Dielectric Properties of Heterogenous Media with Inclusion of Fractal Geometry

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A numerically and experimentally validated analytical model to describe the electrostatic properties of dielectrics with fractal irregularities is developed based on recently proposed fractional-dimensional electromagnetic models. Fractality can be induced into regular integer-dimensional shapes using defined mathematical methods. The fractional dimension of the structures is given by Hausdorff's formula. We induced fractality at different levels in geometries including 3D cube/cuboid (for experiments and simulations) and 2D square/rectangle (for simulations). Full-wave simulations using COMSOL and experiments were employed to verify our analytical models.

Effect of Fractal Plate Distance

Cantor Plates geometry: divide the region between the electrode plates into alternating parallel layers of dielectric and air, with the thickness of each dielectric layer set to the3 length of the components of a Cantor set of certain removal factors (3, 4, 5, 6, and 7) at the 4th iteration.

Fractal Dimension: Fractal dimension (α) of plate distance (d) for a removal factor (r) using Hausdorff's formula: $\alpha = -\frac{\log(2)}{\log\left[\frac{1-1/r}{2}\right]}$

Theoretical scaling to be followed by simulation results of capacitance (C) for fractal dimension (α) and plate distance (d): $C \propto \frac{1}{d^{\alpha}}$

Results: the simulation results and practical experiments satisfied the theoretical scaling.

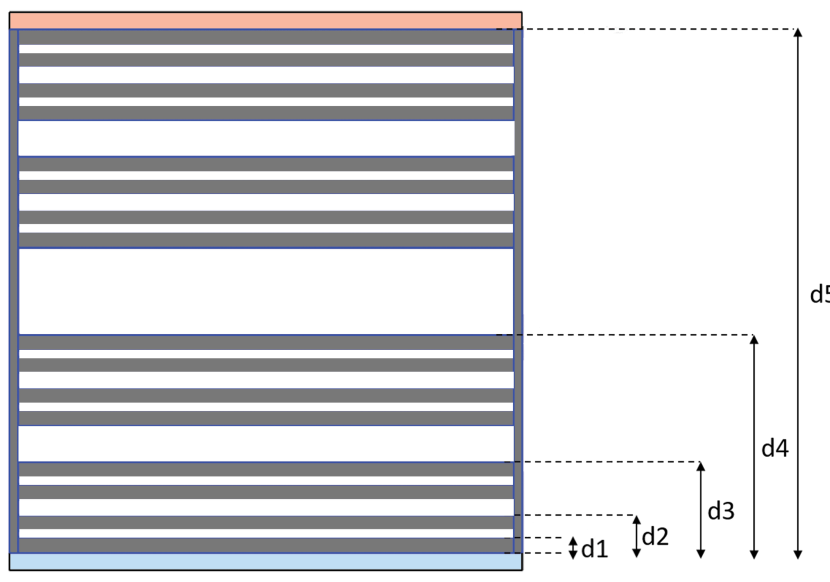

Fig 1: Discrete *d* values within 4th Cantor set iteration cube – varying plate distance in a *fractal* manner

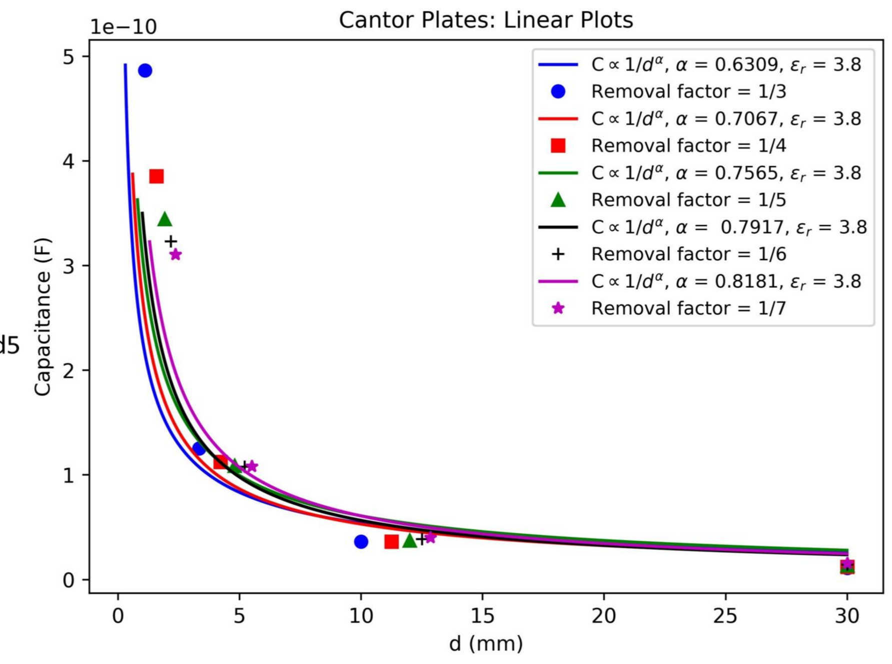

Fig 2: Simulation results for Cantor Plates of different removal factors

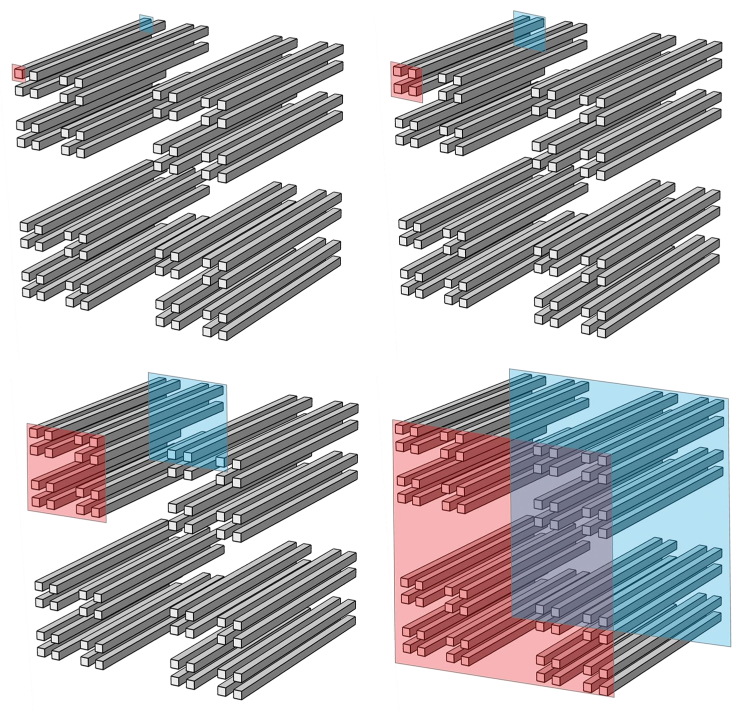

Fig 3: Varying plate area for Cantor Bars

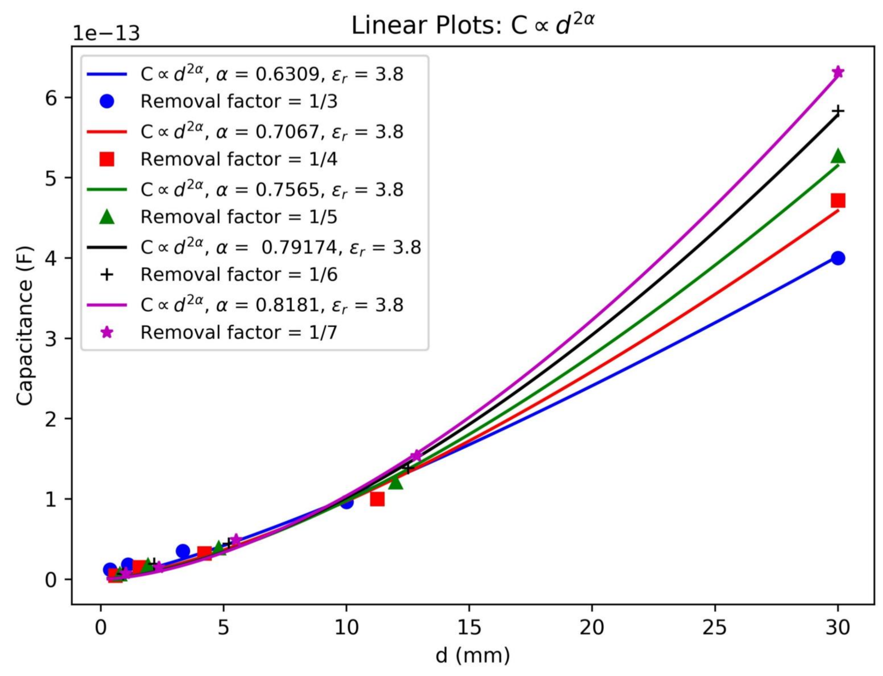

Fig 4: Simulation results for Cantor Bars of different removal factors

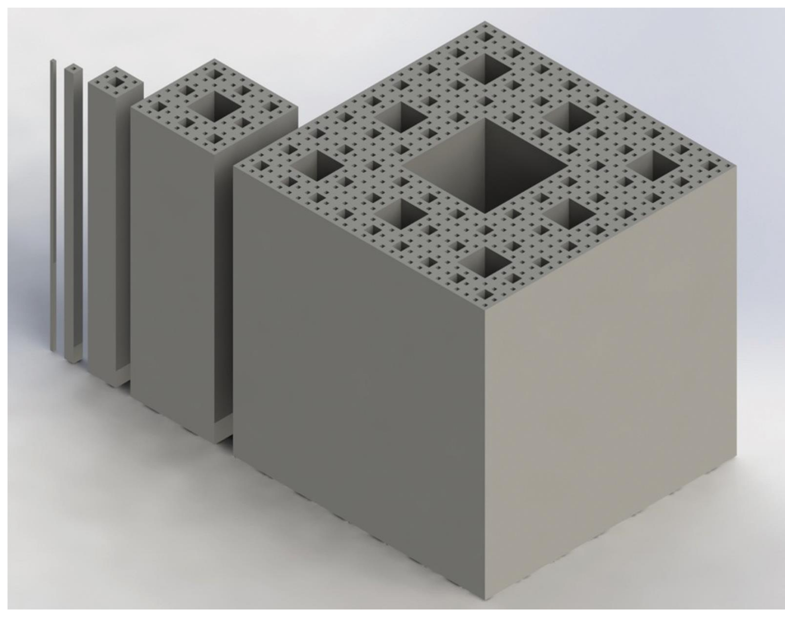

Fig 5: Varying plate area for Sierpinski Carpet

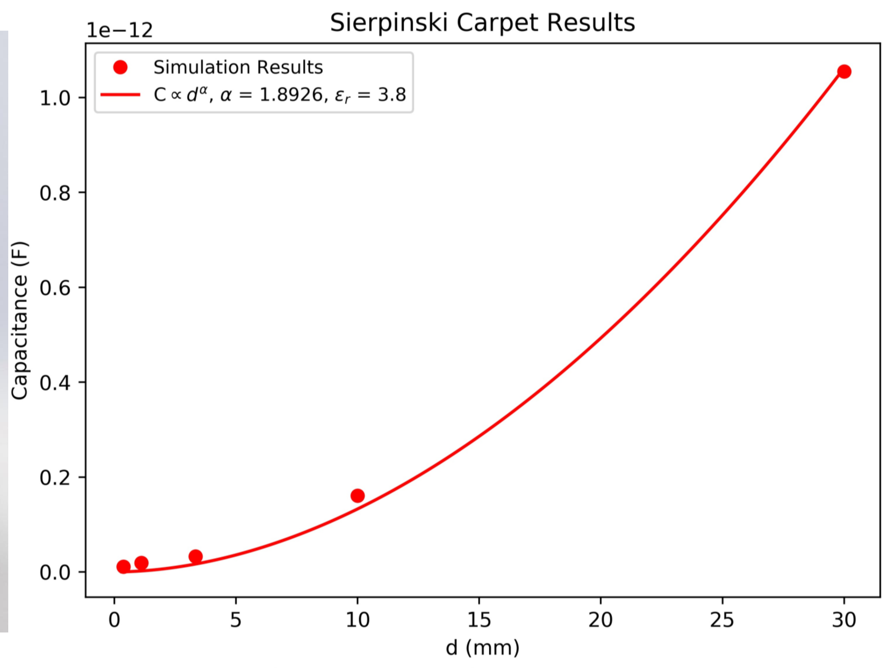

Fig 6: Simulation results for Sierpinski Carpet

Effect of Fractal Plate Area

(A) Cantor Bars geometry: create Cantor sets along two orthogonal directions to form fractal plate area (of a square outer shape) and extrude it along the third perpendicular direction to create non-fractal plate distance.

Fractal Dimension: The plate area is $d^{\alpha} \times d^{\alpha}$, where d is the side of the square, and α is the Hausdorff dimension for a certain removal factor. The fractal dimension of the plate area ($d^{2\alpha}$) is 2α.

Theoretical scaling to be followed by simulation results (C) for Hausdorff dimension for certain removal factor (α) and the distance between plates (d): $C \propto d^{2\alpha}$

(B) Sierpinski Carpet geometry: creating Sierpinski carpet as the plate area and extruding it, thus creating a fractal plate area and non-fractal plate distance.

Fractal Dimension: The fractal dimension for Sierpinski Carpet is a theoretical value determined by Hausdorff's formula.

Theoretical scaling to be followed by simulation results (C) for Hausdorff dimension (α_H) and the distance between plates (d): $C \propto d^{\alpha_H}$

Results: for both cases, simulation results satisfied the theoretical scaling.

Effect of Fractal Plate Distance and Area

(A) Menger Sponge Geometry: the dielectric had the geometry of Menger sponges with the highest iteration being 3. The plate electrodes also have a fractal area.

Fractal dimension: theoretically determined by Hausdorff's formula (α_H). The overall shape of the dielectric is cubic, the plate area has a fractal dimension of $(2/3)*\alpha_H$ and the plate distance has a fractal dimension of $\alpha_H/3$.

Theoretical scaling to be followed by simulation results (C) for Hausdorff dimension (α_H) and the distance between plates (d): $C \propto d^{\alpha_H}$

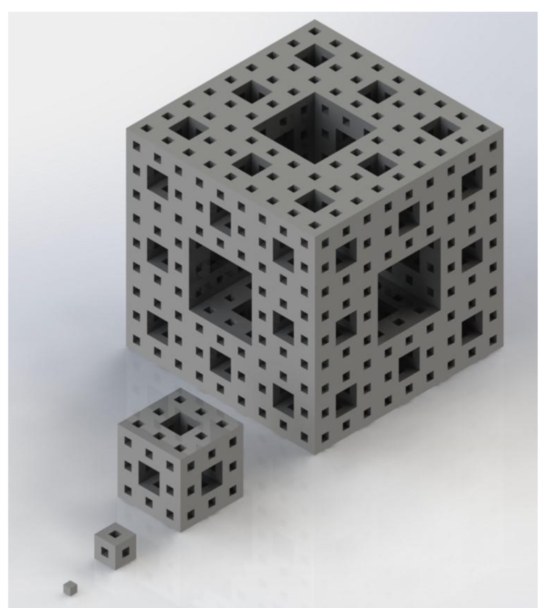

Fig 7: Varying parameters for Menger Sponge

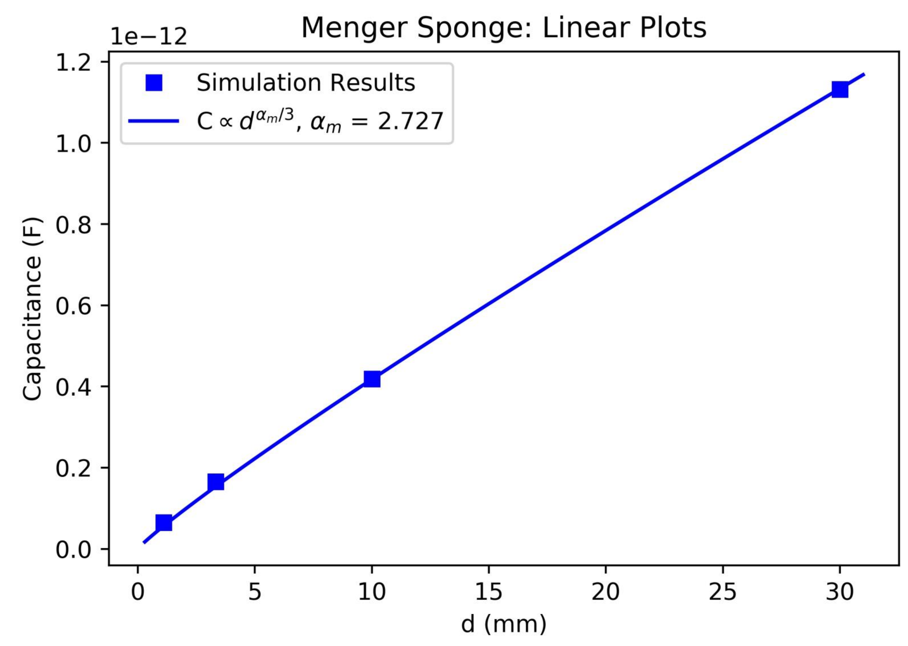

Fig 8: Simulation results for Menger Sponge

(B) Cantor Dust Geometry: create Cantor sets along all 3 orthogonal directions with an overall cubic structure. Thus, both plate area and plate distance are fractal.

Fractal dimension: overall structure is cubic, the plate area is $d^{2\alpha}$ while the plate distance is d^{α}. Hence the overall dimension ($d^{2\alpha} / d^{\alpha} = d^{\alpha}$) is α, where α is the Hausdorff dimension for a certain removal factor.

Theoretical scaling to be followed by simulation results (C) for Hausdorff dimension (α) and side of cube (d): $C \propto d^{\alpha}$

Results: for both cases, the simulation results satisfied the theoretical scaling.

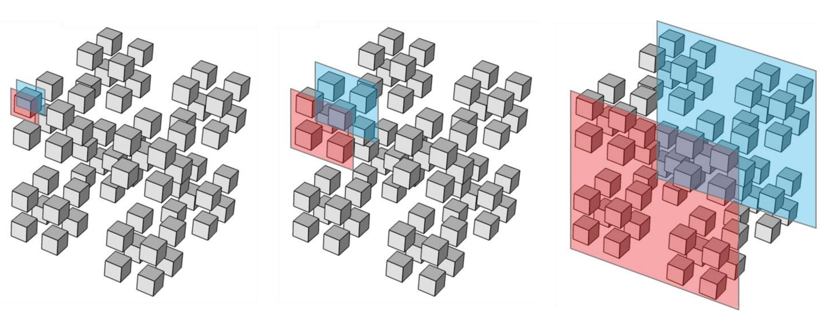

Fig 9: Varying parameters for Cantor Dust

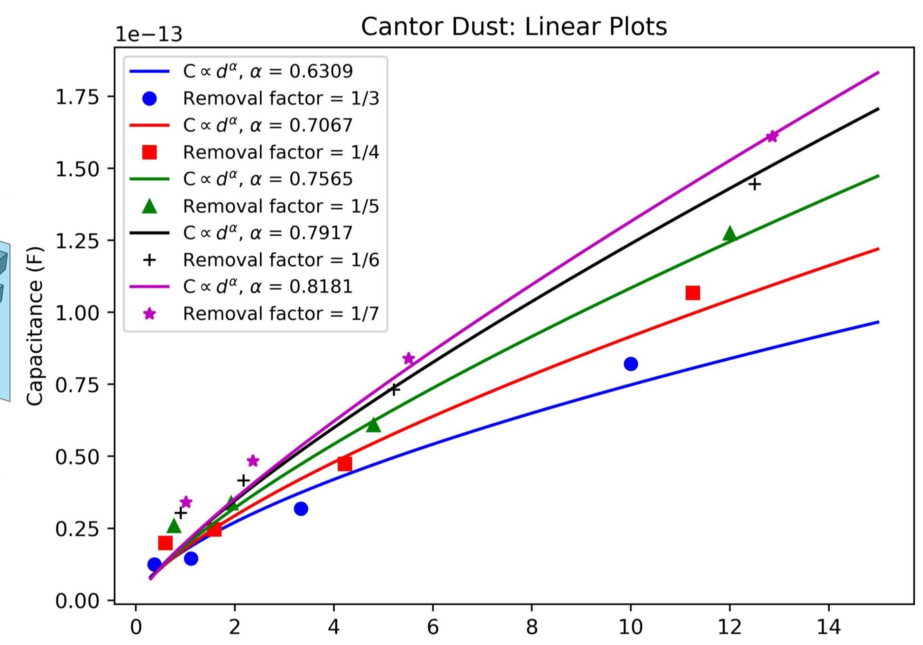

Fig 10: Simulation results for Cantor Dust

References

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