Multiphysics Modelling of Sound Absorbing Fibrous and Porous Materials

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COMSOL

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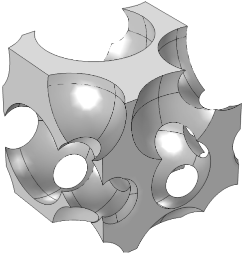
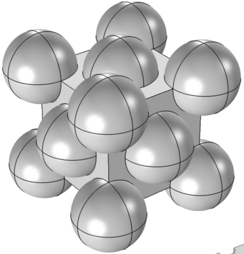
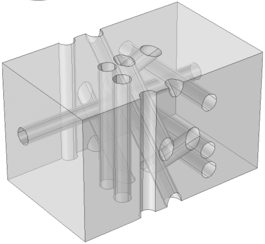
open-cell foams





granular media

fibrous media



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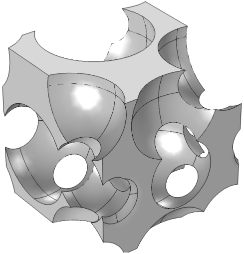

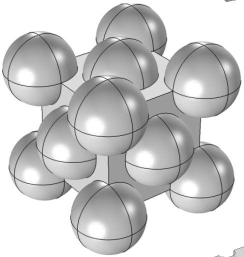

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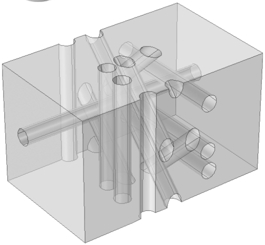


Acoustics of porous media with rigid frame

Fluid-equivalent approach

An effective fluid is substituted for a porous medium. It is dispersive and substantially different from the fluid in pores.

Requirements: **(1)** open-cell porosity, **(2)** rigid (motionless) skeleton, **(3)** wavelengths significantly bigger than the characteristic size of pores.

- The **Helmholtz equation** of linear acoustics (in the equivalent fluid):

$$\omega^2 \hat{p} + \hat{c}^2 \Delta \hat{p} = 0, \qquad \hat{c}^2 = \frac{\hat{K}}{\hat{\varrho}}$$

  $\hat{p}$ – the amplitude of acoustic pressure, $\omega = 2\pi f$ – the angular frequency,
  $\hat{c}$, $\hat{\varrho}$, $\hat{K}$ – the effective speed of sound, density and bulk modulus

- The **effective density and bulk modulus** for a porous medium:

$$\hat{\varrho}(\omega) = \frac{\varrho_0 \, \alpha(\omega)}{\phi}, \qquad \hat{K}(\omega) = \frac{K_0}{\phi \beta(\omega)} \quad \text{where} \quad \beta(\omega) = \gamma - \frac{\gamma - 1}{\alpha'(\omega)}$$

  Here: ϱ_0 is the density and $K_0 = \gamma p_0$ is the bulk modulus of fluid in pores,
  γ – the heat capacity ratio of fluid in pores, p_0 – the ambient mean pressure,
  $\alpha(\omega)$, $\alpha'(\omega)$ – the **dynamic (visco-inertial) tortuosity** and '**thermal tortuosity**',
  ϕ – the porosity; $\phi\hat{\varrho}(\omega)$ and $\phi\hat{K}(\omega)$ are the **dynamic density and bulk modulus**

Model parameters

Johnson-Champoux-Allard-Lafarge (JCAL) model

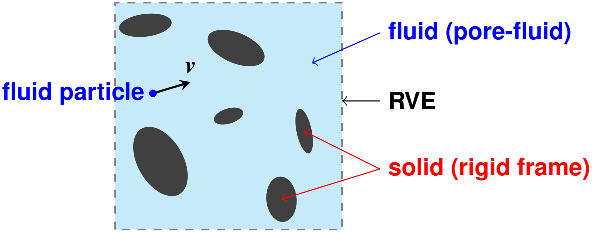
$$\alpha(\omega) = \alpha_\infty + \frac{\nu}{\mathrm{i}\omega}\frac{\phi}{k_0}\sqrt{\frac{\mathrm{i}\omega}{\nu}\left(\frac{2\alpha_\infty k_0}{\Lambda\phi}\right)^2 + 1}, \quad \alpha'(\omega) = 1 + \frac{\nu'}{\mathrm{i}\omega}\frac{\phi}{k_0'}\sqrt{\frac{\mathrm{i}\omega}{\nu'}\left(\frac{2k_0'}{\Lambda'\phi}\right)^2 + 1}$$

ϕ, α_∞, k_0, k_0', Λ, Λ' – the purely geometric parameters of skeleton
$\nu = \mu/\varrho_0$ – the kinematic viscosity of pore-fluid (μ – the dynamic viscosity)
$\nu' = \nu/\mathrm{Pr}$ (Pr – the Prandtl number of pore-fluid)

- **Parameters of the fluid in pores** (the density ϱ_0, heat capacity ratio γ, viscosity μ, and Prandtl number Pr) and the **ambient mean pressure** p_0
- **Transport parameters of the skeleton** of porous medium:

Symbol	*Unit*	*Parameter*
ϕ	[–]	the open porosity
α_∞	[–]	the tortuosity
k_0	[m^2]	the (static) viscous permeability
k_0'	[m^2]	the (static) “thermal permeability”
Λ	[m]	the viscous characteristic length
Λ'	[m]	the thermal characteristic length
α_0, α_0'	[–]	the low-frequency limits for $\alpha(\omega)$ and $\alpha'(\omega)$ (**P**ride *et al.* enhancements; **JCAPL model**)

Small-velocity flow in a porous medium



Small fluctuations of visco-thermal flow

The **velocity field** $\boldsymbol{v}$ describes ***small*** **fluctuations of fluid particles** around their initial (motionless) equilibrium state.
Fluid density, **pressure** and **temperature** are decomposed as follows:

$$\varrho = \varrho_0 + \tilde{\varrho}, \qquad p = p_0 + \tilde{p}, \qquad T = T_0 + \tilde{T}$$

$\tilde{\varrho}$, $\tilde{p}$, $\tilde{T}$ – **small fluctuations** of **density**, **pressure**, and **temperature**, respectively, around their constant, **equilibrium values**: ϱ_0, p_0, and T_0.

Microstructure-based calculations (hybrid appraoch)

Micro-scale level: Solve **3 steady-state BVPs** on the micro-scale:

1. **Stokes flow** (steady, incompressible viscous flow) – then calculate:
   - static viscous permeability, k_0
   - viscous tortuosity at 0 Hz, α_0
2. **Steady heat transfer** – then calculate:
   - static thermal permeability, k'_0
   - thermal tortuosity at 0 Hz, α'_0
3. **Laplace problem** – then calculate:
   - (viscous) tortuosity (at ∞ Hz), α_∞
   - viscous characteristic length, Λ

The **thermal characteristic length**, Λ', and the **porosity**, ϕ, are determined directly from the **micro-geometry**. The thermal length is computed as the ratio of the doubled volume of fluid domain to the surface of skeleton walls.

Macro-scale level: **Use the parameters** calculated (averaged) **from microstructure** for the **Johnson-Allard formulas** to compute the **dynamic tortuosity functions**, and then the **dynamic permeability functions**, and finally, the **effective density and bulk modulus**.

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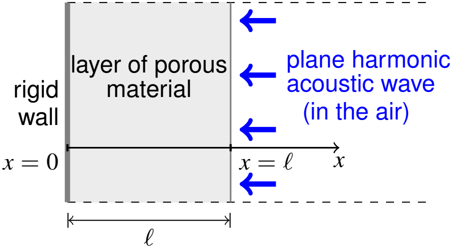
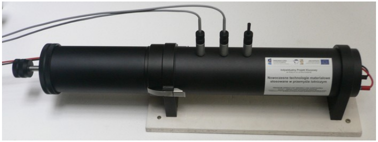
Impedance and absorption of porous layer



Impedance tube
for material testing



- **Surface acoustic impedance**:

$$Z(\omega) = \hat{\varrho}\,\hat{c}\,\frac{\exp(2\mathrm{i}\omega\ell/\hat{c}) + 1}{\exp(2\mathrm{i}\omega\ell/\hat{c}) - 1} = -\mathrm{i}\hat{\varrho}\,\hat{c}\cot\left(\frac{\omega\ell}{\hat{c}}\right) = -\frac{\mathrm{i}Z_0}{\phi}\sqrt{\frac{\alpha}{\beta}}\cot\left(\frac{\omega\ell}{c_0}\sqrt{\alpha\beta}\right)$$

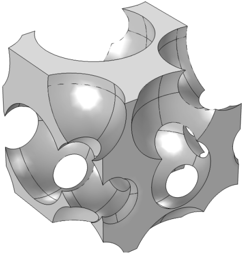
$Z_0 = \varrho_0\, c_0$ – the characteristic impedance of pore-fluid

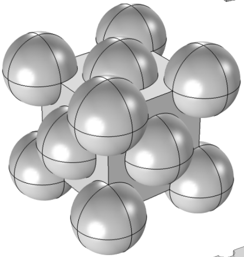
ϱ_0, c_0 – the density and speed of sound in pore-fluid

- Acoustic **absorption** and **reflection** coefficients:

$$A(\omega) = 1 - |R(\omega)|^2, \qquad R(\omega) = \frac{Z(\omega) - Z_0}{Z(\omega) + Z_0}$$

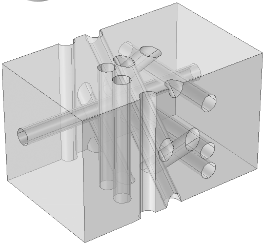
open-cell foams





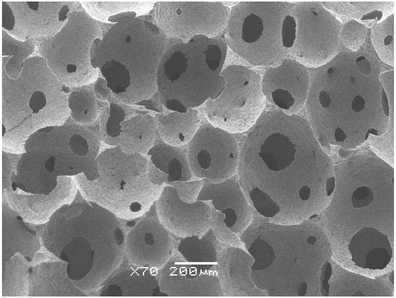
granular media

fibrous media



Foams with spherical pores

Example: A corundum ceramics foam



A foam with spherical pores

Total porosity:	**88 %**
Mean pore size:	**380 µm**
Mean window size:	**60 µm**

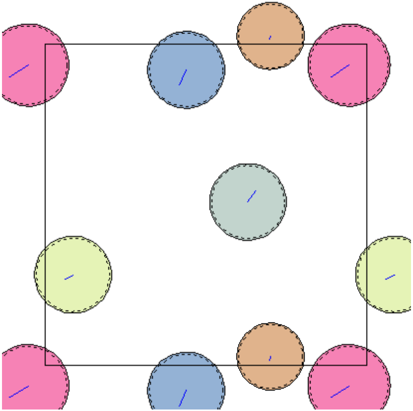
M. POTOCZEK: "Gelcasting of alumina foams using agarose solutions." *Ceramics International*, Vol. 34, pp. 661-667, 2008.

T.G. ZIELIŃSKI, M. POTOCZEK, R.E. ŚLIWA, Ł.J. NOWAK: "Acoustic absorption of a new class of alumina foams with various high-porosity levels." *Archives of Acoustics*, Vol. 38, No. 4, pp. 495-502, 2013.

- **Objective:**
  Automatic generation of **irregular (random) yet periodic** porous microstructures.
- **Assumption:**
  The pores are spherical.
- **Features:**
  Some characteristic (average or macroscopic) features should be controlled, namely:
  - the open **porosity**,
  - the typical **size of pores**,
  - the typical **size of windows** linking the pores.

Bubble dynamics

- A simple **bubble dynamics** can be used to generate some **random distribution of pores** (penetration allowed) or **rigid spheres** (no penetration).
- The approach is similar to the **Lubachevsky-Stillinger compression algorithm** (originally proposed for two-dimensional discs).



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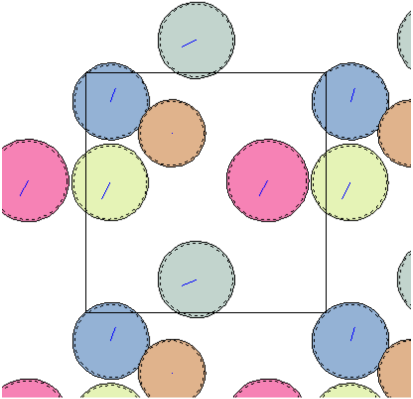
- **Additional constraints** are necessary to ensure **periodicity**.



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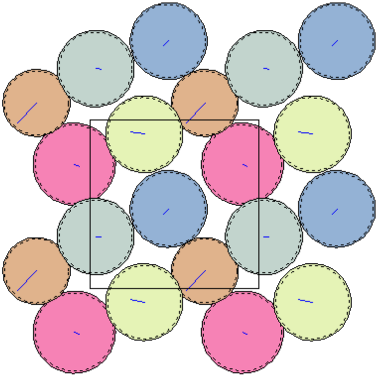
- **Additional constraints** are necessary to ensure **periodicity**.
- Improved **mixing/packing techniques** may involve: bubble shrinking and swelling, gravity, random acceleration, collision energy dissipation, etc.



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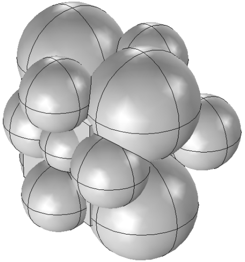
Representative Volume Elements for a foam

Periodic porous microstructure:

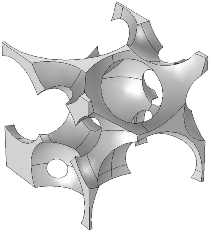
- **5 complete pores** in the cubic cell
- **porosity 88%** (as in the ceramic foam)



periodic arrangement of spherical pores

Observation

More pores with much **more diversified sizes** are necessary to represent a **very complex geometry** of the **ceramic foam!**



solid skeleton

Weakly-representative cells:

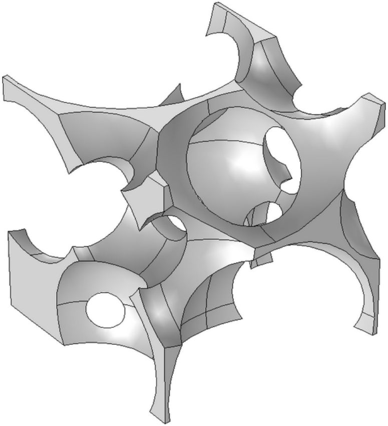
***RVE-1*:** • microstructure with 5 spherical pores and porosity **88%** • average size of pores **380 µm** (as in the ceramic foam) • average size of windows **176 µm** (much larger)

***RVE-2*:** • microstructure with 5 spherical pores and porosity **88%** • average size of pores **300 µm** • average size of windows **139 µm**

Transport parameters of foams with porosity 88%

Transport parameters:

- porosity **[%]**: **87.92**



solid skeleton

Transport parameters of foams with porosity 88%

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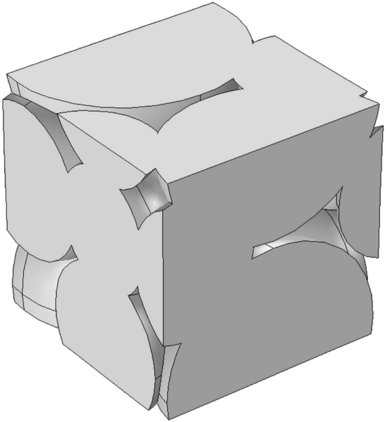
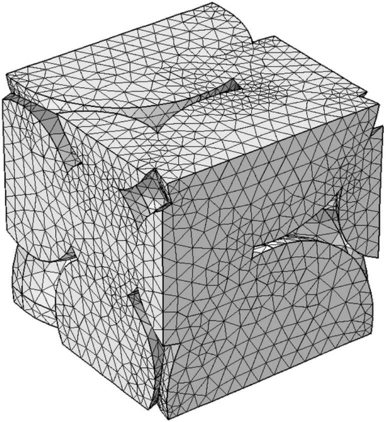


fluid domain

Transport parameters of foams with porosity 88%

Transport parameters:

- porosity **[%]**: **87.92**
- thermal length **[μm]**
  240.9 (*RVE-1*) **190.2** (*RVE-2*)

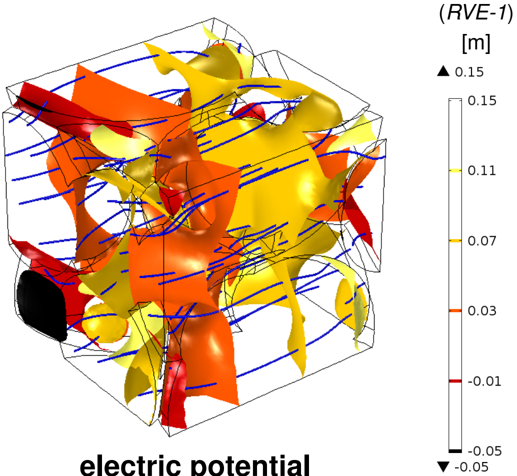


meshed domain

Transport parameters of foams with porosity 88%

Transport parameters:

- porosity **[%]**: **87.92**
- thermal length **[μm]**
  240.9 (*RVE-1*) **190.2** (*RVE-2*)
- viscous length **[μm]**
  125.2 (*RVE-1*) **97.60** (*RVE-2*)
- tortuosity **[–]**
  1.264 (*RVE-1*) **1.266** (*RVE-2*)



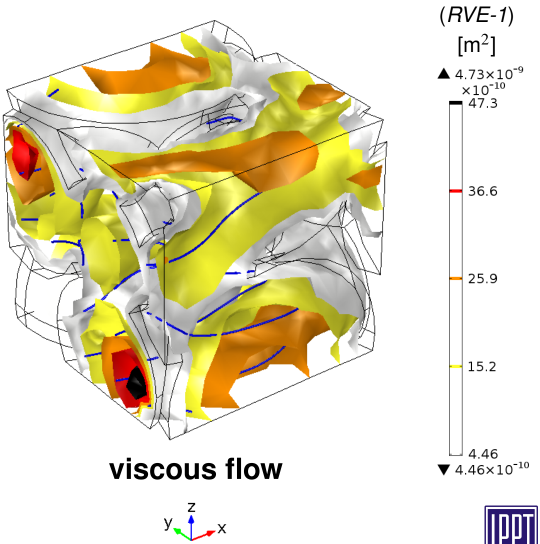
electric potential

propagation direction: X

Transport parameters of foams with porosity 88%

Transport parameters:

- porosity **[%]**: **87.92**
- thermal length **[μm]**
  240.9 (*RVE-1*) **190.2** (*RVE-2*)
- viscous length **[μm]**
  125.2 (*RVE-1*) **97.60** (*RVE-2*)
- tortuosity **[–]**
  1.264 (*RVE-1*) **1.266** (*RVE-2*)
- viscous permeability **[μm^2]**
  1199 (*RVE-1*) **746** (*RVE-2*)
- viscous ‘static’ tortuosity **[–]**
  1.892 (*RVE-1*) **1.898** (*RVE-2*)



viscous flow

propagation direction: X

Transport parameters of foams with porosity 88%

Transport parameters:

- porosity **[%]**: **87.92**
- thermal length **[μm]**
  240.9 (*RVE-1*) **190.2** (*RVE-2*)
- viscous length **[μm]**
  125.2 (*RVE-1*) **97.60** (*RVE-2*)
- tortuosity **[–]**
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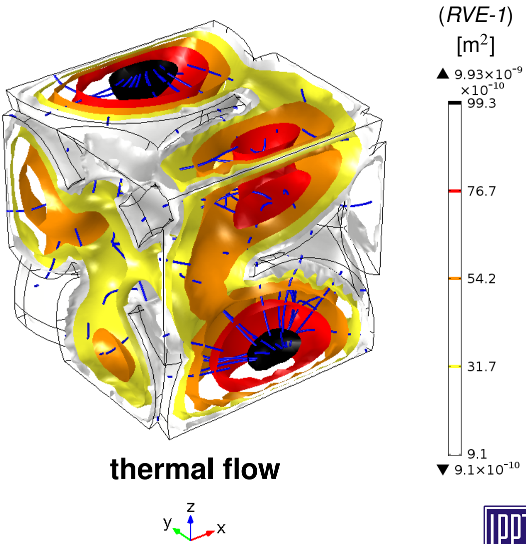
- viscous permeability **[μm^2]**
  1199 (*RVE-1*) **746** (*RVE-2*)
- viscous 'static' tortuosity **[–]**
  1.892 (*RVE-1*) **1.898** (*RVE-2*)
- thermal permeability **[μm^2]**
  3999 (*RVE-1*) **2845** (*RVE-2*)
- thermal 'static' tortuosity **[–]**
  1.346 (*RVE-1*) **1.405** (*RVE-2*)

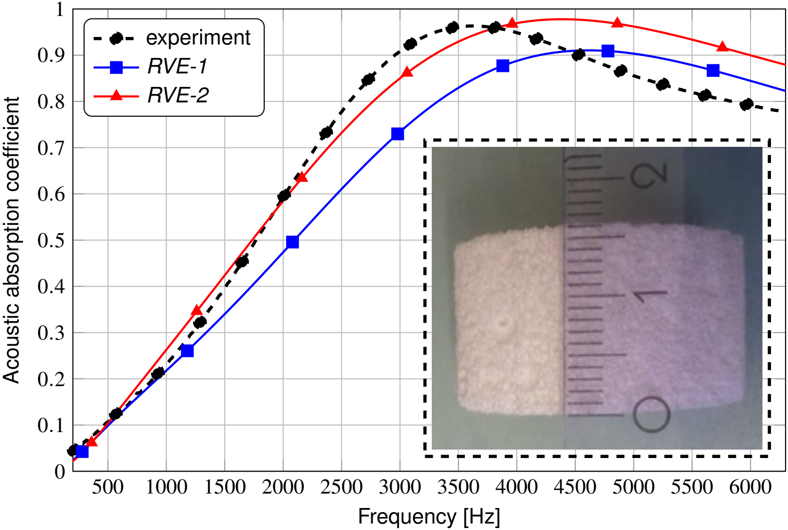


thermal flow

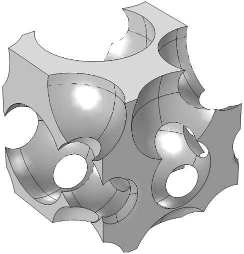
propagation direction: X

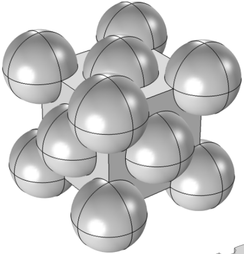
Acoustic absorption of a layer of (ceramic) foam

Results for a **porous layer** (**16.5 mm**-thick) of a **ceramic foam** with **porosity 88%** and two **weakly-representative cells (RVEs)** with spherical pores:



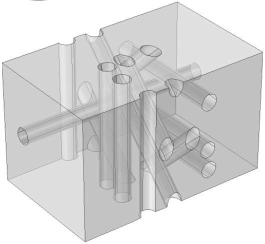
open-cell foams





granular media

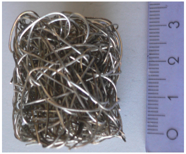
fibrous media



Fibrous samples from copper wire (diameter: 0.5 mm)

sample in the tube

30 mm sample



60 mm sample

- **Two fibrous samples** were manufactured from a silver plated **copper wire of diameter 0.5 mm**.
- The samples were **manually woven** and fitted into the impedance tube of **diameter 29 mm**, and their **heights** are **30 mm** and **60 mm**, respectively.
- The **length of wire** used for the smaller sample was **10 m**, and for the twice taller one it was **20 m**, so that both samples have the same **porosity 90%**.

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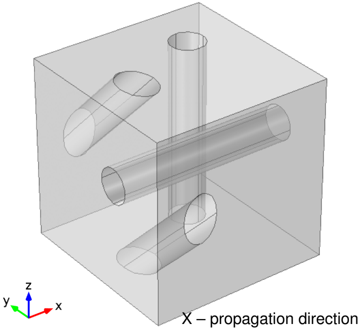
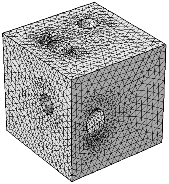
Periodic representations with straight fibres

- **Periodic RVE** with **uniformly-spaced fibres**
- **Straight fibres** with **diameter 0.5 mm**
- **Porosity: 90%**
- **Other transport parameters:**

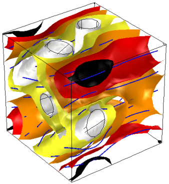
		RVE-1
viscous permeability:	[10^{-8} m^2]	**11.60**
thermal permeability:	[10^{-7} m^2]	**1.56**
(viscous) tortuosity:	[–]	**1.07**
viscous static tortuosity:	[–]	**1.38**
thermal static tortuosity:	[–]	**1.17**
viscous length:	[mm]	**1.43**
thermal length:	[mm]	**2.27**

RVE-1

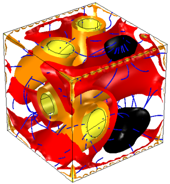




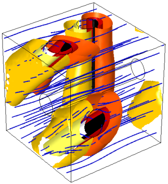
mesh



velocity



temperature



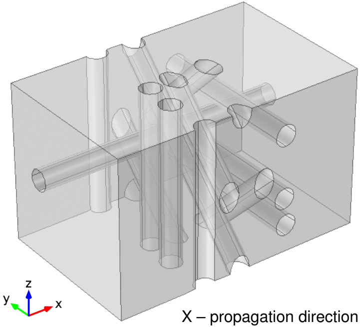
potential

Periodic representations with straight fibres

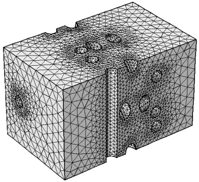
- **Periodic RVE** with **layer-grouped fibres**
- **Straight fibres** with **diameter 0.5 mm**
- **Porosity: 90%**
- **Other transport parameters:**

		RVE-1	**RVE-2**
viscous permeability:	$[10^{-8} m^2]$	11.60	**7.39**
thermal permeability:	$[10^{-7} m^2]$	1.56	**4.83**
(viscous) tortuosity:	[–]	1.07	**1.10**
viscous static tortuosity:	[–]	1.38	**1.47**
thermal static tortuosity:	[–]	1.17	**1.56**
viscous length:	[mm]	1.43	**1.11**
thermal length:	[mm]	2.27	**2.26**

RVE-2



X – propagation direction



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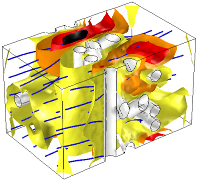
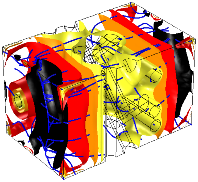
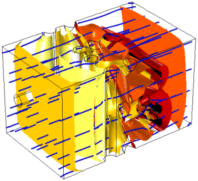


velocity



temperature



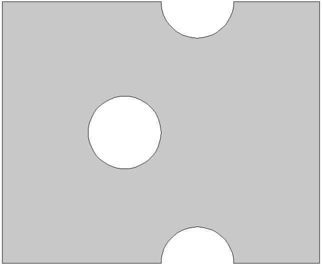
potential

Periodic representations with straight fibres

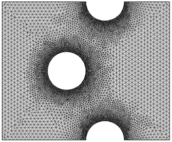
- **Periodic 2D cell** with **layer-grouped fibres**
- **Straight fibres** with **diameter 0.5 mm**
- **Porosity: 90%**
- **Other transport parameters:**

		RVE-1	RVE-2	**2D-cell**
viscous permeability:	[10^{-8} m^2]	11.60	7.39	**4.87**
thermal permeability:	[10^{-7} m^2]	1.56	4.83	**2.04**
(viscous) tortuosity:	[–]	1.07	1.10	**1.12**
viscous static tortuosity:	[–]	1.38	1.47	**1.30**
thermal static tortuosity:	[–]	1.17	1.56	**1.20**
viscous length:	[mm]	1.43	1.11	**1.08**
thermal length:	[mm]	2.27	2.26	**2.25**

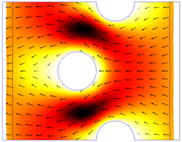
2D-cell



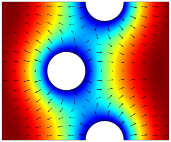
⟶ X – propagation direction



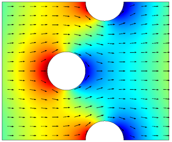
mesh



velocity

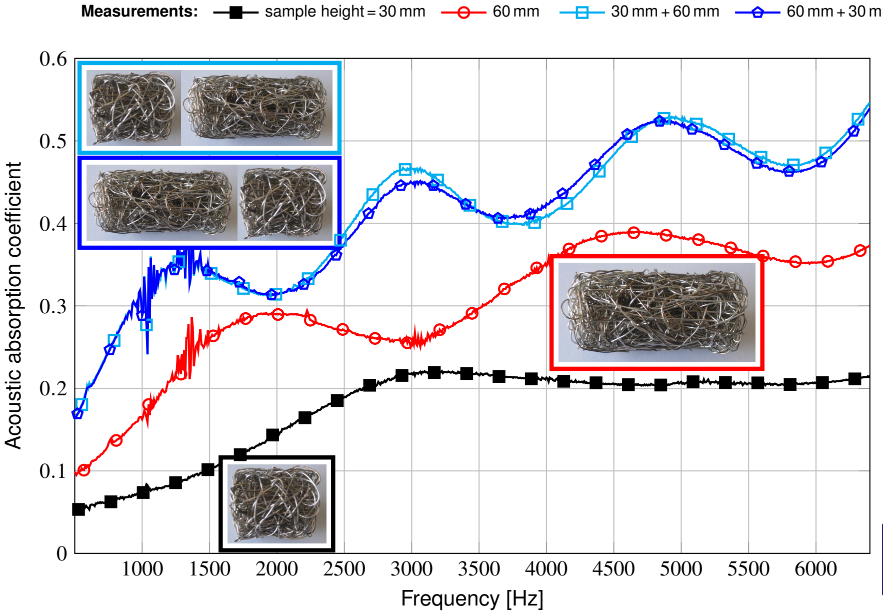


temperature

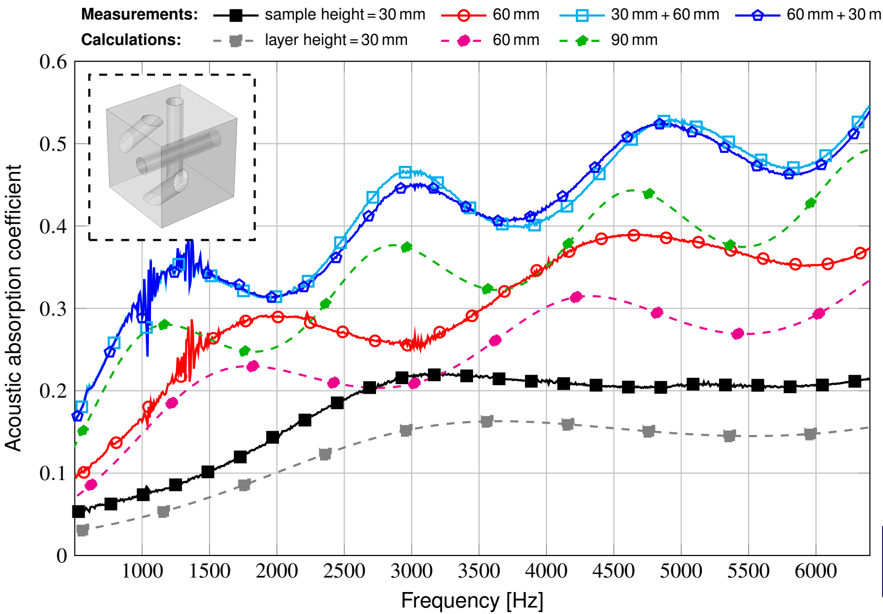


potential

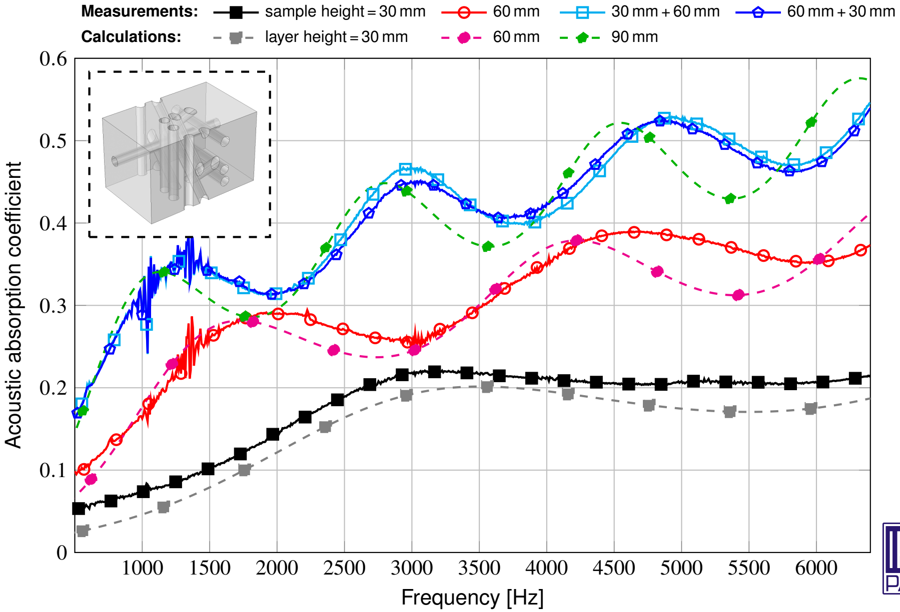
Acoustic absorption of fibrous samples



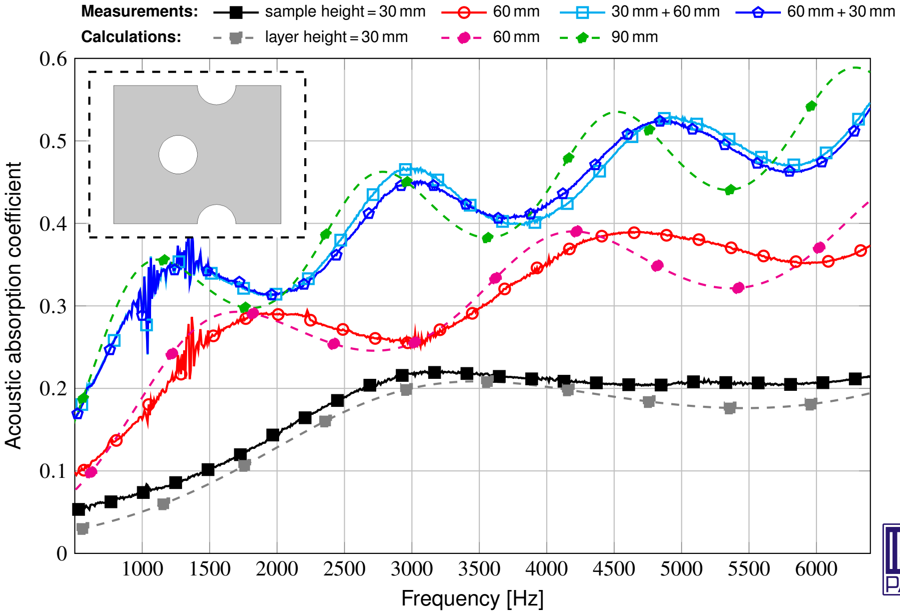
Acoustic absorption of fibrous samples



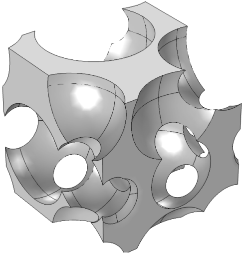
Acoustic absorption of fibrous samples

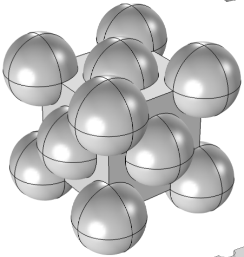


Acoustic absorption of fibrous samples



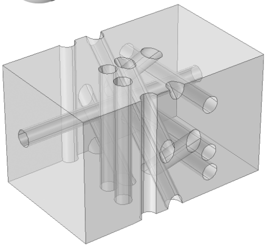
open-cell foams





granular media

fibrous media



Final remarks

IN GENERAL:

- **More than a few pores, grains, or fibres** in a periodic cell (RVE) are necessary **to well represent the geometry** of real porous media.
- More pores (grains, fibres) in a representative cell means **larger RVE** and that would require **more computational power**.
- Moreover, **the size of a large RVE** may (at higher frequencies) become **comparable with the wave-lengths**, which would worsen the accuracy and reliability of estimations, because of a **weak separation of scales**.
- **A few random representations** with **the same features** should be generated to compute **predictions as the average results**.

HOWEVER:

- **Simple representations** may be **useful** for some materials: for example, in the case of some **fibrous materials** even **2D cells** can give **good estimations**.

Final remarks

MODELLING IN COMSOL MULTIPHYSICS:

- **Finite-element calculations** are on the **micro-scale level**.
- **Periodic boundary conditions** are extensively used (in case of each of three micro-scale problems).
- **Symbolic expressions** and **equation-based modelling** allowed to program the **weak forms** of the scaled problems defined the on **micro-scale domains**. Alternatively, the *PDE Form (Laplace Problem)*, *Fluid Mechanics* and *Heat Transfer Modules* could be used (re-scaling to be done afterwards).
- On the **macro-scale level**, the **analytical solutions** for the **plane-wave propagation** in layered media are programmed in MATLAB. In general (i.e., for a more complex propagation), **FE analyses** should be applied using COMSOL *Acoustics Module*.
- ***LiveLink to MATLAB*** allows to integrate the procedures for generation of periodic representative geometries with FE computations on the micro-scale level and some calculations on the macro-scale.

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Some of the discussed results have been recently published in:

- T. G. ZIELIŃSKI **(2015)** "Generation of random microstructures and prediction of sound velocity and absorption for open foams with spherical pores." ***Journal of the Acoustical Society of America***, Vol.**137**, No.4, pp.1790-1801. DOI:10.1121/1.4915475

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