finite number of fixed positions at the rim (Dirchlet boundaries) may strongly depend on both, number and position of fixed boundary points at the rim of the device. In particular, the frequencies as well as the amount of deformation can be adjusted by suitable positioning of the fixing points of a mounting.

Furthermore, the anisotropy of the Silicon wafer material may cause a complex dependence on the wafer's orientation within a mounting.

The control or even suppression of oscillations in Silicon wafers is of large importance for the optimization of the mounting and handling of wafers during processing, transport and storage. The results of this work are thus relevant for future developments.

7. References

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