# Validation of the Acoustic Finite Element Model of a Very Light Jet Cavity Mock-up

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**Abstract:** This paper presents an important step in developing a combined active noise- and audio system for a (very) light jet. To prepare its installation both a wooden mock-up and a finite element model of the investigated cavity were created. Sensitivity analysis and key parameters selection were done in order to validate the model. Error analysis between measured and computed data closes the investigation. The simulations were performed using COMSOL Multiphysics vers 3.4. The validated model could be used to optimize the loudspeaker- and microphone positions.

**Keywords:** FE-model, acoustic, model validation, frequency response.

## 1. Introduction

Without special treatment, the cabins of small aircraft (very light jet, VLJ) are particularly noisy. The results of in-flight measurements taken inside an aircraft of type Cessna Citation Bravo show that certain places in the cabin are subjected to total sound pressure levels (SPL) of up to 91,4 dB ([1] p. 825).

To prepare the installation of a combined audio and active noise control (ANC)-system for a VLJ, a wooden aircraft mock-up was built. The complete system must meet requirements to supply both high quality audio entertainment and effective reduction of unwanted noise. The project is conducted at the University of the Federal Armed Forces Hamburg in cooperation with Innovint Aircraft Interior GmbH (Hamburg, Germany).

Several acoustic problems in the aircraft cabin cannot be analytically described due to complex functional interrelations (for example, the problem of the optimal positioning of loudspeakers and microphones). In these cases numerical methods are often used. These computations enable the prediction of acoustic behaviour for complex geometric problems. Therefore, a validated finite element model is needed.

The paper describes several steps of the model validation procedure via comparison between the measured sound pressure field (taken in a wooden aircraft mock-up) and the calculated pressure field (from a COMSOL-model). A short overview of the validation methods and the benefits of the method used here are presented. Its implementation in COMSOL/MATLAB and the interpretation of the obtained results is also emphasized in this paper.

## 2. Overview on Validation Procedures

Model validation is a confirmation process that a model can adequately predict the underlying physics with a satisfactory level of accuracy. In ([2] p. 588) three model validation approaches, i.e., *graphical comparison*, *confidence interval approach* and *r² approach,* are presented.

### 2.1 General Approaches

a) Graphical Comparison

This approach is the most straightforward approach used in many applications although the format may be different. One may plot the experimental and predicted results with different symbols in order to distinguish between these two groups on the same graph using model parameters as the axes and then directly compare the results. The plot therefore provides an overall qualitative view about the validity of the model. In this approach, uncertainty associated with model parameters is not considered but some information can be extracted.

b) Confidence Interval Approach

In this approach, probabilistic results from the uncertainty analysis and a specified significance level are used to create an allowable variation (confidence interval) of predicted response corresponding to acceptable variations of input parameters. If all corresponding experimental results are found within the confidence intervals created, the model is not considered to be invalid at the design space tested with the specified significance level, i.e., error measure is statistically zero (the model is good enough under those conditions). This approach may not be convenient in cases where there are several design points since there will be several plots to analyze.

c) $r^2$ Approach

To address the concern in the confidence interval approach on the examination of each individual confidence interval at each design point, the $r^2$ approach eliminates the individual examination at each design point by calculating the distance between physical tests and simulated results of multiple design points all at once in one metric, $r^2$. By considering n design points, $r^2$ is defined in ([3] p294) as follows:

$$r^2 = \left[p_1 - p_{mean1}\ p_2 - p_{mean2} \cdots p_n - p_{meann}\right] COV^{-1} \begin{bmatrix} p_1 - p_{meann} \\ p_2 - p_{mean2} \\ \vdots \\ p_n - p_{meann} \end{bmatrix} \quad (1)$$

where $p_i$ is an average of experimental results for the $i$th design point, $p_{meani}$ is a mean of predicted results for the $i$th setting point and COV is an $n \times n$ covariance matrix of the predicted results for $n$ design points.

The metric $r^2$ is theoretically applicable when the probability density function of the simulated response is normally distributed in a satisfactory manner ([2] p. 589). If the calculated $r^2$ is less than a defined critical value $r^2_{critical}$, the model is considered to be not invalid at the tested space with the given significance level.

## 2.1 Validation Procedures for Vibro-Acoustic Problems

As listed in sec. 2.1, model validation has two classes of methods, i.e., direct comparison without the association of uncertainty and methods with the association of uncertainty.

Cavity acoustic FE-models are usually validated by direct graphical comparison (without the association of uncertainty). The sound field distribution in the cavity can be clarified only by mapping the sound field. Thus a large quantity of design points results from the mapping, which make the consideration of uncertainty associated with model parameters more difficult. While the distance between two design points remains small, a qualitative investigation for FE-model validation is adequate. Two validation types of acoustic FE-models can be differentiated. They are based on similar approaches and differ only in the type of the parameters used for the correlation examination.

a) First Type: Validation on the Basis of Modal Parameters

Since the characteristics of a dynamic system can be fully described with its modal values (natural frequencies, modal damping and mode shape), this procedure is adequate for validating structure dynamic (vibro acoustic) systems. Thereby the system is excited at different places and the frequency response function (FRF) is recorded in the entire measurement area. Then an experimental modal analysis (EMA) is done, in order to extract the system modal parameters from the measured FRFs. The used curve fitting algorithm (to fit the FRF) represents the core idea of the EMA. A lot of software for EMA exists on the market. Afterwards, a FE-model eigenvalue analysis is conducted and the calculated modal values are compared with modal values extracted from measurements. FE-model setting parameters are varied until the test-analysis correlation is successful.

The success of this procedure depends on the quality of the curve fitting. Existing software are limited to processing systems with low modal densities. Since for structure dynamic problems their modes are not located very close to each other in the low frequency range, the application of this method is very suitable.

The application of this method in room acoustics is used for small volumes only and in the low frequency range (e.g. VLJ Cavity). In a square room the number of modes obeys the following rule ([4] p. 164):

$$N_f = \frac{4\pi}{3} V \left(\frac{f}{c}\right)^3 \quad (2)$$

where:

$V$ is the volume of the room;

$f$ is the highest considered frequency ;

$c$ the speed of sound.

b) Second Type: Validation on the Basis of Operating Oscillation Parameters

In this case the experimental environment is reproduced in the FE-model as accurately as possible. At each measuring position, the measured FRF is compared with the FRF determined from the FE-computation. This method is very effective for room acoustic problems and supplies accurate results if different measurement planes and variable excitation positions are considered. While the processing of the EMA represents the main difficulty in the first procedure type, the accurate reproduction of the excitation as well as the exact positioning of the measurement points in the FE-model is the greatest challenge. Also the adjustment parameters of the FE-model are varied until the computed FRFs agree with the measured FRFs.

Fig. 1 shows the main steps in the model validation process.

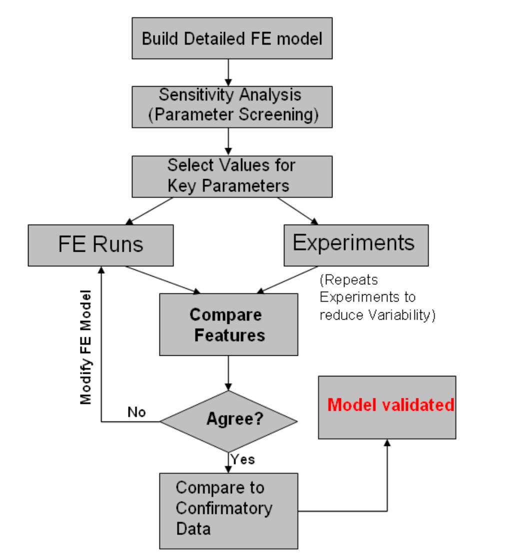


**Fig. 1**: Model validation approach: ideal vs. actual [5]

After the detailed FE-model is built, a sensitivity analysis is to be accomplished. The sensitivity analysis is needed to determine the influence of the possible model adjustment parameters on the system responses. Then the values of key parameters are selected. The FE simulation is started with the selected key parameter values and its results are compared with the measurement results. The selected key parameter values should be varied until the examination of the test-analysis correlation is successful. A discrepancy investigation between the validated and the test model is conducted, in order to define the FE-model accuracy and its field of application.

## 3. Validation of an Acoustic Model for a VLJ

### 3.1 Description of Test Rig and Experiment

The sound field in the mock-up cavity was mapped for one excitation position and three mapping areas. A loudspeaker (Chassis type 6ND430 Eighteen Sound) was used to excite the system with a band limited white random noise (Frequency range: 0-800 Hz). An accelerometer recorded the membrane acceleration of the loudspeaker. Fig. 2 shows the interior of the mock-up and Fig. 3 presents the excitation position and the three measurement areas. The green measurement area is at the ear level of a sitting passenger. The measurement device consists of four equidistant microphones.

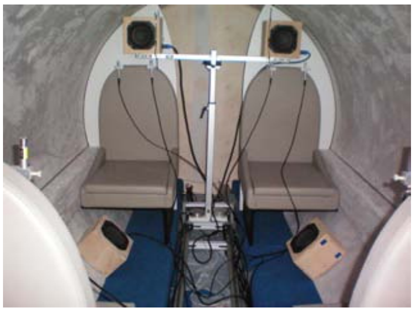

**Fig. 2**: Interior of the acoustic mock-up

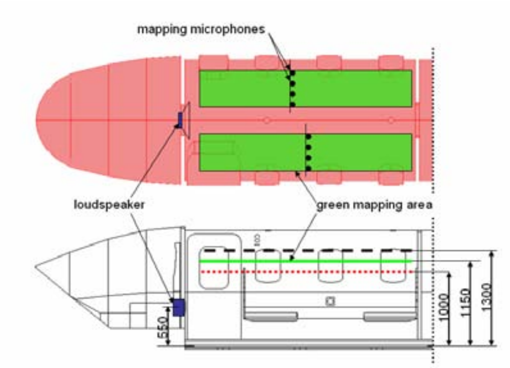


**Fig. 3**: Excitation position and measurement areas

### 3.2 Description of the Finite Element Model of the Acoustic Cavity

a) Geometry and Mesh

Using technical drawings of the mock-up, a simplified model geometry of the cavity was created in COMSOL MULTIPHYSICS. It represents the air volume in the aircraft interior. Figs. 4 A) to C) show a part of the model geometry. In A), an external view of the area inside of the cabin structures is given. The image in B) shows the accounted internal structure including passenger seats and 2 seats in the cockpit. In C) the model mesh is shown.

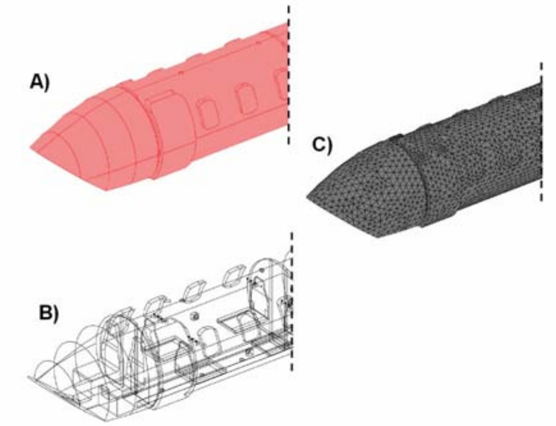


**Fig. 4**: Model geometry and mesh.

For the mesh generation in 2D or 3D with COMSOL MULTIPHYSICS the Delaunay algorithm ([6] p. 190) was used. Tetra 2nd-order Lagrange elements were generated.

Choosing the mesh parameters, attention was given to generate a "regular" mesh, i.e. the size of the elements is evenly distributed over the geometrical model because the accuracy of the results depends on meshing methods. Also as recommended in ([7] p. 22), the number of elements per wavelength was set to at least 3.

For the current project the max. frequency is 500 Hz. The shortest relevant wavelength is:

$$\lambda = \frac{c}{f} = 0{,}68\ \mathrm{m} \qquad (3)$$

where,

$c$ = 343 m/s is the speed of sound,

$f$ = 500 Hz is the frequency.

Considering that it is necessary to have at least 3 elements per wavelength, the largest element must have a maximum size of 0.22 m for frequencies up to 500 Hz. The model consists of 60989 elements and 92162 degrees of freedom.

b) Sensitivity Study

For analyzing the system sensitivity, COMSOL allows to set the following parameters:

- geometry
- wave number
- reflection factor of the sound reflecting surfaces
- loudspeaker correction factor.

The geometry was built with a high accuracy level and could not be varied during the sensitivity test.

The introduction of a complex wave number can influence the sound damping in the cavity. The complex wave number is defined in ([8] p. 7) as follows:

$$k_c = \frac{\omega}{c}(1 - j \cdot \beta) \qquad (4)$$

where, $\beta$ is the damping number, $\omega$ the angular frequency and $j^2 = -1$.

The response due to an increase of the frequency corresponds to a decrease of the speed of sound. Fig. 5 shows the influence of the damping number on the frequency response in a specific microphone ($f$ = 100Hz). The complex pointers of the measured frequency responses ($Z_{MEAS}$) and the computed frequency responses ($Z_{FEM}$) are presented. The variation of the damping number is marked in green. Fig. 6 shows also the influence of the speed of sound variation on the system response.

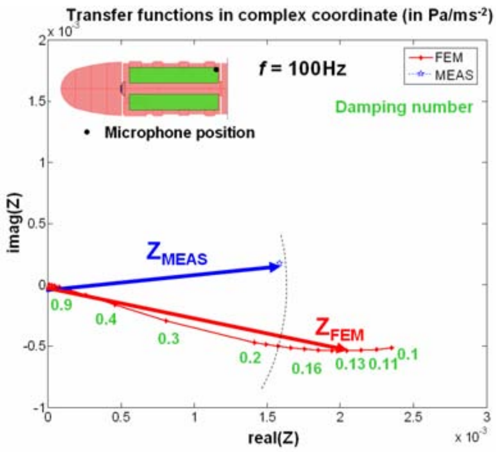


**Fig. 5**: Influence of the damping number on the system response

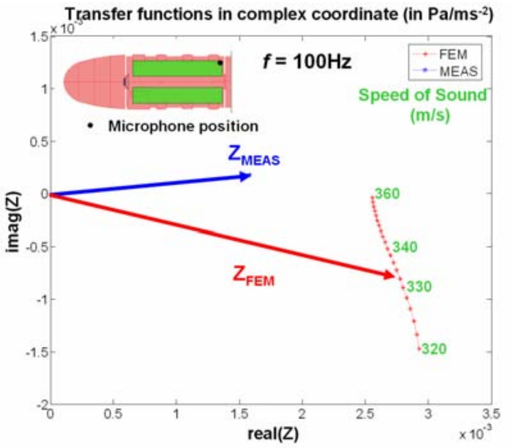


**Fig. 6**: Influence of the speed of sound on the system response

The damping number has a large influence on the amplitude but only small influence on the phase of the frequency response. The speed of sound affects both the amplitude and the phase of the frequency response.

The reflection factor affects both the amplitude and the phase of the FRF. For a given refection factor $r$ of a surface, the characteristic sound impedance is defined as follows ([9] p. 60):

$$Z_r = \frac{1+r}{1-r}\rho c \tag{5}$$

with $$r = \hat{r}e^{j\theta} \tag{6}$$

where $\rho$ is the air density and $\theta$ the reflection phase.

A variation of the reflection phase causes a phase shift of the FRF (Fig.7).

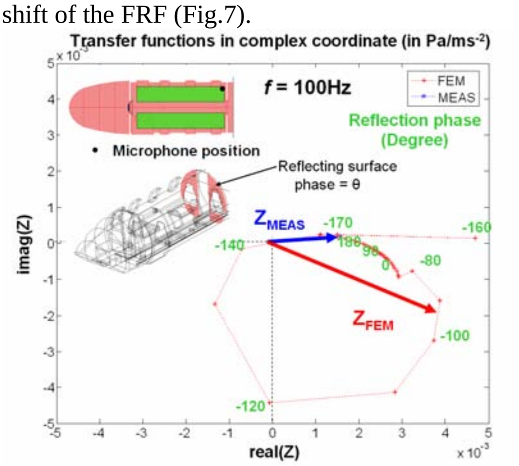


**Fig. 7**: Influence of the reflection phase on the system response

The loudspeaker correction factor is a transfer function, which represents the relationship between the modeled loudspeaker effect and the real loudspeaker effect. Since the effect results from a normal acceleration, it is defined as follows:

$$K_{Ls} = a \cdot e^{j\phi} \tag{7}$$

where $a$ and $\Phi$ adjust the magnitude and the phase of the normal acceleration respectively.

### 3.3 Description of the used validation procedure

a) Fit of Model Parameters

- Wave Number

The wave number can be formulated as:

$$k_c = \frac{\omega}{c}(1 - j \cdot \beta) = \hat{k}e^{j\vartheta} \tag{8}$$

If the optimal wave number is known, then both the optimal speed of sound $c_{opt}$ and the optimal damping number $\beta_{opt}$ can be deducted. The following cost function is needed to determine the optimal wave number:

$$J = \alpha \frac{(e_{Norm}{}^{H} e_{Norm})}{M \cdot max(e_{Norm})^2} + (1-\alpha)\frac{(e_{Phase}{}^{H} e_{Phase})}{M \cdot max(e_{Phase})^2}$$

(9)

Where,

$e_{Norm} = |Z_{MEAS}| - |Z_{FEM}|$;

$e_{Phase} = angle(Z_{MEAS}) - angle(Z_{MEAS})$;

$Z_{MEAS}$ = Column matrix of the measured FRFs

$Z_{FEM}$ = Column matrix of the computed FRFs

M = Number of measurement points

α = 0.5(Magnitude/Phase weighting factor).

The cost functions were calculated for three selected frequencies and different values of the wave number. An example of a cost function plot is shown in Fig. 8.

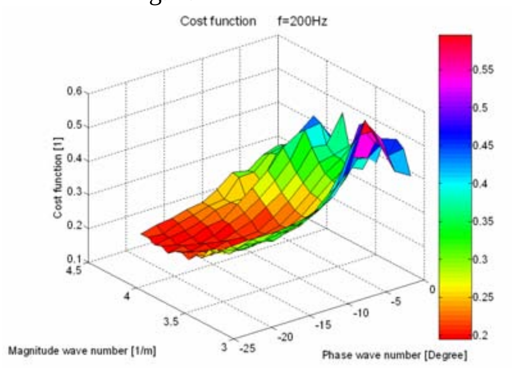


**Fig. 8**: Cost function as function of the wave number

The optimal wave number corresponds to the values of the cost function minimum**.**

- Reflection Factors

The Sound field can be arbitrary modified by changing the lining material reflection factor. It is possible to modify both the value of the reflection phase and its magnitude. The selected surfaces to perform those changes are presented in Fig. 9.

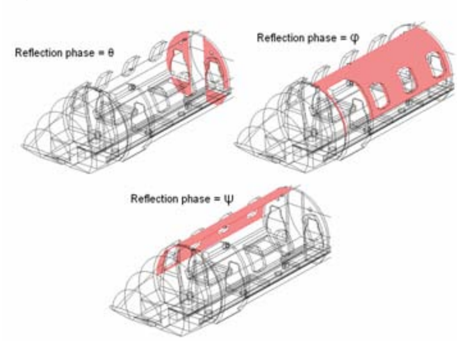


**Fig. 9**: Used surfaces for the advanced sound field optimization

Tab.1 summarizes the fitting results of the model parameters for the frequencies 100Hz and 300Hz.

| | Parameters | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | wave number | | Reflection factors (Magnitude and phase) | | | | | | Loudspeaker correction | |
| | c[m/s] | β[1] | rθ [1] | θ [°] | rφ[1] | φ [°] | rψ[1] | ψ [°] | a[1] | Φ [°] |
| 100 Hz | 350 | 0,1 | 0,8 | 0 | 0,8 | 0 | 0,8 | 0 | 0,4 | 30 |
| 300 Hz | 370 | 0,09 | 0,99 | 0 | 0,99 | 35 | 0,99 | 35 | 1 | -10 |

**Tab. 1**: Model setting parameters for two different frequencies

b) Validation of Model Performance

After FE-Model adjustment, the FRFs of the measurement area can be plotted and compared with the plot of the computed FRFs. Fig. 10 shows the result after adjustment of the FRFs magnitude (in $dB/20Pas^2m^{-1}$) in the green mapping area for the frequency 100Hz. The FRFs phase [in degree] is represented in Fig. 11 for the same frequency. Figs. 12-13 show the FRF magnitude and phase (measured and computed) in the green and the red mapping areas for the frequency 300Hz.

The histogram of the difference between measured and computed FRFs shows that the results of the FE-simulation present a high accuracy level in low frequency range (e.g. at 100Hz). In this case, the FE-model can be used both for qualitative and quantitative analysis. For the frequency 300Hz, highly inaccurate results appear at many points of the mapping area. In this case, only an application for qualitative investigation is appropriate.

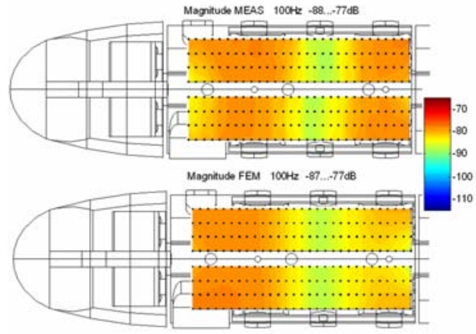


**Fig. 10**: Magnitude of the frequency responses in the green mapping area (100Hz)

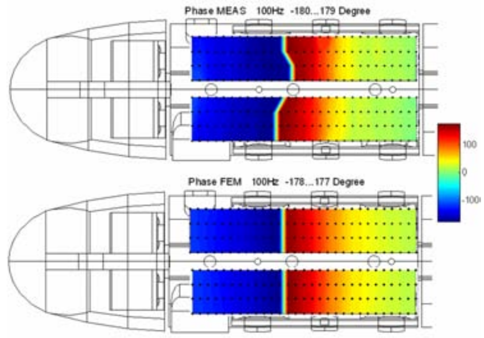


**Fig. 11**: Phase of the frequency responses in the green mapping area (100Hz)

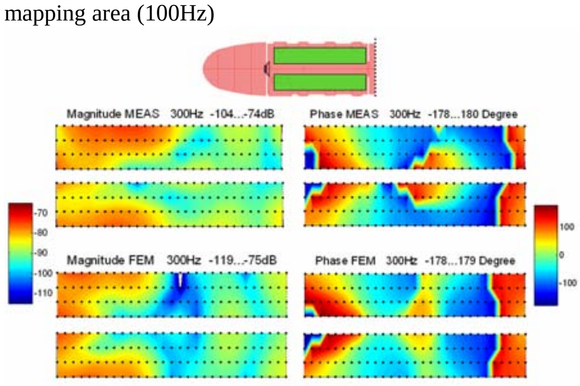


**Fig. 12**: Frequency responses in the green mapping area (300Hz)

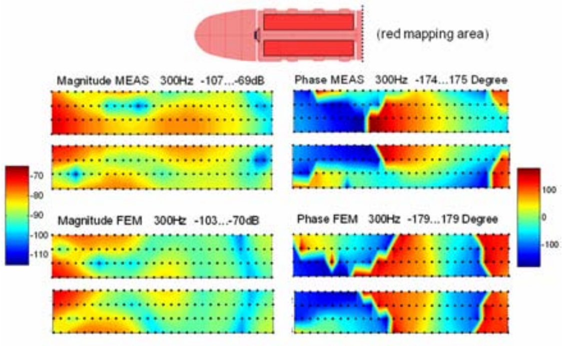


**Fig. 13**: Frequency responses in the red mapping area (300Hz)

## 4. Conclusions

In this paper a FE-model of an acoustic mock-up cavity was presented. The model geometry was created in COMSOL MULTIPHYSICS 3.4. To validate the FE-model, a short overview of model validation procedures was presented. A direct method for comparison without the consideration of uncertainty was used. The sound field in the mock-up cavity was mapped for one excitation position and three mapping areas. Furthermore, a sensitivity test was conducted in order to select the model adjustment parameters. Three main groups of adjustment parameters were defined: the parameter depending on the wave number (sound speed and damping number), the reflection factor for model boundaries (magnitude and phase) and the loudspeaker correction factor (magnitude and phase). A cost function was defined to fit the parameter of the first group. The other parameters were determined through both a visual comparison of the FRF plots and comparison of the cost function (defined by the mean square error). After adjustment, the results of the measurements and the FE-simulation for the frequencies 100Hz and 300Hz were presented. The frequency 100Hz present a high accuracy level and can be used for both a qualitative and quantitative investigations. In high frequency range (>300Hz) the FE-model can only be used for qualitative investigations.

Upcoming research will focus on the improvement of the FE-model accuracy using the Monte Carlo Method to fit all adjustment parameters at the same time. Furthermore, a sequential algorithm can be used to optimize the loudspeaker and the microphones positions.

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