# A Study of Laser Doppler Anemometer Using COMSOL Multiphysics

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Abstract: Laser anemometers based on application of Doppler Effect have been developed and are used in-flight, on aircrafts for measurement of air flow parameters, mainly its speed versus the airplane. The air speed measurements are vital for safe flights. The main basic idea of Doppler techniques consists in measuring the frequency of scattered light. In this paper, we propose a study of a laser Doppler Anemometer using COMSOL Multiphysics. The main purpose of this study is to provide essential data for an improved design of this type of laser devices. The use of Laser Doppler Anemometer COMSOL Multiphysics simulations is under continuous development by using more and more realistic values of input constructive laser device parameters.

**Keywords:** Laser Doppler Effect, air particle scattering.

# 1. Introduction

The Doppler Effect using laser as radiation source has important applications for design and building anemometers (LDA) used for in-flight air speed measurements based on application of have been developed and are used on aircrafts for measurement of air flow parameters, mainly its speed versus the airplane. The air speed measurements are vital for safe flights. The main basic idea of Doppler techniques consists in measuring the frequency of scattered light. Light is scattered from a moving particle thereby altering the frequency of the incident laser signal. This altered frequency of light can be determined directly with frequency-dependent optical processing using a spectrometer or an optical filter. The common approach for detection is converting light frequency changes into light intensity fluctuations using interference of light. In this paper, we propose a study of a laser Doppler Anemometer using COMSOL Multiphysics. The experimental setup of the investigated laser Doppler Anemometer is presented in Figure 1. The main purpose of this

study is to provide essential data for a proper design of a device of this type [1-9].

# 2. Theory

The schematic of experimental LDA setups are presented in Figures 1 and 2. It is worth to be pointed that these are schematic of common LDA which are in use, onboard on flying airplanes. Both examples of LDA setup schematic have as a common detail the fact that laser radiation is focused at some distance from the device (the airplane) into a volume of flowing air [1-5].

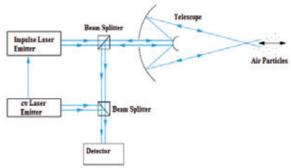


Figure 1 - Example of LDA setup schematic.

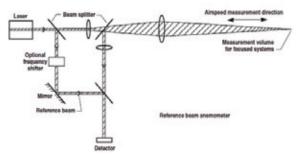


Figure 2 - Example of LDA setup schematic

The Doppler effect was observed for the first time by the physicist Christian Doppler. It consists in the followings:

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- A moving radiation source, a time variable signal source generates a signal of frequency f and wavelength  $\lambda$ ;
- A stationary observer will measure a frequency  $f_m$  for the signal emitted by the moving radiation source, different of f.

The measured signal frequency,  $f_m$  is defined as:  $v_m = v + \Delta v$  (1)

where  $\Delta v$  is the frequency shift measuring the magnitude of Doppler Effect.

In the case of a LDA, the radiation emitter and detector are in the same geometrical position, location. This means that frequency shift is doubled, as defined by the equation:

$$\Delta v = \frac{2v}{\lambda} \tag{2}$$

where V denotes the speed of flowing air particles [1-7].

The common approach for LDA devices consists in using a detection system based on converting light frequency changes into light intensity fluctuations using interference of light. The altered frequency of laser light scattered by particles of flowing air can be determined directly with frequency-dependent optical processing, i.e., by using a spectrometer or an optical filter. Practically, this means the use of an interferometer setup like the one presented in Figure 3. In Figure 3 a Mach-Zehnder interferometer setup is presented.

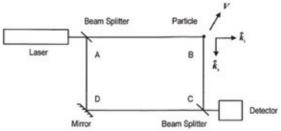


Figure 3 - Mach-Zehnder Laser Doppler Anemometer configuration

The starting point of this frequency shift measuring setup consists in considering the light scattered by a single particle denoted by the subscript m. A proportion of this light propagates around the path ABC to form the object arm of the interferometer and is received at the detector. If the wave-length and frequency of the illuminating laser are  $\lambda$  and  $\nu$ , respectively, the complex amplitude,  $U_m$ , incident on the detector due to this component can be written in the form [3-7]

$$U_{m} = A_{m} e^{\left[j2\pi\left(vt - \frac{x}{\lambda}\right)\right] - ti\theta}$$
(3)

where x is the path length and  $A_m$  is the amplitude of the scattered radiation at the detector.

If the particle is moving with constant velocity, *V*, for small excursions from this basic geometry the path length can be written in the form:

$$x = x_0 - n(\vec{K}_s - \vec{K}_i) \cdot \vec{V}t \tag{4}$$

where  $\vec{K}_s$  and  $\vec{K}_i$  are unit vectors in the directions of the illuminating and scattering directions respectively, n is the refractive index of the flowing medium, and  $x_0$  is a constant.

The detected signal complex amplitude is then:

$$U_{m} = A_{m}e^{\left[j2\pi\left(vt - \frac{n(\vec{K}_{s} - \vec{K}_{t})\vec{V}t}{\lambda}\right)\right] - \frac{x_{0}}{\lambda}}$$
 (5)

It is clear from this equation that an additional term is added to the frequency of the scattered light:

$$\Delta v = \frac{n(\vec{K}_s - \vec{K}_i) \cdot \vec{V}t}{\lambda} \tag{6}$$

This is the Doppler shift,  $\Delta v$ .

The reference beam propagates around the path ADC and the complex amplitude,  $U_R$ , of this beam at the detector can be written as:

$$U_{R} = A_{R} e^{\left[j2\pi\left(vt - \frac{x_{R}}{\lambda}\right)\right]}$$
 (7)

where  $A_R$  is the reference beam amplitude and  $x_R$  is the path length.

The signal output by the detector is proportional to the incident intensity, *I*, which can be written as:

$$I = A_m^2 A_R^2 + 2A_m A_R \cos(2\pi\Delta v t + \phi_m)$$
 (8)

where the phase constant,  $\varphi_m$ , is given by

$$\phi_m = \frac{2\pi(x_R - x_0)}{\lambda} \tag{9}$$

If the frequency of this signal is measured directly, a sign ambiguity in the Doppler frequency is observed. The ambiguity is most easily resolved by introducing a frequency shift,  $\Delta v_R$ , to the reference beam such that the detector output can be written as:

$$I = A_m^2 A_R^2 + 2A_m A_R \cos(2\pi(\Delta v - \Delta v_R)t + \phi_m)$$
 (10)

In this way, a carrier frequency is introduced and, provided that its magnitude is greater than the Doppler shift, it allows the particle velocity to be determined unambiguously.

All in-flight optical airflow velocity measurements use light scattering. Light is scattered on in-homogeneities in the optical refractive index of a medium from both air molecules and aerosols entrained in the air. The characteristics of molecular scattering and aerosol scattering are quite different.

There several observations to be briefly underlined regarding the air aerosol scattering issue. The main issue of this kind consists in the fact that elastic scattering processes in the atmosphere are attributed to different processes which can be classified into three categories:

### RAYLEIGH SCATTERING

It is a non-resonant scattering by air particles with dimensions much smaller than the wave-length of the incident light,  $\lambda$ . In air the  $N_2$  and  $O_2$  molecules play an important role in this kind of scattering. Its intensity varies as  $\lambda^{-4}$ .

For 1 =770 nm, the differential back-scattering cross section can be defined as:

$$\frac{d\sigma_R}{d\Omega} = 1.373 \times 10^{-32} \frac{m^2}{sr} \tag{11}$$

If molecular anisotropy is neglected, differential cross-section is related to the total cross-section  $\sigma R$  via phase function by

$$\frac{d\sigma_R}{d\Omega}\bigg|_{\theta} = \frac{\sigma_R^3}{4\pi^4} \Big( 1 + (\cos\theta)^2 \Big)$$
 (12)

#### MIE SCATTERING

It consists in scattering by aerosols with a size similar to the wavelength of the incident light ( $\lambda$ ). In air the  $N_2$  and  $O_2$  molecules play an important role in this kind of scattering. For larger particles, its intensity increases more slowly with decreasing wavelength (it varies as  $\lambda^{-1}$  to  $\lambda^{-2}$ ). The backscatter cross-section  $\sigma_{\text{Mie}}$  increases strongly with the particle size, but it is also dependent on shape and material. Information about the size distribution of atmospheric aerosol can be derived from the wavelength dependence of the backscatter coefficient. For very small particles (below ~100 nm) the Mie cross-section converges towards the Rayleigh value.

## RESONANCE SCATTERING

It can occur by resonant absorption and elastic reemission if the wavelength emitted coincides exactly with a transition line of an atmospheric constituent  $\rightarrow$ the cross-sections can be very large (15 or more orders of magnitude larger than  $\sigma_R$ ), they can be used to quantify the occurrence of trace gases as in the case of mesospheric metal atoms. In Figure 4, the Mie scattering - Characteristics for  $0.81\mu m$ wavelength incident laser radiation is presented.

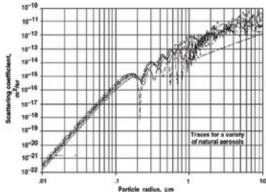


Figure 4 – Mie scattering characteristics for 0.81 mm wavelength incident laser radiation.

For short range applications (less than 150 meters) focusing is another technique used to define the measurement volume. By concentrating the optical energy in a small focal region, the vast majority of detected light is scattered from the aerosols within the measurement volume and propagates from this region back to the detector. Although not commonly applied, the optics used to concentrate scattered light on the detector can put an additional limit on the size of the measurement volume. Light is also scattered from aerosols located in regions of low energy density outside the focal region. When this light reaches the detector it adds small amounts of noise to the signal.

For focused systems, the measurement volume is delineated by the focal region of the light source, combined with phase effects of scattered light and reference-beam light for coherent systems (Vaughan and Forrester 1989).5 The focal region is defined as the volume bounded by the surface where optical energy density has decreased to 1/e2 of the maximum intensity level encountered at the center of the volume. For a laser beam in the fundamental transverse mode the intensity profiles can be approximated as a Gaussian distribution. The focal region is an ellipsoid with the long axis coaxial with the axis of the focusing lens. The beam waist diameter w0, for a light beam focused at distance r, with a lens or telescope with diameter D, is under ideal conditions:

$$w_0 = \frac{4\lambda r}{\pi D} \tag{13}$$

The measurement volume length  $\Delta L$  is at 50 percent of maximum intensity:

$$\Delta L = \frac{s \cdot \lambda \cdot r^2}{\pi \cdot D^2} \tag{14}$$

Optical principles dictate that for a fixed lens diameter, the focal region increases as the distance to the focal point increases. Note that ideal conditions are never encountered in practice. The focal region is larger when the laser is not in the fundamental transverse mode, when the Gaussian beam diameter does not match with the telescope diameter, and when the energy beam radiated is from a distributed light source (rather than from a point) or departs from a collimated condition.

# 3. Results and Discussion

The COMSOL Multiphysics program is used to simulate the propagation of laser beam inside a focusing volume. The variations of observed laser signal frequency induced by the scattering on the flowing air particles is considered.. The procedure was the following: we select **3D** as the **Space** 

Dimension, then in the list of Physical Models the following menu link COMSOL Multiphysics > PDE Modes > Classical PDEs > PDE, General Form is selected. We built the geometry of the measuring focusing volume. In the next steps we fixed the boundary settings, the mesh parameters (Figures 5 and 6) and compute the final solution, namely the variation of laser signal frequency with air speed.

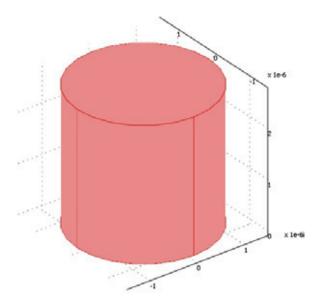


Figure 5 - Geometry of the LDA interaction focusing volume for  $\lambda = 1.06~\mu m$  incident wavelength.

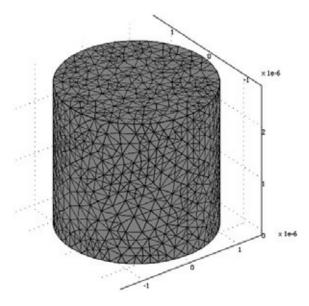


Figure 6 - Geometry of the LDA interaction focusing volume for  $\lambda=1.06~\mu m$  incident wavelength. The COMSOL mesh grid can be observed.

The parameters of the laser light – air particles interaction volume were defined using Eqs. (13) and (14). The case of using a solid state Nd:YAG laser was considered. Thais means an emission wavelength of 1.064 mm. The transverse laser intensity beam distribution was approximated as Gaussian with an waist of ~45 mm at a distance of 150 m from the laser emitter. The estimated values of  $w_0$  and  $\Delta L$  were of  $10^{-5}\, m$  as order of magnitude.

#### 4. Conclusions

In this paper we have demonstrated the versatility of COMSOL Multiphysics regarding the modeling and simulation of in-flight laser Doppler anemometer based on use of Doppler effect and Fabry - Perot etalon composed of Bragg grating reflectors.

The obtained COMSOL Multiphysics models are under development for fulfillment of aeronautic industry design needs. The considered development includes comparison with experimental results.

# 5. References

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