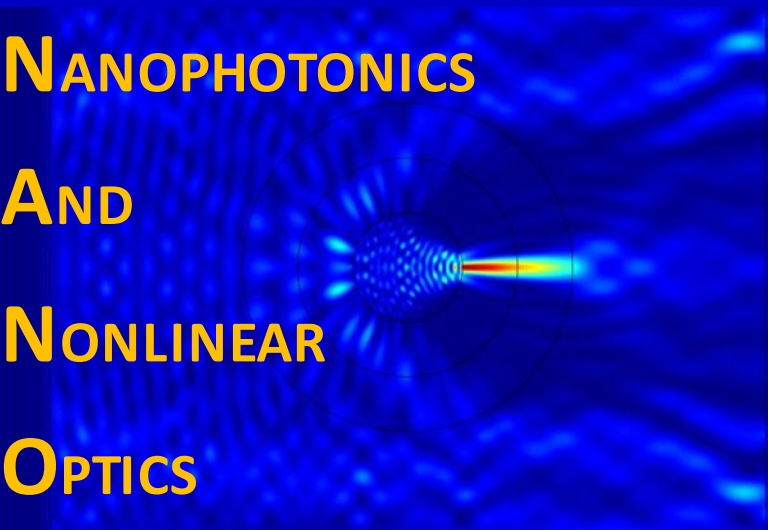


**NANOPHOTONICS**

**AND**

**NONLINEAR**

**OPTICS**



**Presented at the 2011  
COMSOL Conference in Boston**

**Simulation of Field  
Enhancement in Anisotropic  
Transition Metamaterials  
using COMSOL**

**Apra Pandey and Natalia M. Litchinitser**  
**University at Buffalo, The State University of New York**



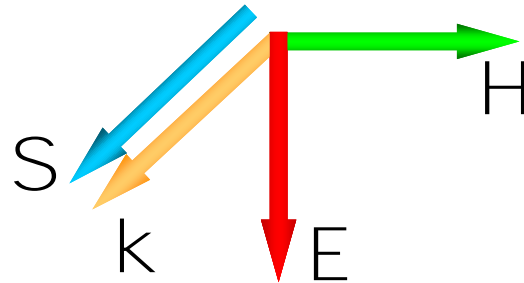
# Left-Handed Materials or Negative Index Materials

Right-Handed or Positive Index Material (PIM) if

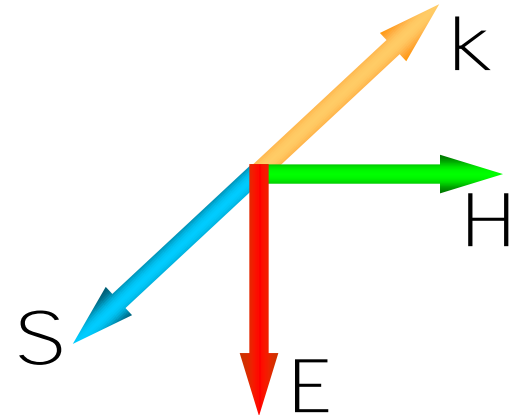
Left-Handed or Negative Index Material (NIM) if

$$\left. \begin{aligned} [kE] &= \frac{\omega}{c} \mu H \\ [kH] &= -\frac{\omega}{c} \epsilon E \end{aligned} \right\}$$

$$\epsilon > 0, \mu > 0$$

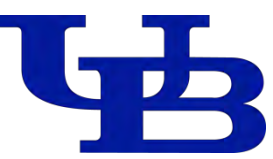


$$\epsilon < 0, \mu < 0$$



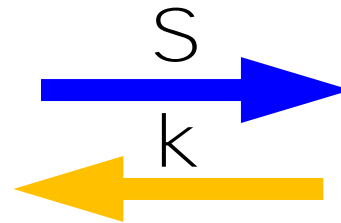
$$S = \frac{c}{4\pi} [EH]$$

Poynting vector



# Negative Index Materials: definition

Materials with antiparallel  $k$  and  $S$



Can be realized in

Double negative  
metamaterials

$$\epsilon < 0, \mu < 0$$

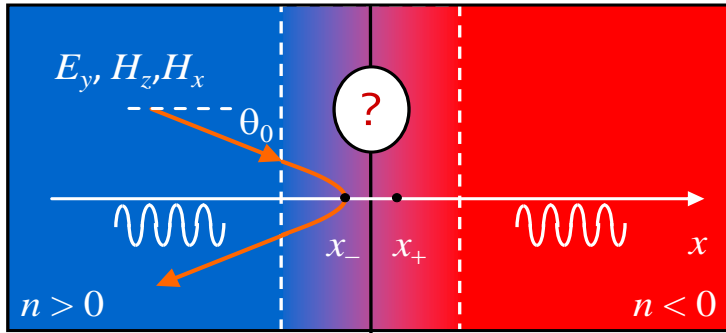
V. Veselago, Sov. Phys. Usp.  
10, 509(1968)

Anisotropic hyperbolic  
metamaterial waveguides

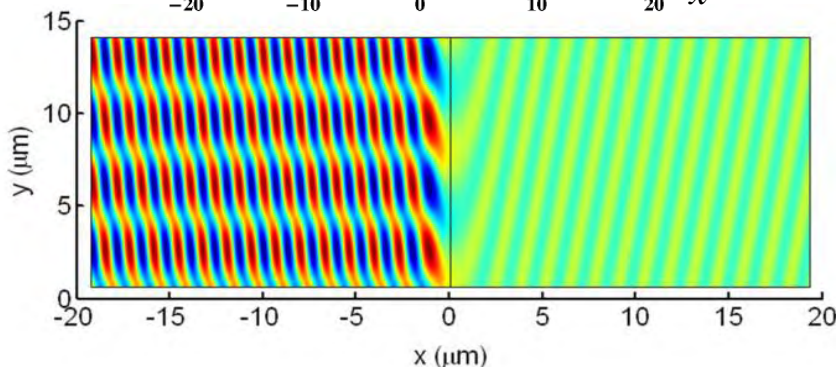
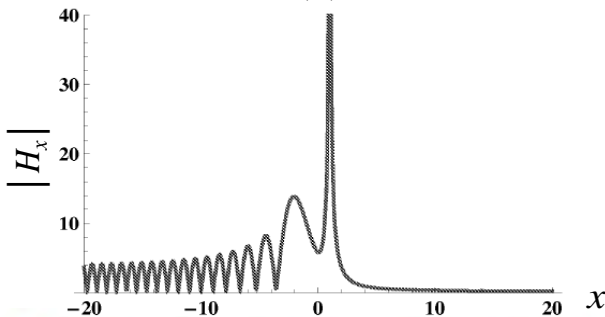
$$\epsilon_x > 0, \epsilon_y < 0$$

D. R. Smith, D. Schurig, PRL 90, 077405 (2003)  
R. Wangberg, J. Elser, E. E. Narimanov, V. A.  
Podolskiy, JOSA B 23, 498 (2006)

# Transition Metamaterials



$$\text{Re}(n) = 0$$



Transition metamaterial is a graded-index metamaterial with refractive index varying from positive to negative values (or vice versa)

- Resonant electromagnetic field enhancement and resonant absorption at oblique incidence in the vicinity of the point where  $\epsilon$  and  $\mu$  (or  $n$ ) change signs
- Potential applications: low-intensity nonlinear optics, polarization sensitive devices, wave concentrators

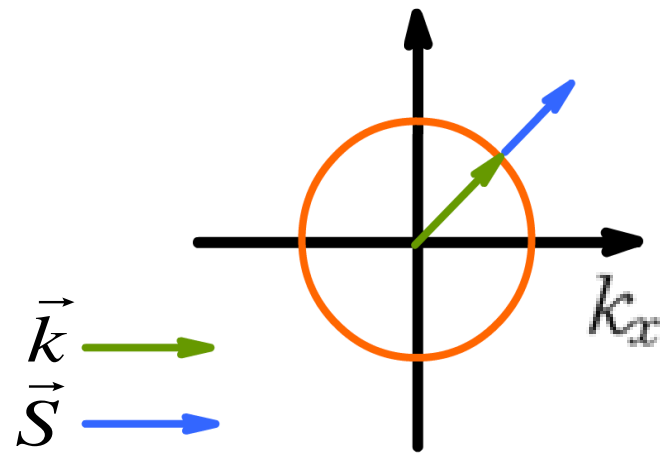
Spatial distribution of Electric field.



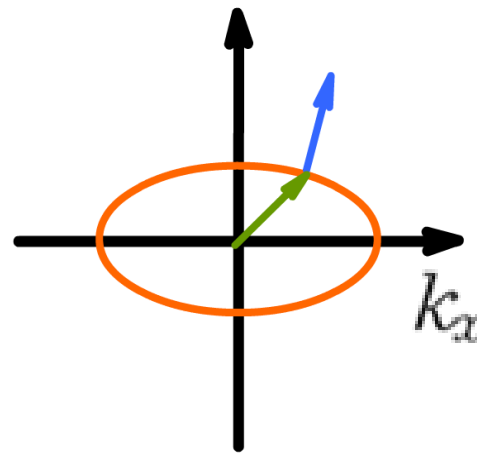
# Motivation

- Previous studies focused on double-negative metamaterials, i.e. both dielectric permittivity  $\epsilon$  and magnetic permeability  $\mu$  were assumed to change sign
- However, designing  $\mu$  is possible but challenging
- Our goal is to investigate propagation of electromagnetic waves in anisotropic graded-index metamaterials with  $\mu = 1$

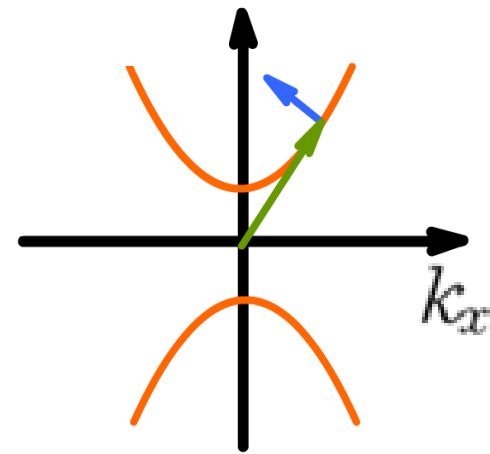
# Anisotropic Materials



Isotropic material  
 $\epsilon_x = \epsilon_y$



Anisotropic material  
 with  $\epsilon_x, \epsilon_y > 0$   
 $\epsilon_x \neq \epsilon_y$



Anisotropic material  
 with  $\epsilon_x > 0, \epsilon_y < 0$

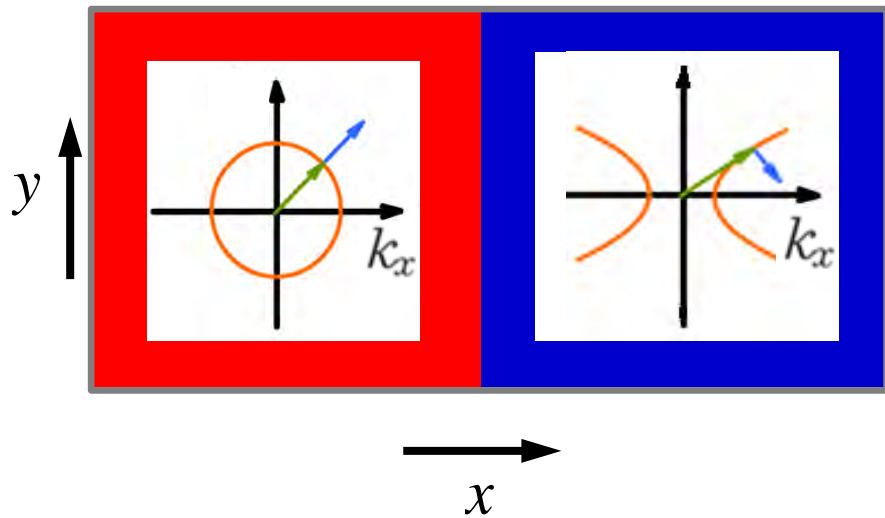
$$\frac{k_x^2}{\epsilon_y} + \frac{k_y^2}{\epsilon_x} = \frac{\omega^2}{c^2}$$



# Isotropic-Anisotropic Interface

Case 1: When  $\varepsilon_x$  changes sign

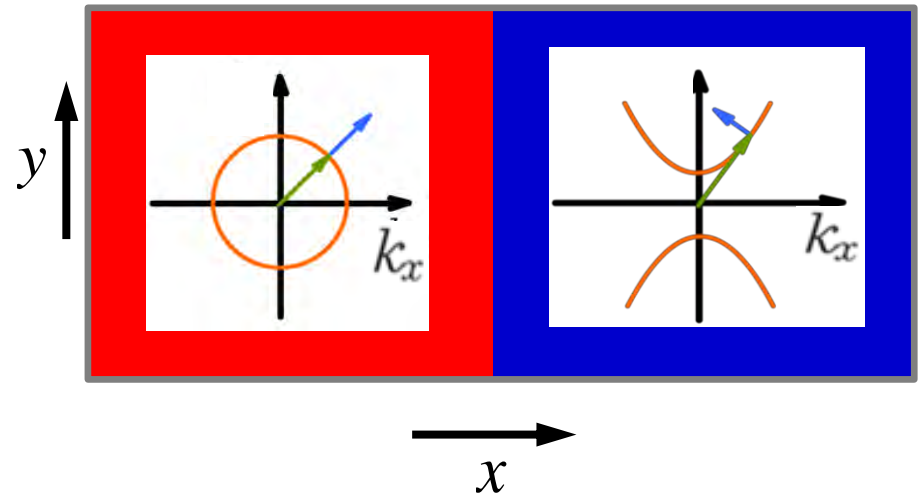
Case 2: When  $\varepsilon_y$  changes sign



$$\varepsilon_x = \varepsilon_y > 0$$

$$\varepsilon_x < 0$$

$$\varepsilon_y > 0$$



$$\varepsilon_x = \varepsilon_y > 0$$

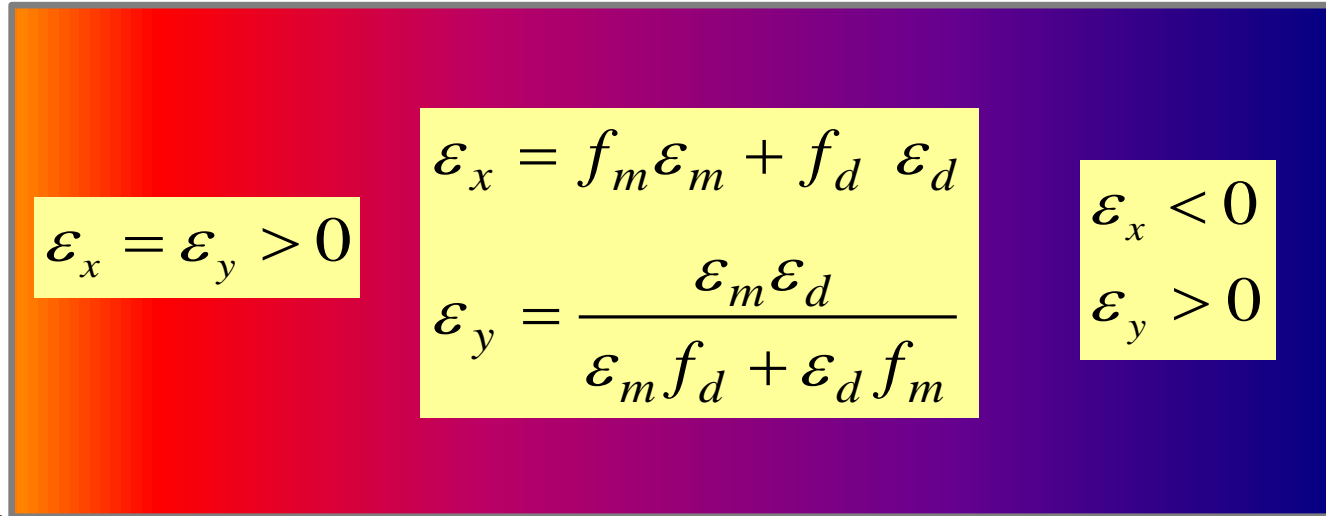
$$\varepsilon_x > 0$$

$$\varepsilon_y < 0$$



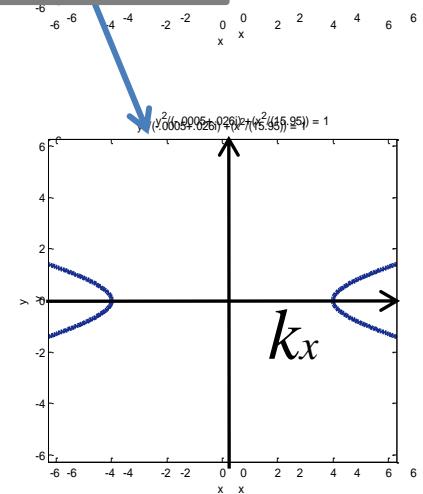
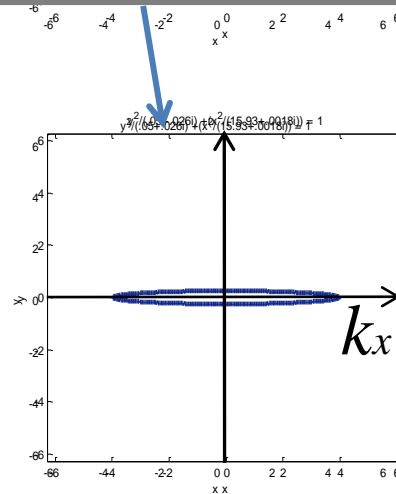
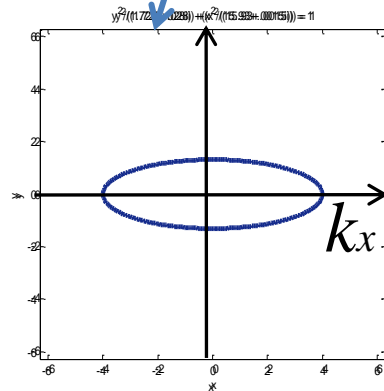
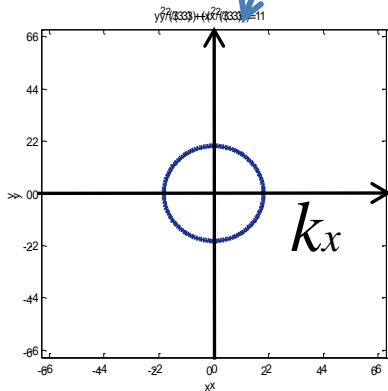
# Evolution of Dispersion Relation

Case 1:  $\epsilon_x$  is graded

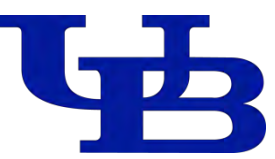


$y$

$x$

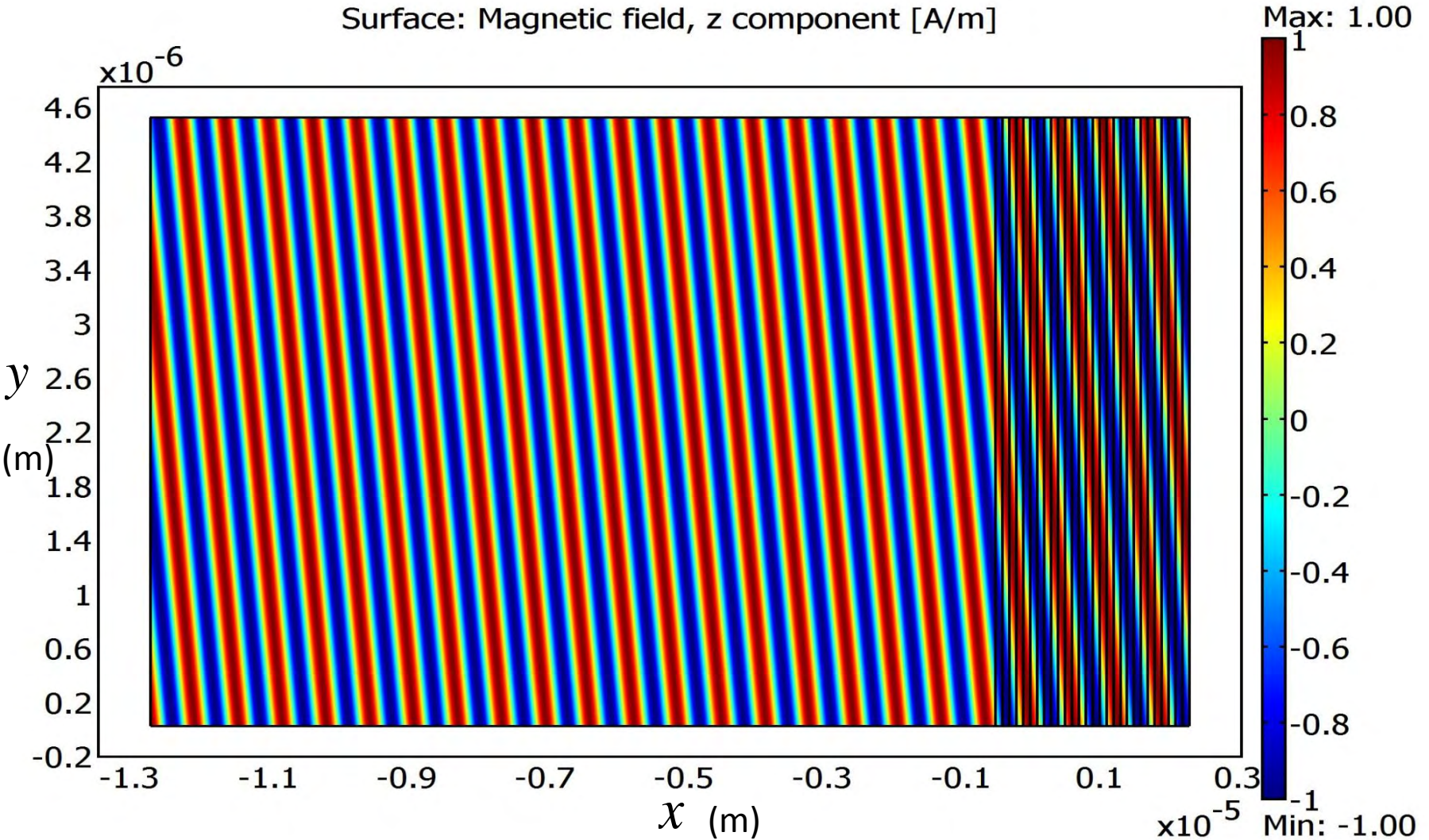






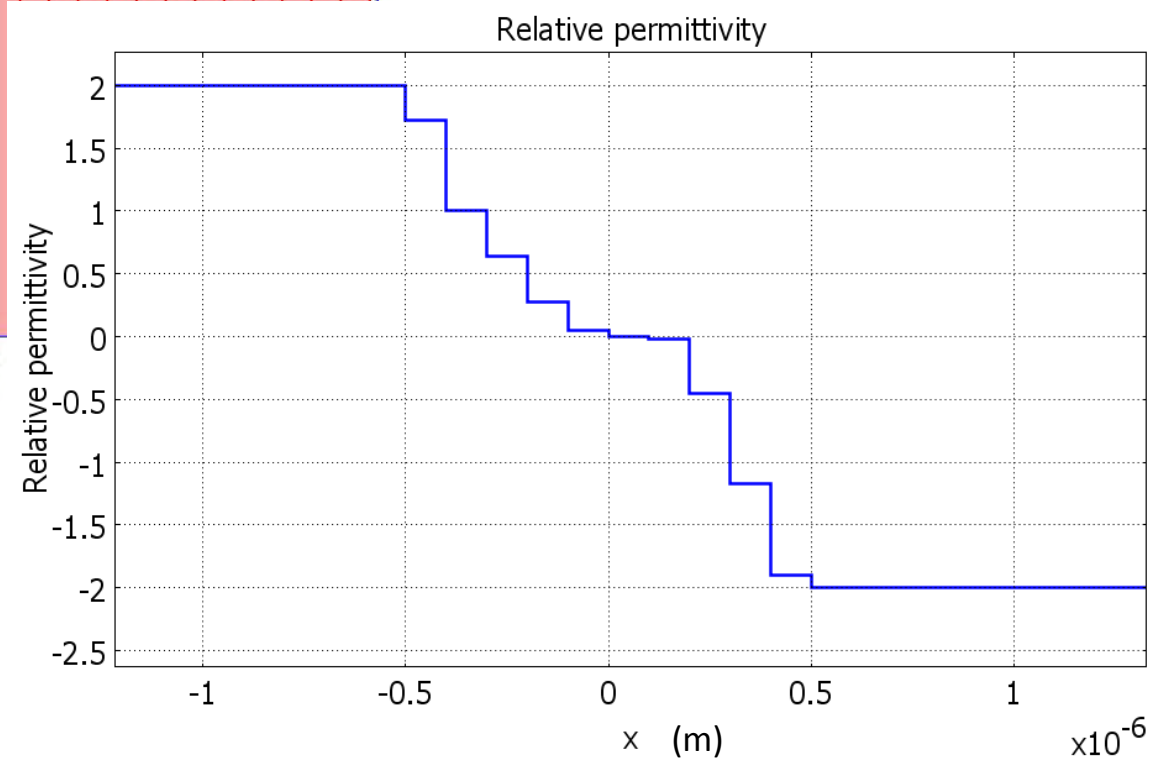
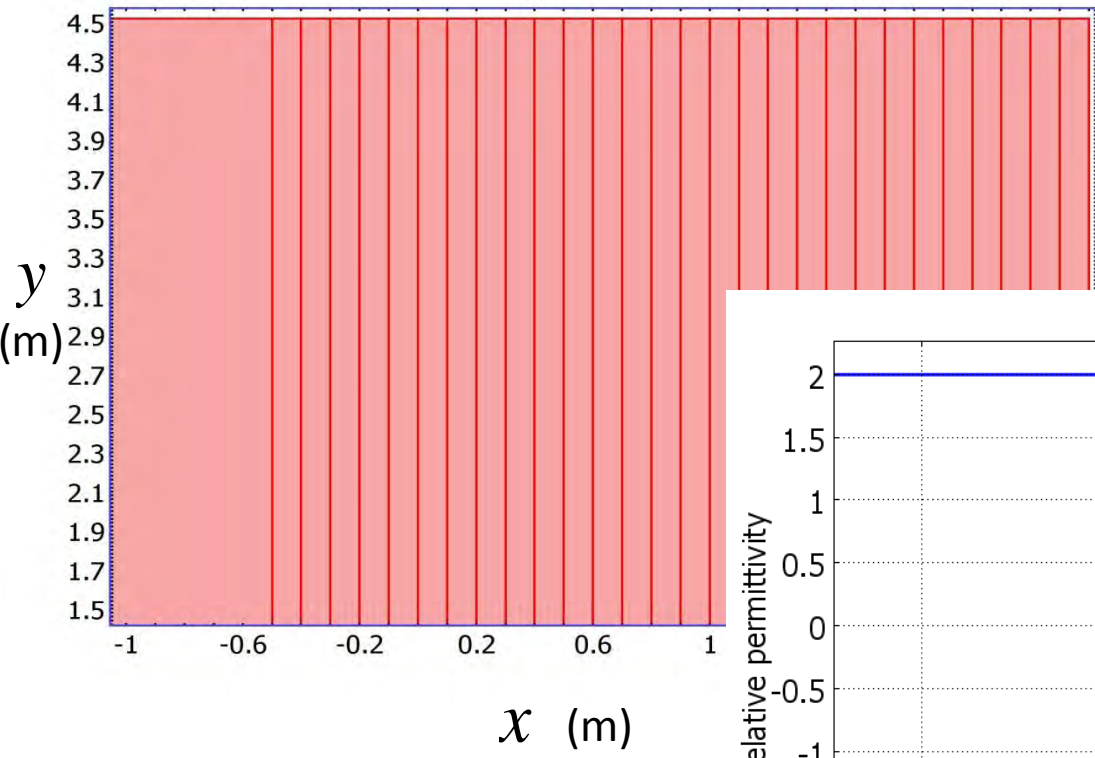
# Oblique incidence in isotropic dielectric medium

Surface: Magnetic field, z component [A/m]





# Design of Hyperbolic Transition Metamaterial

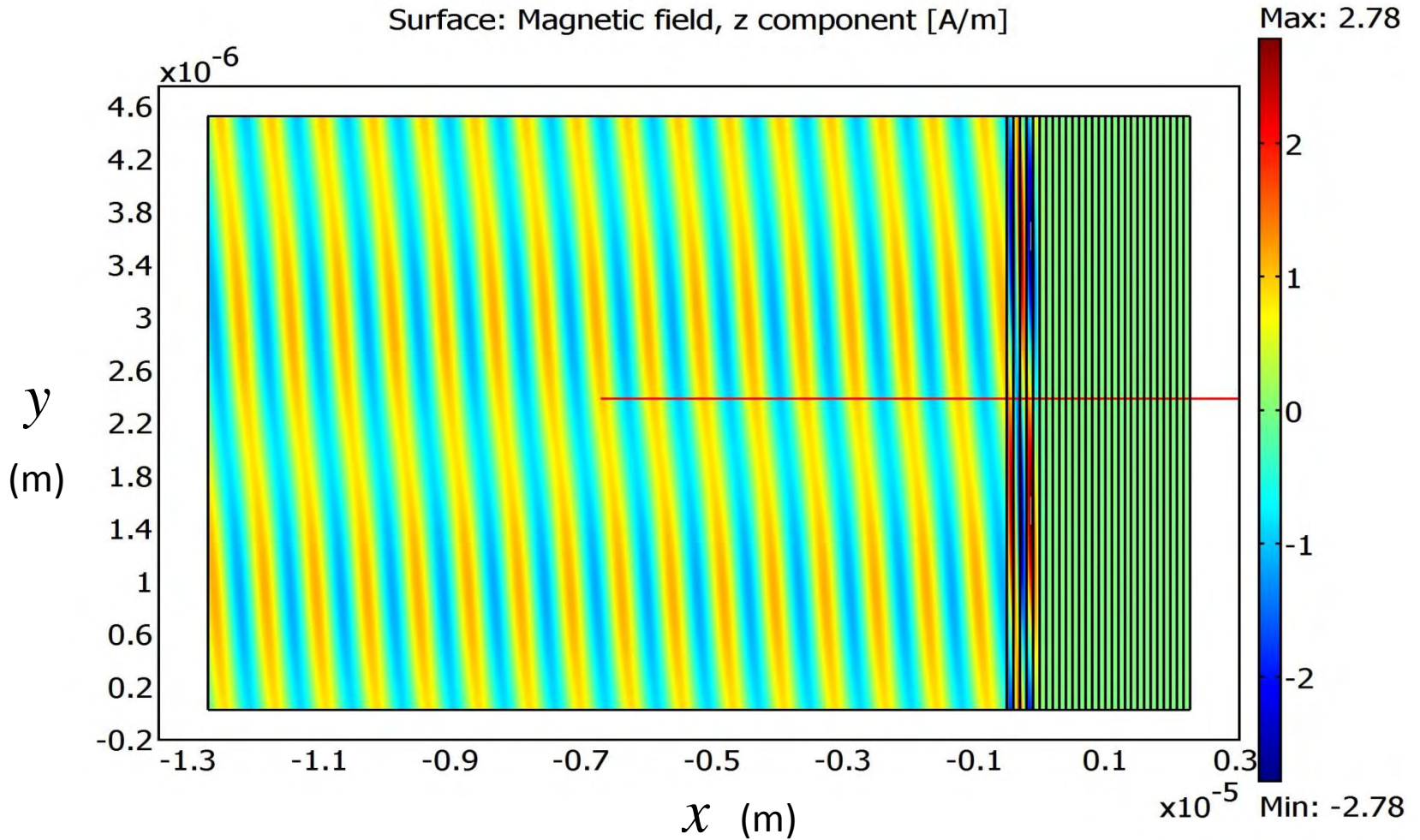




# Oblique incidence

$$H_z$$

Surface: Magnetic field, z component [A/m]

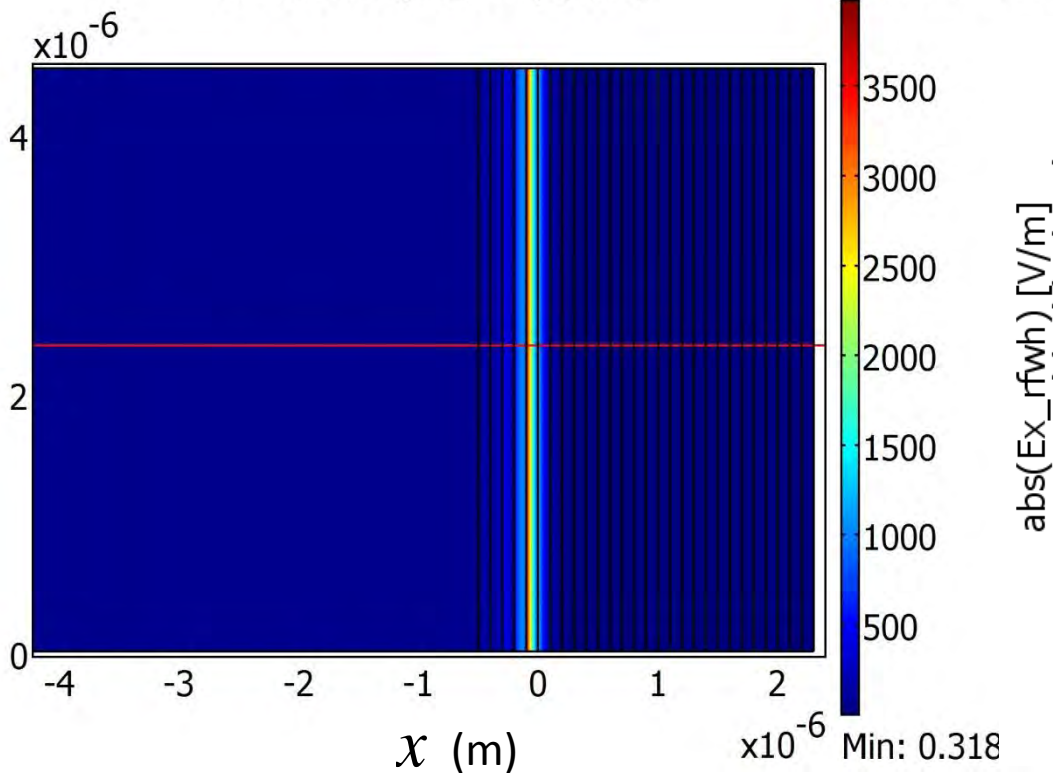




# Enhancement in case of Oblique Incidence

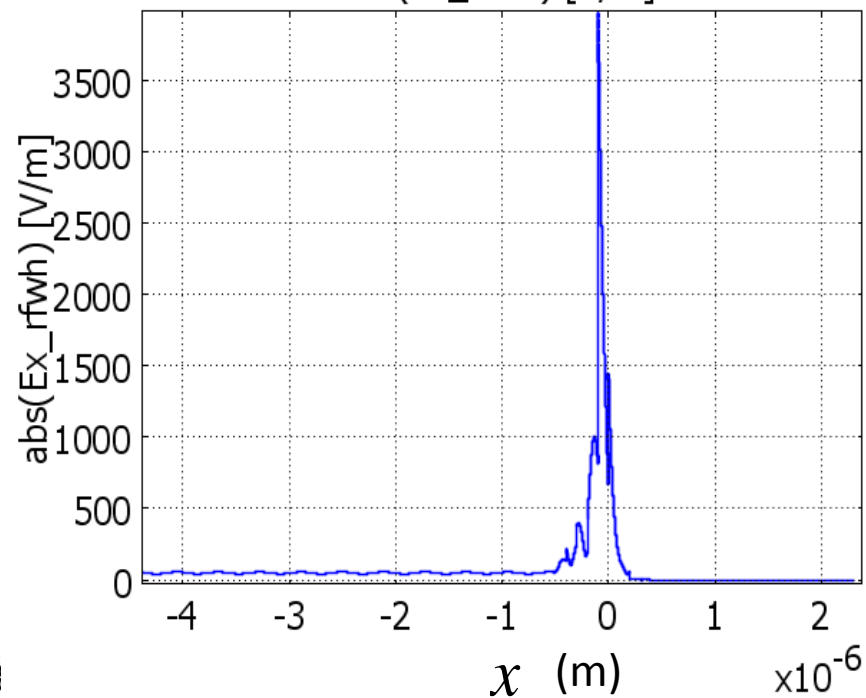
$$|E_x|$$

Surface: abs(Ex\_rfwh) [V/m]



$$|E_x|$$

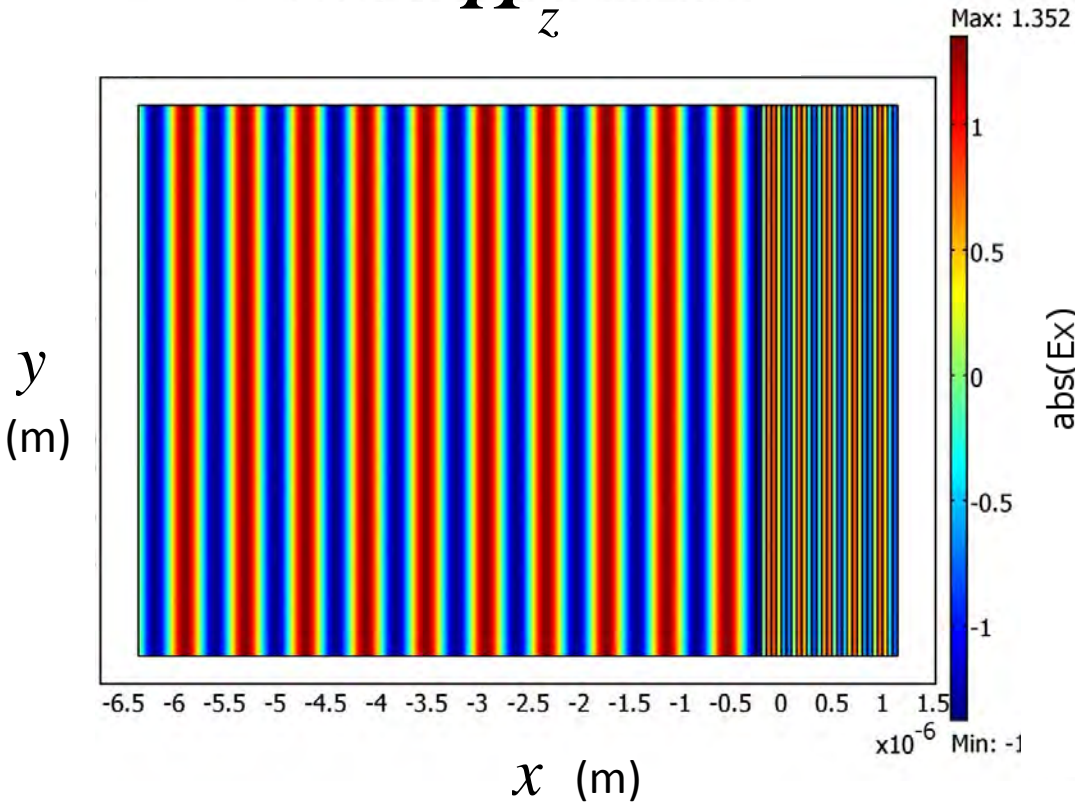
abs(Ex\_rfwh) [V/m]



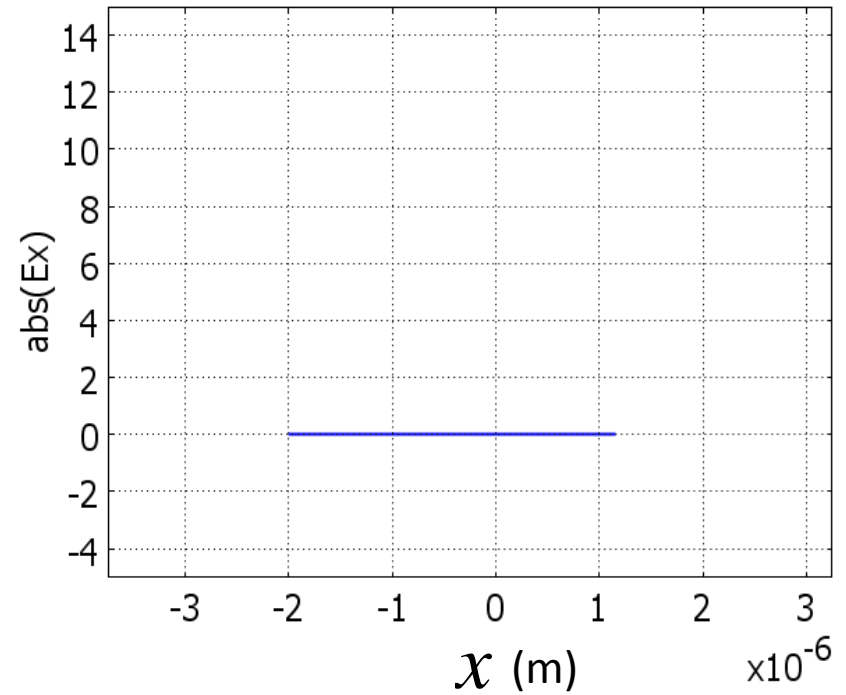


# Normal Incidence

$$H_z$$



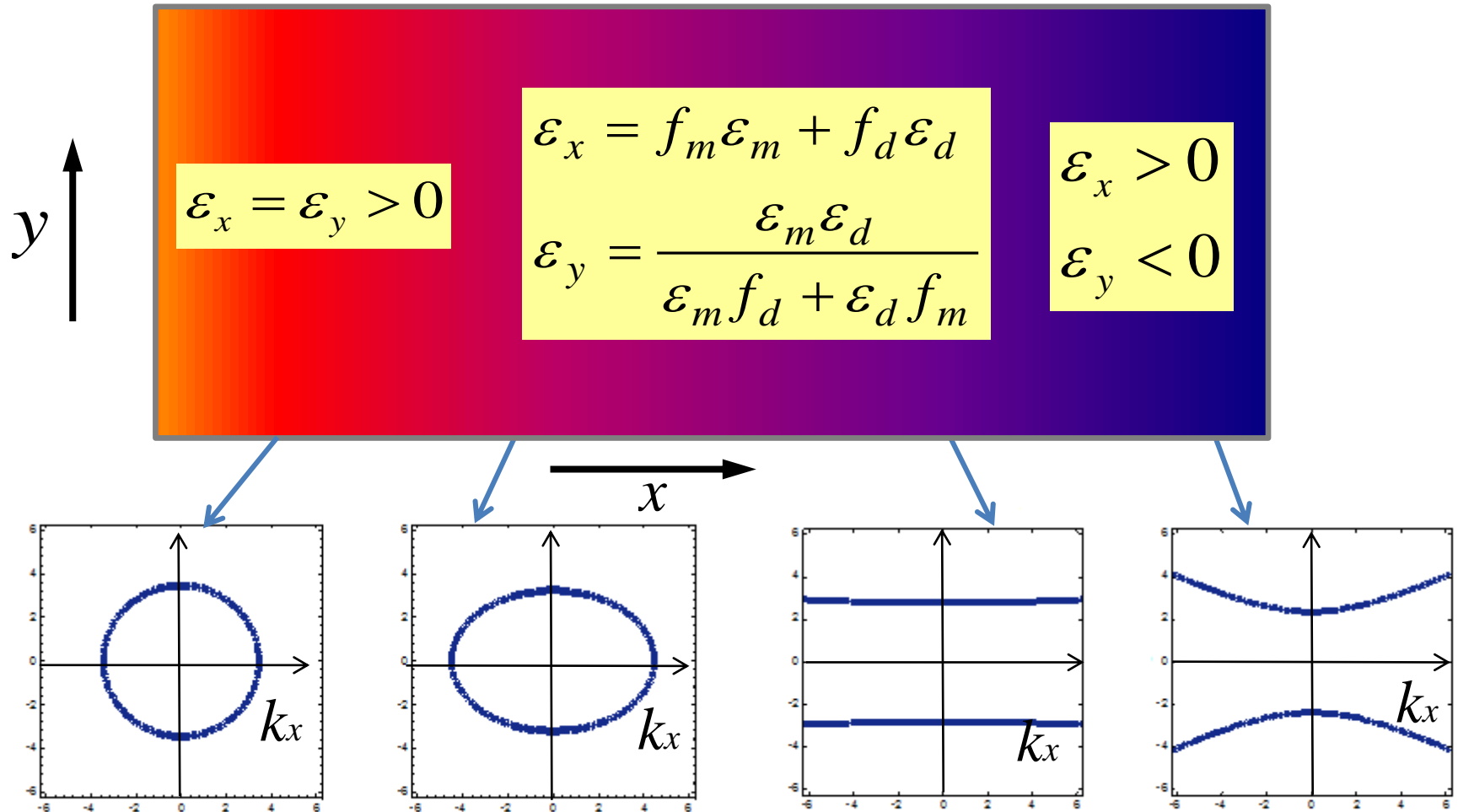
$$|E_x|$$





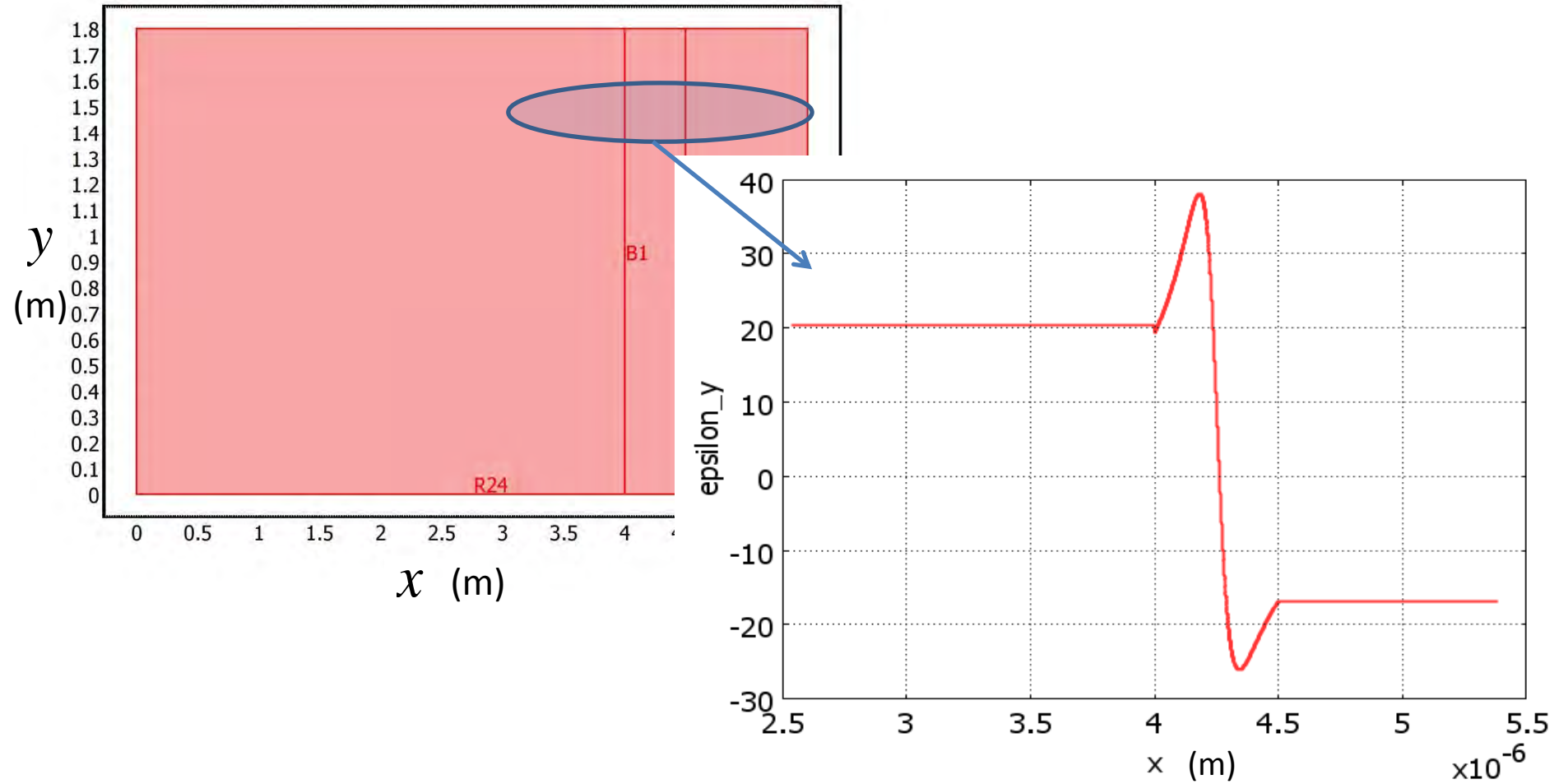
# Evolution of Dispersion Relations

Case 2:  $\epsilon_y$  is graded





# Design

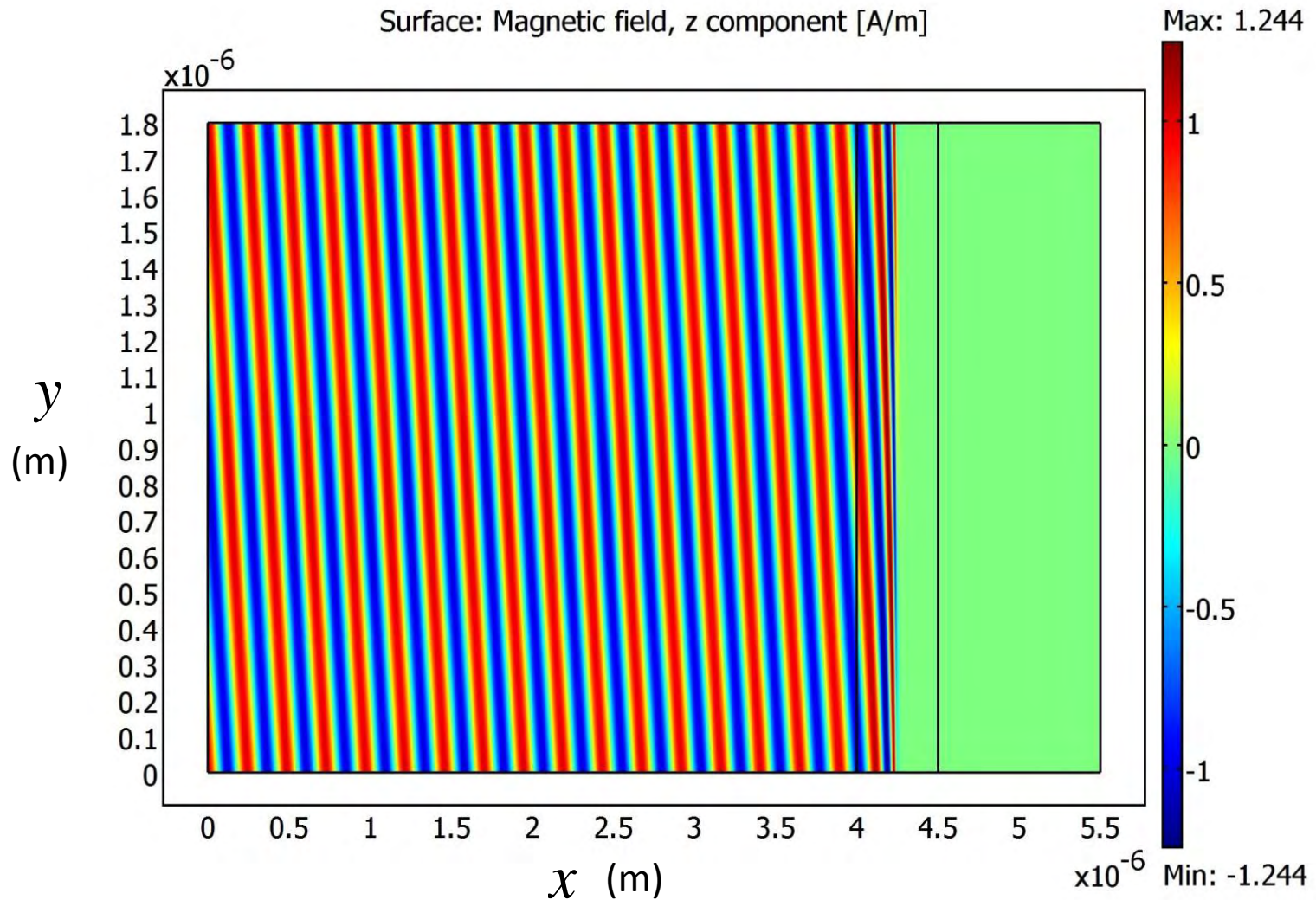


Real part of  $\epsilon_y$  permittivity along x-direction

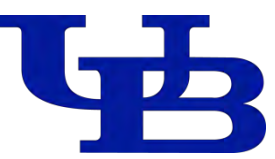


# Oblique Incidence

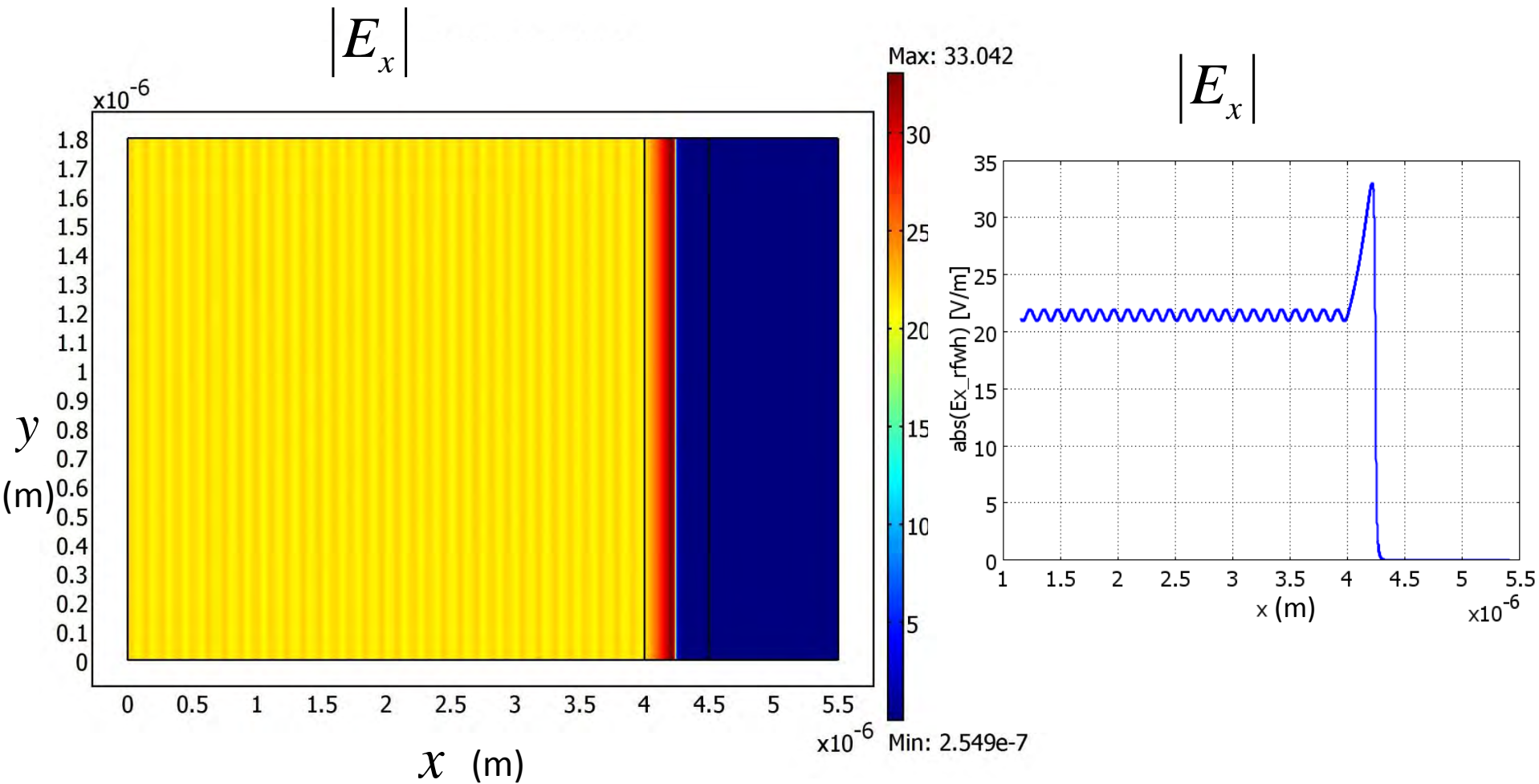
$$H_z$$







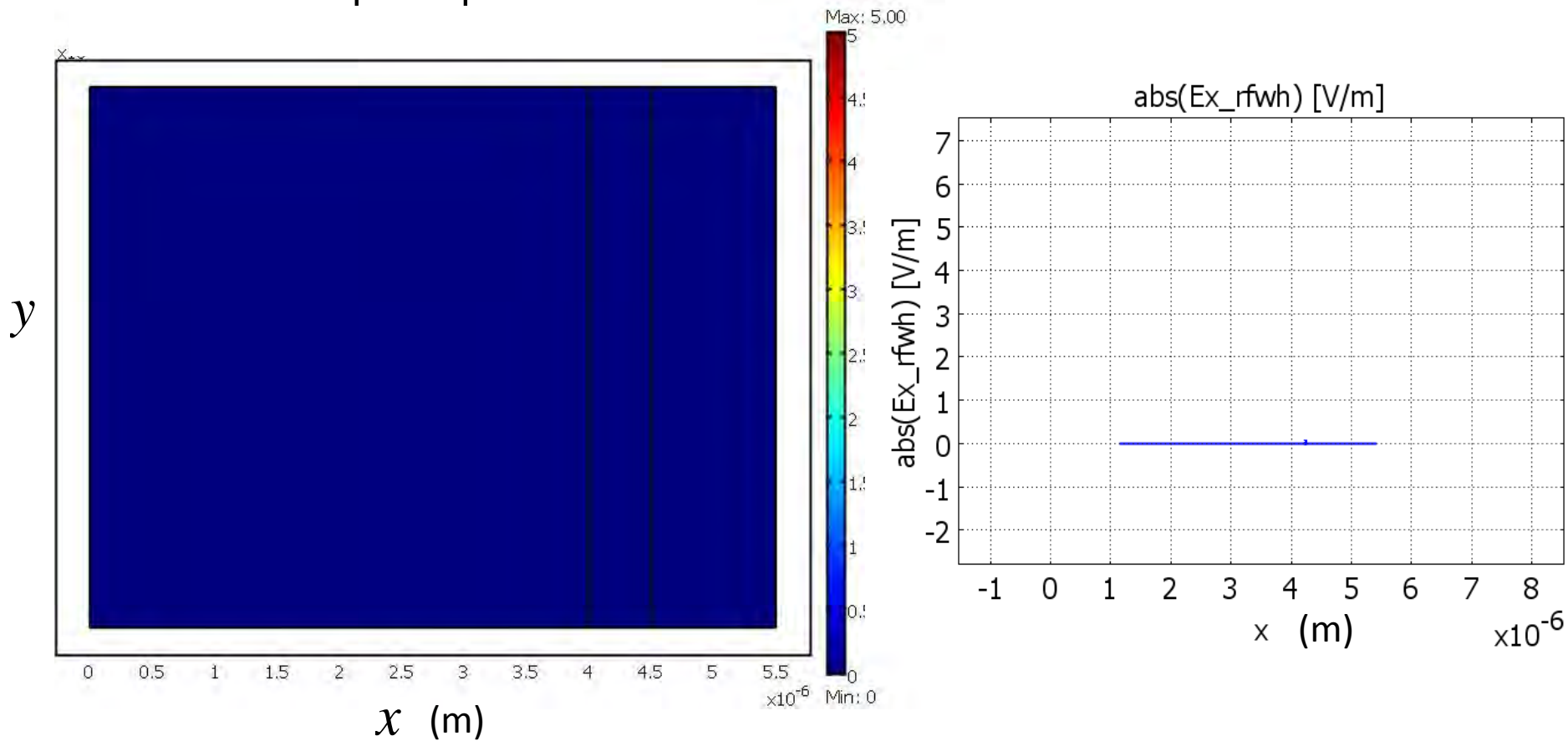
# Enhancement in Case of Oblique Incidence

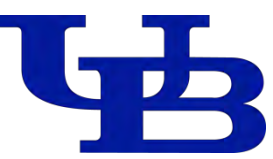




# Normal Incidence

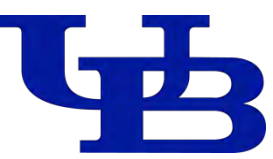
$$|E_x|$$





# Conclusion

- We investigated EM wave propagation in graded anisotropic metamaterial structures.
- Dispersion relation transitions from elliptical to hyperbolic
- We demonstrate enhancement of  $x$ -component of electric field in anisotropic transition region in case of oblique TM wave



# Acknowledgements

- I would like to thank Gayatri Venugopal for helpful discussions.
- This work was supported by US Army Research Office Award # W911NF0910075

**THANK YOU!**